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THE GLASGOW TEXT BOOKS OF CIVIL ENGINEERING. Edited by G. MONCUR, B.Sc. M.Inst.C.E., M.Am.Soc.C.E. *Professor of Civil Engineering in the Royal Technical College, Glasgow.*

PLANE AND GEODETIC SURVEYING FOR ENGINEERS

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THE GLASGOW TEXT BOOKS
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PLANE AND GEODETIC SURVEYING

FOR ENGINEERS

VOL I

PLANE SURVEYING

BY

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PREFACE TO THE FIRST EDITION

THIS textbook is designed to form a complete treatise on plane surveying with such parts of geodetic work as are of interest to the civil engineer. The author would emphasise at the outset that he does not claim that a knowledge of geodesy is a very essential part of the equipment of the engineer The execution of surveys of such extent and character as to necessitate the general methods of geodetic surveying and levelling does, however, occasionally fall within his province For this reason, a knowledge of its principles is required in the examinations of the Institution of Civil Engineers and of universities and colleges

In a general text on surveying there is little room for originality, except in treatment. Although this work is intended to serve as a reference book for practising engineers and surveyors, the chief aim has been to cover ground suitable for a degree course, and, while it is hoped that the book will prove of value to those pursuing a college course, the needs of the self-taught student have been specially kept in view. In consequence, many explanatory notes and practical hints have been inserted, particularly with reference to the more common surveying operations The latter are not meant to take the place of practice in the field, which, needless to say, is an essential part of a training in surveying, but are intended as a guide to the reader with limited opportunities for field practice, and are mainly suggested by the author's experience of the initial mistakes of young engineers in practice and of students undergoing field training in camp

The subject matter is presented in two volumes The first covers in ten chapters the more common surveying operations of the engineer, and the second deals with astronomical and geodetic work and the methods employed in large surveys generally

In the arrangement of the present volume it has been thought desirable, for convenience of reference, to group descriptions of the more commonly used instruments and their adjustments to form Chapter I. It is hoped that the detailed method of treating the subject of adjustments will afford a sound understanding of the geometrical principles in each case. In Chapters II to VI the subjects of Chain Surveying, Theodolite and Compass Traversing, Ordinary Levelling, Plane Table Surveying, and Contouring are described as applied to cadastral and engineering surveys. Chapter VII deals with the office work of computing areas and volumes, the latter with particular reference to the measurement of earthwork quantities. The practice of setting out works is treated in Chapter VIII. The setting out of railways is the only branch of this subject meriting detailed description, and problems in connection with the setting out of simple, compound, reverse, and transition curves are treated. The principles and practice of Tacheometry are given in Chapter IX, and Hydrographical Surveying, including Marine Surveying and Stream Measurement, is dealt with in Chapter X. Owing to the number of texts available on mine surveying, no special reference has been made to that subject

Sets of illustrative numerical examples and answers are given for practice For permission to reproduce questions set by the Institution of Civil Engineers, the University of London, and the Royal Technical College, Glasgow, the author desires to express his thanks to the authorities concerned

Lists of references have been inserted, after the appropriate chapters, on such subjects as readers might wish to pursue further. The author would gratefully acknowledge the assistance he has derived from these and other books and papers on Surveying

D C.

PREFACE TO THE SECOND EDITION

In the preparation of this edition it has been found desirable to enlarge the text and to re-write a considerable part of the book. The number of illustrations has also been increased The eight years which have elapsed since the issue of the First Edition have witnessed extensive developments in the design and manufacture of surveying instruments, and an endeavour has been made to bring the book up to date in respect of these. For permission to reproduce questions set by the Institution of Civil Engineers, the University of London, the Royal Technical College, Glasgow, and Trinity College, Dublin, the author desires again to express his indebtedness. He would also express his thanks to Messrs. C. F. Casella and Co , Ltd., Cooke, Troughton and Simms, Ltd., George Russell and Co., Ltd., E. R. Watts and Son, Ltd., Henry Wild Surveying Instruments Supply Co. Ltd., and Carl Zeiss for placing at his disposal information regarding their recent instruments. He gratefully acknowledges his further indebtedness to Messrs. Cooke, Troughton and Simms for permission to reproduce Figs. 71, 72, and 301, to Messrs. C. F. Casella and Co. and Henry Wild Co. for Fig. 69, and to Messrs. Carl Zeiss for Fig. 102.

D. C.

Trinity College,
Dublin, 1931.

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PLANE AND GEODETIC SURVEYING FOR ENGINEERS

PLANE SURVEYING

INTRODUCTION

SURVEYING is the art of making such measurements of the relative positions of points on the surface of the earth that, on drawing them down to scale, natural and artificial features may be exhibited in their correct horizontal or vertical relationship

Less comprehensively, the term, "surveying," may be limited to operations directed to the representation of ground features in plan Methods whereby relative altitudes are ascertained are distinguished as "levelling," the results being shown either in vertical section or conventionally in plan

Plane and Geodetic Surveying.—A plan is a projection upon a horizontal plane, and in its construction all linear and angular measurements drawn down must be horizontal dimensions It is impossible to give a complete representation of distances following the undulations of the ground other than by a scale model Now a horizontal plane is one normal to the direction of gravity as indicated by a plumb line, but, on account of the spheroidal form of the earth, the directions of plumb lines suspended at different parts of a survey are not, in normal circumstances, strictly parallel, and the horizontal plane at one point does not precisely coincide with that through any other point No reference is here made to surface irregularities, the effect being due to the curvature of the earth's mean surface, which is taken to be that corresponding to mean sea level supposed extended round the globe

In surveys of small extent the effect of curvature is quite negligible, and it is justifiable to assume that the mean surface of the earth is a horizontal plane within the area covered Surveying methods based on this supposition are comprised under the head of *Plane Surveying.* The assumption becomes invalid in the accurate survey of an area of such extent that it forms an appreciable part of the earth's surface Allowance must then be made for the effect of curvature, and the operations belong to *Geodetic Surveying.*

No definite limit can be assigned for the area up to which a survey may be treated as plane since the degree of accuracy required forms the controlling factor The sum of the interior angles of a geometrical figure laid out on the surface of the earth differs from that of the corresponding plane figure only to the extent of one second for about every 76 square miles of area, so that, unless extreme accuracy is required, plane surveying is applicable to areas of several hundreds of square miles

Plane Surveying.—Plane surveying is of wide scope and utility, and its methods are employed in the vast majority of surveys undertaken for various purposes, such as engineering, architectural, legal, commercial, scientific, geographical, exploratory, military, and navigational As applied to civil engineering, all surveying methods are utilised in the various surveys required for the location and construction of the different classes of works within the province of the engineer These surveys embrace rapid reconnaissances of an exploratory character undertaken to facilitate the selection of an approximate site for the work These are followed by more detailed surveys of the selected region, in which a much greater degree of accuracy is sought, and from which the best location is ascertained. The obtaining of various data required in the design of the proposed works forms part of the preliminary operations, and may involve surveying methods of a specialised character Previous to and during construction, the surveyor's duties also include the routine of setting out the lines and levels of the works and the measurement of areas and volumes

Geodetic Surveying.—Geodetic surveys are usually of a national character, and are undertaken as a basis for the production of accurate maps of wide areas, as well as for the furtherance of the science of geodesy, which treats of the size and form of the earth The most refined instruments and methods of observation are employed, and the operations are directed to the determination of the absolute positions on the earth's surface of a series of points which serve as controls for all other surveys

Field and Office Work.—A record of measurements made on the ground is usually, in plane surveys at least, of little or no service until the dimensions are laid down to scale on a drawing The usual stages in the production of the finished drawing may be summarised as

Field Work · (*a*) A preliminary examination, or reconnaissance, of the ground to discover how best to arrange the work

(*b*) The making of the necessary measurements.

(*c*) The recording of the results in a systematic form.

Office Work . (*a*) The making of any calculations necessary to transform the field measurements into a form suitable for plotting.

(*b*) The plotting or drawing down of those dimensions.

(*c*) The inking-in and finishing of the drawing

(*d*) The calculation of quantities to be shown on the drawing for special purposes, such as areas of land, etc.

Principles of Surveying.—The fundamental principles upon which various survey methods are based are themselves of a very simple nature, and may be stated here in a few lines.

It is always practicable to select two points in the field and to measure the distance between them. These can be represented on paper by two points placed in a convenient position on the sheet and at a distance apart depending upon the scale to which it is proposed to plot the survey. From these initial points others can be located by two suitable measurements in the field and laid down in their relative positions on the sheet by means of appropriate drawing instruments Points so obtained serve in turn to fix the positions of others.

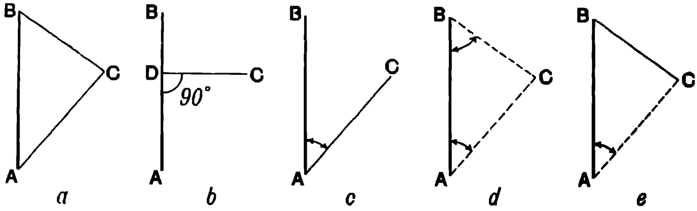


FIG 1

The more direct methods of locating a point C, with respect to two given points A and B, are illustrated in Fig. 1.

(*a*) Distances AC and BC are measured, and C is plotted as the intersection point of arcs with centres A and B and radii scaling the measured distances This system is employed in linear or chain surveying (Chap II).

(*b*) Perpendicular CD and the position on AB of its foot D are measured, and C is plotted by the use of a set-square. This method, termed offsetting, is used, in combination with other surveying systems, for locating subsidiary points not required for extending the survey, but for defining details.

(*c*) Distance AC and angle BAC are measured, and C is plotted either by means of a protractor or trigonometrically The traverse method of surveying (Chap III) is founded on this principle.

(*d*) Angles BAC and ABC are measured, and C is plotted by solution of the triangle or by use of a protractor. Distance AB being known, C is located without further linear measurement, and, in consequence, the method, known as triangulation, is applicable to the most extended surveys, in which it is desirable to reduce linear work to a minimum.

(*e*) Angle BAC and distance BC are measured, and C is plotted trigonometrically or by protracting the angle and swinging an arc from B. This method is of minor utility, and is required only in exceptional cases.

The same methods could be employed in measuring relative altitudes as well as positions in plan, but for this purpose methods (*b*), (*c*), and (*d*) only are of practical utility If the diagrams are regarded as elevations, with AB horizontal, ordinary spirit levelling (Chap. IV) is illustrated by (*b*). The elevation of C relatively to that of A is obtained by establishing instrumentally a horizontal line AB through A, such that AB and C are in the same vertical plane, and measuring the vertical distance CD Trigonometrical levelling (Vol II, Chap V) is exemplified in (*c*) and (*d*)

Several of the above systems, both for horizontal and vertical location, may be employed in the same survey, and various types of instruments are available for the angular and linear measurements A survey may therefore be executed in several ways by different combinations of instruments and methods, and some parts of the work may require different treatment from others The principal factors to be considered are the purpose of the survey and the degree of accuracy required, the nature of the country, the extent of the survey, and the time available for both field and office work To select the methods best suited to a particular case demands on the part of the surveyor a degree of skill which can be acquired only by experience

Nature of the Subject.— By virtue of the simplicity of the underlying principles of surveying, there is little of theory to be studied, and a training in the subject must be chiefly directed towards a thorough working knowledge of field methods and the associated instruments, as well as of office routine Success in the field is the outcome not only of skill in solving the larger problems connected with the general organisation of surveys, but also of attention to the methodical performance of the numerous details of field work Frequent practice in the field under expert guidance saves the beginner much memorising of these details, makes for skill and speed in manipulating instruments, and promotes systematic habits of work Numerous minor problems requiring special treatment are likely to be encountered in field work, and to the beginner these are apt to assume a more difficult aspect on the ground than they do on paper A little experience is necessary before one can entirely overcome the distractions of field conditions, especially when these are aggravated by physical fatigue and bad weather

Even in favourable circumstances there are many ways in which errors may enter into the work, and it is important to realise that *absolute precision can never be attained.* Any desired degree of refinement of practical utility may, however, be secured by the use of suitable instruments and methods of observation The surveyor must keep in view the uses to which his results will be put, and must select those methods which will yield sufficient accuracy for the purpose. Much time may be wasted in needless refinements, and the necessary judgment must be cultivated by a study of the nature and relative importance of the various sources of error affecting different surveying operations

Errors.—While a knowledge of the theory of errors, as dealt with in Vol II, Chap IV, is not required in connection with small surveys, it is desirable at the outset to distinguish between the different kinds of error to which all survey operations are subject These may be classified as mistakes, systematic or cumulative errors, and accidental or compensating errors

Mistakes arise from inattention, inexperience, or carelessness. Since an undetected mistake may produce a very serious effect upon the final results, the surveyor must always arrange his work to be self-checking, or take such check measurements as will ensure the detection of mistakes On discovering that a mistake has been made, it may be necessary to repeat the whole or part of the survey. It should, however, always be possible for the surveyor to guarantee that his completed work is free from mistakes

Systematic Errors are those which are recognised as existing in the performance of any particular survey operation Their character is understood, and their effects can be eliminated either by the exercise of suitable precautions or by the application of corrections to the results obtained Such errors are of a constant character, and are regarded as positive or negative according as they make the result too great or too small Their effect is therefore cumulative thus, if, in making a measurement with a 50-ft linen tape which is 1 in too long, the tape is extended ten times, the error from this source will evidently be 10 in The effects of constant errors may become very serious, and the precautions to be adopted against them in various field operations are treated in detail throughout the text

Accidental Errors include all unavoidable errors which are present notwithstanding that every precaution may have been taken. They arise from various causes, such as want of perfection of human eyesight in observing and of touch in manipulating instruments, as well as from the lack of constancy in the conditions giving rise to systematic errors Thus, in the example cited above the error of 1 in in the tape may fluctuate a little on either side of that amount by reason of small variations in the pull to which it is subjected or even by changes in the humidity of the atmosphere Such errors are of minor importance in ordinary surveying operations They are of a compensating character, since they are unlikely to be of the same sign throughout, and therefore to a large extent balance out in the final results In consequence, it is needless to adopt elaborate precautions against the occurrence of errors of this type while possibly overlooking the propagation of serious cumulative errors.

CHAPTER I

INSTRUMENTS—CONSTRUCTION AND ADJUSTMENT

BEFORE proceeding to consider the actual work of surveying it is desirable that a knowledge of the instruments employed should be obtained, and in this chapter are described the instruments in most regular use by the engineer and surveyor in the everyday work of chain and angular surveying and levelling. Those of a more specialised character are treated later under the branches of surveying with which they are related

While the necessity for a thorough understanding of instruments is self-evident, it is not suggested that the surveyor need at first make an exhaustive study of the construction of all the fittings which go to make up his instruments The more detailed his knowledge, however, the better qualified will he be to effect temporary repairs—an important matter when working far from headquarters In general, these individual fittings can be treated in groups forming definite and essential parts of the instrument, and the fundamental principles underlying the arrangement, use, and adjustment of the instrument are to be studied by reference to the functions and relationships of those parts.

The Nature of Adjustments.—Adjustments are of two kinds—*Temporary* and *Permanent*. The former are those which have to be made at each setting up of the instrument, and, as they form part of the regular routine in using the instrument, need no preliminary explanation The so-called permanent adjustments, on the other hand, are directed to eliminating errors of workmanship or defects which may develop either through wear or accident, and in general consist in setting essential parts into their true positions relatively to each other.

The method of making these adjustments in any instrument is entirely dependent upon the arrangement of its essential parts and the geometrical relationships they are designed to bear to each other. The adjustment of an instrument is simply a practical problem in geometry. Unless so regarded, the operation can only be performed mechanically, and the methods will be much more readily forgotten

Note—The examination of an instrument from the geometrical standpoint should lead the student to a consideration of —

(1) The nature of the errors of measurement made by the instrument due

to (*a*) erroneous relationships between adjustable parts, (*b*) defects in non-adjustable parts

(2) The method of manipulation whereby these errors may be reduced to a minimum

(3) The manner in which the instrument should be tested to discover whether it is in adjustment or not.

(4) The nature of the correction necessary to eliminate an error discovered in the test

The Principle of Reversal.—In testing for instrumental errors, considerable use is made of the method of reversal. Such tests are usually directed to examining whether a certain part is truly parallel, or perpendicular, to another, and on reversing the one part relatively to the other, erroneous relationship between them is made evident

To take the simple instance of examining the perpendicularity between two edges of a set-square, let the line BC (Fig 2) be drawn with the set-square in the first position, and BC′ after reversal The reversal constitutes the test, for if C and C′ coincide, the required condition that $ABC = 90°$ obtains If not, let e be the angular error Then clearly $CBC' = 2e$, or the apparent error on reversal is *twice* the actual error, and therefore, if correction is possible, the error to be eliminated is only half the amount of the apparent error The example further illustrates how good results are obtainable from a defective instrument, *viz* by reversing and taking the mean of the two erroneous results As a second example of the effect of reversal, the level tube (page 30) should be studied

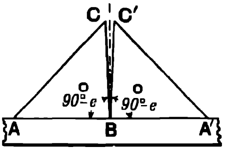


FIG 2

Notes on Adjustments.—1 Most instruments have several relationships to be established by adjustment, and it is important that these should be performed in such sequence and in such a manner (*a*) that those first executed will not be disturbed in making subsequent adjustments, (*b*) that, in performing any adjustment, possible errors of the adjustments to follow will be balanced out, if they are such as to influence the accuracy of that under examination

2. As it is sometimes troublesome to ensure that a particular adjustment will be quite independent of the others, it is well to repeat the adjustments from first to last and so gradually perfect them all, especially if serious errors have been discovered.

3 In making an adjustment, it is difficult to eliminate an error completely at the first trial, and the test and correction should be repeated a number of times, using greater refinement as the error decreases.

4. Instability of the instrument makes it almost impossible to adjust it satisfactorily, and the instrument should therefore be

placed upon firm ground In adjustments involving the taking of test sights, parallax (page 25) must be carefully eliminated.

5 Adjusting screws must be left bearing firmly, so that they will not slacken on being accidentally touched, but at the same time they should never be forced The nuts are commonly provided with holes into which an adjusting pin may be fitted, and, being rotated in the manner of a capstan, are usually referred to as capstans In using them, consideration must be given to the required direction of rotation

6. The length of time an instrument will maintain its adjustments to the accuracy with which they are made depends both upon the instrument and the manner in which it is handled Some adjustments are of much greater importance than others, and require more frequent examination In precise work these should be tested every day To obtain the best results in surveying, it is assumed that no instrument is free from error, and the routine in observing must be arranged to eliminate the effects of residual errors left after adjustment, latent errors which may have developed since adjustment, and inherent defects in the non-adjustable parts.

PARTS COMMON TO SEVERAL INSTRUMENTS

Before proceeding to a consideration of particular instruments, some features common to various instruments will be treated separately Parts thus dealt with are the telescope, level tube, vernier, magnetic needle, and tripod stand

THE TELESCOPE

In the majority of surveying instruments the several fittings may be broadly classed according to their functions as optical parts and measuring parts In many instruments, such as the theodolite and the level, the optical fittings take the form of a telescope, the use of which is to assist the eye in obtaining distinct vision of distant objects, as well as to furnish a definite line of sight of which the direction and inclination may either be known or are to be measured The provision of a telescope in such instruments is not absolutely essential, but the utility and accuracy of the instrument would be greatly impaired without it Some instruments, however, such as the tacheometer, the photo-theodolite, and the sextant, depend for their action upon certain laws of optics, and in these the optical parts are essential and assume the character of measuring parts.

A knowledge of the principles underlying the optical arrangement of the telescope is helpful both in the everyday use and the adjustment of the instruments of which it forms a part, and the student should therefore have some acquaintance with the elements of the science of optics. To introduce a description of the surveying telescope, the elementary principles relating to image formation by lenses will here be summarised

Definitions.—A *Lens* may be defined as a portion of a transparent substance enclosed between two surfaces of revolution which have a common normal, termed the *Principal Axis* of the lens The curved surfaces employed are of spherical form, but one of the surfaces may be plane

Lenses are classed as convex or concave The various forms are shown in Fig. 3, these being distinguished as (*a*) double convex or biconvex, (*b*) plano-convex, (*c*) concavo-convex or positive meniscus, (*d*) double concave or biconcave, (*e*) plano-concave; (*f*) convexo-concave or negative meniscus In the case of *a* and *d* the curvature of the two surfaces may be equal or unequal

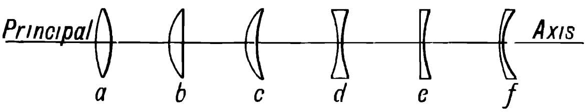


FIG. 3

The *Optical Centre*, or simply the centre, of a lens is that point on the principal axis through which pass all rays having their directions parallel before and after refraction A ray passing through this point undergoes a small lateral displacement, but, unless the lens is very thick, it is convenient to assume that the emergent ray is in the same straight line as the incident ray The optical centre is so situated that its distances from the surfaces are directly proportional to their radii In the case of double convex and concave lenses the centre lies within the thickness, in plano-convex and plano-concave lenses it is situated on the curved surface, while in meniscus lenses it is outside the lens and on the same side as the surface of smaller radius

A *Secondary Axis* is any straight line, other than the principal axis, passing through the centre of a lens

Refraction through Lenses.—The nature of the refraction of rays traversing a convex lens is such that a beam of light is on emergence more convergent or less divergent than at incidence Such lenses may be referred to as converging lenses in contradistinction to concave or diverging lenses, which have the opposite effect The influence of the curvature of the surfaces is to produce different angles of deviation on the individual rays of a beam according to the position of their points of incidence, and such that if all the incident rays pass through a point on the principal axis, the refracted rays, produced if necessary, will pass more or less exactly through some other point on the axis, In Fig 4 an axial parallel beam of light is shown incident upon a convex lens, the source of the light being an infinitely distant point on the principal axis The constituent rays after refraction converge to a point F on the principal axis This point is termed the *Principal Focus* of the lens, and in the case of thin lenses its distance from O, the optical centre, is called the *Focal Length,* a quantity which will be designated by f In the case of the concave lens (Fig 5) the parallel rays are, after refraction, made to diverge from the principal focus, which lies on the same side of the lens as the incident beam

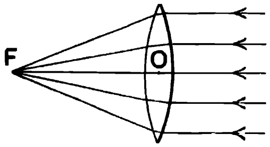


FIG 4

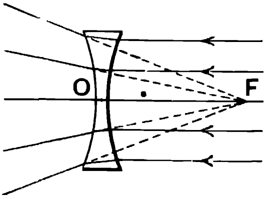


FIG 5

The principal focus may be regarded as the point at which is formed an image of the distant source of light In the case of the convex lens, the refracted rays actually pass through F, and the image is real, so that it could be received on a screen With a concave lens, however, the diverging rays have to be produced backwards to locate the point of divergence F The image in this case has no real existence, and it is said to be a virtual image

The *Power* of a lens is the reciprocal of its focal length, and is considered to be positive for a convex lens and negative for a concave The unit of power is the diopter, which is the power of a lens having a focal length of one metre Thus, a convex lens of 20 cm focal length has a power of 5 diopters

In the case of a lens formed of two or more individual thin lenses in contact, the power of the combination is the algebraic sum of the powers of the separate lenses If the focal lengths of the several lenses in contact are f', f'', etc , the relationship,

$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f''} + \cdots ,$$

enables the focal length f of the combination to be obtained Thus, if a convex lens of $7\frac{1}{2}$ in focal length is cemented to a concave lens of 30 in. focal length, the focal length of the combination is given by

$$\frac{1}{f} = \frac{1}{7\frac{1}{2}} - \frac{1}{30},$$

whence $f = +10$ in , the resulting lens being converging.

Conjugate Foci.– If rays of light proceeding from a point P on the principal axis are refracted through a lens, the emergent rays pass through another point P′, also on the principal axis (Fig 6) The object point P and the image P′ are reciprocal, and are termed conjugate foci If the distances from O to P and P′ are denoted by f_1 and f_2 respectively, it may be shown that

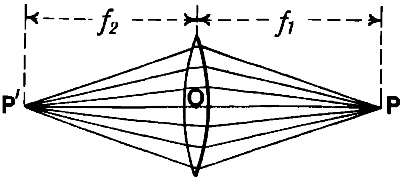


FIG 6

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f},$$

provided the following sign convention is observed. The focal length of a convex lens is positive, and that of a concave lens is negative A distance f_1 from the optical centre of the lens to the source P, measured in the direction opposed to that of travel of the light, is regarded as positive ; but if measured in the direction of the light, it is negative A distance f_2 from the optical centre of the lens to the image P′, measured in the direction of travel of the light, is positive , if measured against the direction of the light, it is negative The object distance f_1 is negative only in the case where the lens receives rays converging to a point P behind the lens, P belonging to a virtual object

Notes —(1) The above equation for conjugate focal distances is applicable only to cases in which the thickness of the lens is sufficiently small to be negligible in comparison with the conjugate distances, but it is sufficient for our requirements The equation does not apply to conjugate focal distances along secondary axes.

(2) The reader may accustom himself to the sign convention by verifying the following positions of the image for the given positions of an object point in the cases of lenses of 10-in focal length When the object point is behind the lens, it is to be understood that the rays incident upon the lens converge towards that point, but do not actually pass through it

Object Point at	Convex Lens Image at	Concave Lens Image at
Infinity in front	10 in behind	10 in in front.
20 in ,,	20 in ,,	6 7 in ,,
10 in ,,	Infinity	5 in ,,
10 in behind	5 in behind	Infinity
20 in ,,	6 7 in. ,,	20 in in front
Infinity ,,	10 in ,,	10 in ,,

The Image of a Body.—Considered optically, a body is simply a collection of points. Cones of rays proceeding from those points, if incident on a perfect lens along any axis, are so refracted that they are brought to a focus, thereby forming an image of the point and in the aggregate an image of the body. To determine the position of the image of any point on the body, it is evidently sufficient to trace the paths of any two rays from it to their intersection after refraction The two rays most easily located are those which pass through the optical centre and the principal focus respectively These are used in the case of Fig 7, which illustrates the formation by a convex lens of a real inverted image of an object situated at a distance from O greater than f The distances f_1 and f_2 along the principal axis are again related by the equation for conjugate focal distances, and it is evident, from the similarity of triangles Oab and OAB, that the ratio of the size of the image to that of the object is f_2/f_1.

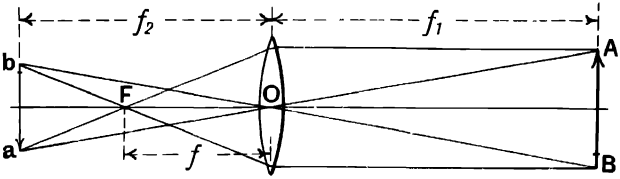


Fig 7

Defects of a Single Lens.– A simple lens is found to have several inherent imperfections affecting the formation of the image, and it is necessary to consider the means adopted to reduce these defects in order to understand the construction of the telescope Some of the errors to which lenses are subject need not be emphasised as, although their correction is of considerable importance in the photographic lens, they are of little consequence in the surveying telescope, the aim of which is to afford a good image of the central portion only of the field of view

The principal defects are (*a*) Chromatic Aberration , (*b*) Spherical Aberration , (*c*) Coma , (*d*) Astigmatism , (*e*) Curvature of Field ; (*f*) Distortion The first two are by far the most important, since they affect the sharpness of the image at the principal axis The others do not influence the image at the centre of the field, but are included on account of their possible effect in stadia tacheometry and in certain astronomical observations in which the image of a star is observed outside the principal axis of the lens system of the telescope

Chromatic Aberration.—White light may be regarded as composed of a series of undulations, each of different wave length Difference of wave length corresponds to difference of colour and also of refrangibility, and, as a consequence, a ray of white light after refraction at a single surface has its components separated from each other The visible rays which have the shortest wave length and greatest refrangibility are those of a violet colour, and wave length increases, and refrangibility decreases, through the range of the colours of the spectrum—violet, blue, green, yellow, orange, and red, Chromatic aberration is that defect of a lens whereby rays of white light proceeding from a point are each dispersed into their components and conveyed to various foci, forming a blurred and coloured image In Fig 8, which illustrates the dispersion produced by a simple convex lens, it will be seen that a sharp image of the radiant point is nowhere formed A screen placed at V receives an image surrounded by a halo of all the colours of the spectrum and bounded by a red fringe, and on removal of the screen to R the halo has a violet margin If the rays of white light, instead of proceeding from a point, emanate from a white body, the middle portion of the image of the body is white, because the individual colours are recombined there, and the chromatic effects are seen only at the boundary of the image

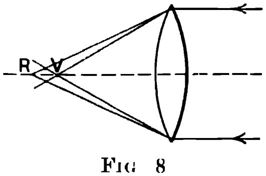


Fig 8

The circumstance that in different varieties of glass dispersive power bears no relation to refractive power renders the use of compound lenses available as a means of correcting the defect Two kinds of glass are employed, crown and flint, both of which are manufactured in a considerable range of optical qualities Flint glass has slightly the greater refracting power, but its dispersive power is roughly double that of crown By combining a crown glass convex lens with a concave lens of flint glass, as in telescope objectives (Fig 14), the proportions are arranged not only to yield the required focal length, but also so that the dispersion produced by the convex lens is neutralised at emergence from the concave

Practically, however, complete achromatism cannot be achieved by the use of only two kinds of glass, since the ratio of their dispersive powers varies at different parts of the spectrum A certain amount of residual colour, or secondary spectrum, is unavoidable, but does not have a serious effect on the sharpness of the image. A lens which is corrected for two colours of the spectrum is said to be achromatic, although it is only partly so

In eyepieces the correction for chromatic aberration is commonly made without compound lenses by the use of two plano-convex glasses of such proportions, and at such a distance apart, that the dispersion produced at the first is eliminated by the second.

Spherical Aberration.—Spherical aberration is a defect whereby the component rays of a beam proceeding from a point on the principal axis are not refracted to pass through a single point, but are differently focussed according to their positions of incidence on the lens It arises from the use of spherical surfaces, which are generally adopted because of the ease with which they may be accurately produced.

Fig 9 illustrates spherical aberration in a convex lens, and shows that rays passing through the margin are brought to a focus nearer the lens than are the central rays, so that the image formed is not sharp. In this case the aberration is regarded as positive, that of a concave lens being of opposite effect or negative. The relative amount of error present in lenses of a given power is largely dependent upon the relationship between the curvatures of the two surfaces and upon whether that having the larger or the smaller curvature receives the incident rays. The surface of smaller radius should face the more nearly parallel rays. Thus, if parallel light is incident upon the curved surface of a plano-convex lens, the aberration produced is only about a quarter of what it would be if the plane surface faced the source of light.

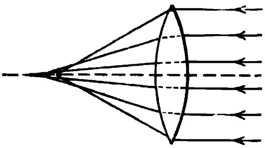


Fig. 9

Spherical aberration may be reduced to any required extent by diminishing the aperture so as to cut out the marginal rays, but this has the serious disadvantage of diminishing the brightness of the image. The correction is actually effected by replacing the simple lens by a combination of two lenses such that the positive aberration of one is neutralised by the negative aberration of the other. The combination may consist of a convex lens in contact with a concave, as in telescope objectives, or of two plano-convex lenses placed at a definite distance apart, as in eyepieces. A lens or combination of lenses in which the defect is eliminated is said to be aplanatic.

Coma.—The spherical aberration described above refers to images produced on the principal axis, and may be distinguished as axial spherical aberration. The remaining defects to be noticed result from the spherical aberration of oblique pencils of light, the axes of which are secondary axes of the lens.

Rays emanating from a point on a secondary axis do not fall upon the lens in the symmetrical manner of an axial pencil. In consequence, the resulting image is neither sharp, nor has its confused outline a circular form as in axial aberration. The image of a point source of light takes various forms, such as pear- and comet-shaped, with an axis of symmetry directed towards the principal axis of the lens. This effect of oblique spherical aberration is known as coma.

The defect impairs the sharpness of an image away from the principal axis. It may be reduced considerably by a moderate decrease in the aperture of the lens, but in the surveying telescope the resulting loss of brightness would be more objectionable than imperfection of the image towards the margin of the field.

Astigmatism.—This effect of oblique spherical aberration is caused by the lens refracting the rays from an extra-axial point so that, instead of passing through a focal point, they pass through a focal line. The directions of the refracted rays are such that, on travelling onwards, they pass through a second focal line at right angles to the first and radial to the principal axis. Between these two lines the best image of the point is obtained as a circular disc

If the object consists of a sheet of paper on which are ruled two lines, one radial and the other tangential, a satisfactory image of one of the lines will be received on a screen placed at one of the focal lines, but the image of the other object line will be confused. On moving the screen to the second focal line, the lack of definition applies to the image of the first object line Astigmatism therefore prevents sharp definition in all directions simultaneously. Its correction in a system corrected for axial aberration necessitates the use of at least three component lenses. A lens combination corrected in this respect is said to be anastigmatic

Curvature of Field.—Curvature is a further effect produced by the spherical aberration of oblique rays whereby the image of a plane surface normal to the principal axis is formed as a curved surface When received on a plane screen, such an image is not equally sharp all over, but, according to the position of the screen, will be distinct at the centre or the margin or along an intermediate circle. The two sets of line foci, as well as the discs of least confusion, formed by a lens uncorrected for astigmatism really lie upon curved surfaces, but curvature may also be present in the anastigmatic lens.

If the object glass or eyepiece of a telescope possessed the defect in a marked degree, it would become necessary to adjust the focus to obtain a clear view of the different parts of the field Curvature may be reduced by the use of a diaphragm or by means of two lenses placed at a suitable distance apart, as in the eyepiece of the surveying telescope

Distortion.—Distortion is the defect, arising from the same cause as the last, whereby straight lines on an object are not reproduced as straight lines in the image. It is always present in single lenses and in achromatic combinations of two lenses

The error can be reduced by covering the margin of the lens by a diaphragm placed on the surface of the glass, but it is accentuated if the stop is at some distance from the lens. The action of a stop set at a distance from the lens is illustrated in Fig 10, which shows that rays passing through or near the centre of the lens may be intercepted by the stop S and prevented from contributing to the formation

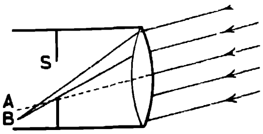
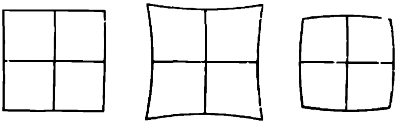


FIG 10.



a b c

FIG. 11.

of the image at its true position A, while the marginal rays pass through the aperture of S, and, because of their greater refraction, cause the image of the point to be formed at B If the object is rectilinear in outline (Fig 11*a*), the sides of the image will in these circumstances be convex inwards, giving rise to what is termed pin-cushion distortion (*b*). The opposite effect is obtained when the stop is placed in front of the lens, and barrel distortion (*c*) is then produced Since the defect does not influence the centre of the field, and is therefore of minor importance in the measuring telescope, the use of two compound lenses with a stop between is not warranted in the telescope objective as it is in the rectilinear photographic lens

Types of Telescopes.—Considered in its simplest form, a refracting telescope consists optically of two lenses, the principal axes of which coincide to form the optical axis of the telescope The lens nearer the body to be viewed is convex, and is termed the object glass or objective, the function of which is to collect a portion of the light emanating from the body The rays transmitted by it, on passing onwards, suffer a second refraction at the other lens, called the eyepiece or ocular, and are then suitably presented to the eye According to the form of the eyepiece and its position relatively to the object glass, a telescope may belong to one or other of two main types.

In *Kepler's* or the *Astronomical* telescope (Fig 12), rays from the object AB are, after refraction at the objective O, brought to a focus before they enter the eyepiece E, and, in consequence, a real inverted image *ba* is formed in front of the eyepiece. If this lens is so placed that *ba* is situated within its focal length, the rays after refraction at E appear to the eye to proceed from *b'a'*, a virtual image conjugate to *ba*. The object AB thus appears magnified, inverted, and placed at *b'a'*.

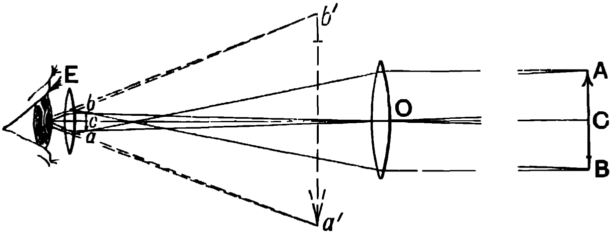


FIG 12

In *Galileo's* telescope (Fig 13) the rays refracted by the objective O are intercepted by a concave eyepiece E before a real image is formed. On entering the eye, they therefore appear to diverge from the virtual image *ab*, which is magnified and erect.

Both these arrangements meet one of the requirements of a telescope for use in surveying, since they afford distinct vision of distant objects, and from this point of view Galileo's telescope has the advantage that the image is erect But the telescope, as an adjunct to a measuring instrument, must also be capable of furnishing a definite line of sight, and it will be shown that only the Kepler telescope fulfils this condition

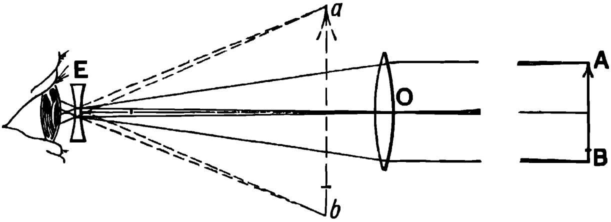


FIG 13

The Line of Sight.—By a line of sight of a telescope is meant any line passing through the optical centre of the objective, traversing the eyepiece, and entering the eye *The* line of sight, or line of collimation, is one such line further and visibly defined as passing through a point marked by the intersection of cross-threads stretched in front of the eyepiece in a plane at right angles to the axis

In Fig 12, let it be supposed that these threads are situated in the plane of the image *ba*, with their intersection at the point *c* on the optical axis The line of sight *c*O meets the body in a certain point C, the real image of which is formed at *c* on *ba*, and therefore the intersection of the hairs coincides with the real image of that particular point on the object focussed which is situated on the line of sight Now, on viewing the image *ba* through the eyepiece, the cross-hairs are simultaneously seen focussed in *b′a′*, and the coincidence of the plane of the hairs with that of the real image ensures that the observed position of the hairs relatively to points on the virtual image *b′a′* is the same as on the real image The hairs and the image *ba* are equally magnified, and distortion or other defect produced in the passage of the rays through the eyepiece affects both to the same degree The observer consequently sees the intersection of the hairs apparently coinciding with that point on the object which lies in the line of sight

The establishment of a telescopic line of sight therefore involves two essential conditions

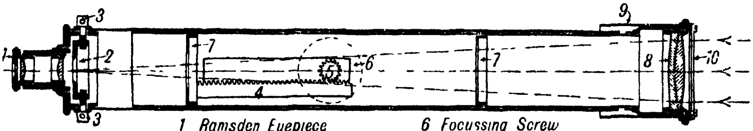
(*a*) A real image must be formed in front of the eyepiece.

(*b*) The plane of the image must coincide with that of the cross-hairs (see Parallax, page 25)

In Galileo's telescope no real image is produced, and it is therefore useless for quantitative observations. Cross-hairs are sometimes provided in such telescopes, but they serve merely to indicate the centre of the field. Galileo's telescope may therefore be regarded as a viewing, as distinct from a measuring, instrument. It is adapted for field glasses, and is used in sextants to facilitate the sighting of distant points

The Surveying Telescope.—The telescope of a surveying instrument is therefore of the Kepler type, and the construction of the external-focussing pattern is illustrated in Fig 14 The body is formed of two tubes, one capable of sliding axially within the other by means of a rack gearing with a pinion attached to the milled focussing screw It is essential that the movement should be strictly axial and free from shake

The objective, the focal length of which serves to designate the size of the telescope, is invariably a compound lens, of which the outer component is a double convex lens of crown glass and the inner is of flint glass and convexo-concave In the example illustrated, the objective cell is screwed to the inner tube, so that the focussing movement, required for placing the real image in the plane of the cross-hairs, is effected by movement of the objective relatively to the outer fixed tube carrying the cross-hairs and eyepiece , but in cases where the objective is mounted on the outer tube the eye end moves in focussing



1 Ramsden Eyepiece
2 Diaphragm
3 Diaphragm Adjusting Screws
4 Focussing Rack
5 Focussing Pinion
6 Focussing Screw
7 Stops
8 Object Glass
9 Ray Shade
10 Shutter

FIG 14 —EXTERNAL FOCUSSING TELESCOPE

In the majority of modern surveying telescopes focussing is effected without movement of either the objective or eyepiece by the introduction between them of a concave lens *11* (Fig 15) which is moved by the focussing screw to bring the image into the plane of the cross-hairs.

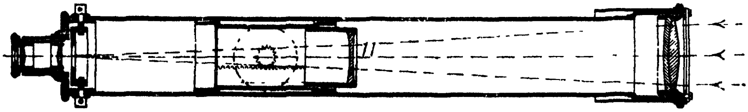


FIG 15 —INTERNAL FOCUSSING TELESCOPE

To examine the effect of the introduction of the internal focussing lens, let O and I (Fig. 16) be the optical centres of the object glass and internal lens, and let f and f' be their respective focal lengths. The distance x between the lens centres is variable, and for any observation the internal lens must be adjusted into such a position that the image of the distant object is formed in the plane of the cross-hairs at P_1, which is at a constant distance l from O.

Considering the formation of the image of an object P distant f_1 from the object glass, the latter would itself form an image at P′, the position of which is obtained from the equation for conjugate focal distances,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

The rays after refraction by the object glass are, however, intercepted by the concave lens as they are converging towards P′, and

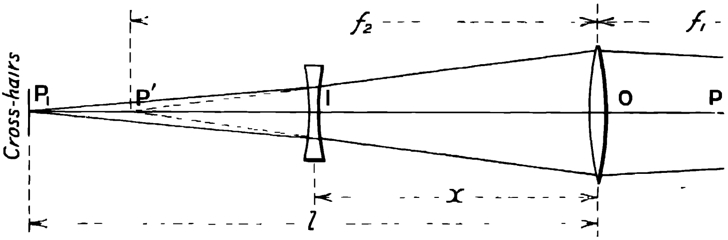


FIG 16

P′ is the position of a virtual object, the image of which is formed by the internal lens at P_1 The distances IP′ and IP_1 are therefore conjugate, so that

$$\frac{1}{(f_2-x)} + \frac{1}{(l-x)} = \frac{1}{f'},$$

(f_2-x) and f' being negative, according to the sign convention of page 11

These two conjugate focal equations show the required distance x between the lenses, if l, f, and f' are fixed. If the object is infinitely distant, f_2 equals f, and x has its minimum value. The amount of movement of the internal lens towards the eyepiece required to focus a near object is greater for a low power focussing lens than for one of high power The focussing lens may be a simple one, as shown, or it may consist of an achromatic combination

A telescope with internal focussing is more likely to be proof against the entrance of water and dust than one with a draw tube The chief merit of the system, however, lies in the avoidance of errors caused by change of position of the line of sight in focussing with an objective or eyepiece slide which has developed looseness through wear If the movement of an internal focussing slide becomes non-axial, the resulting error in sighting is very much less. The disadvantage of internal focussing is the reduction in brilliancy of the image caused by the passage of the rays through the additional lens.

With the object of improving the distinctness of the image in both types of telescope, the interior of the body is painted dull black, to prevent reflection from the internal surfaces, and annular stops may be fitted to intercept rays not required in the formation of the image In inferior instruments the front stop may be found placed near the objective for the purpose of diminishing spherical aberration, but the reduction of aperture thereby occasioned is a serious defect To shield the objective from the rays of the sun, there is fitted a sun cap or shade capable of being extended when required. The shade is closed by a hinged shutter, which serves to protect the lens against rain and dust

The end of the body remote from the objective is threaded to receive the tube in which the eyepiece is fitted to move axially for the purpose of focussing the cross-hairs (page 23). A cap to protect the eyepiece is useful, but is commonly omitted For making solar observations, a sun glass is required this consists of a piece of very dark glass mounted in a rim which can be fitted on the outside of the eyepiece In front of the eyepiece is mounted the diaphragm, across which are stretched the cross-threads The diaphragm and eyepiece call for further remark

The Diaphragm and Reticule.—The diaphragm carrying the reticule, or graticule, of cross-lines is formed of a flanged brass ring held in place in the telescope tube by four capstan-headed screws, as shown in Fig 17 These are screwed into the flange, but pass through slots in the telescope tube, so that, when slackened, they permit movement of the diaphragm horizontally and vertically as well as a small rotation about the axis of the telescope The heads of the screws, instead of being exposed, may be protected by a cover to prevent their being turned accidentally

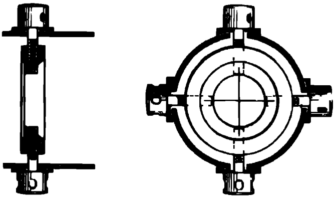


FIG 17 —DIAPHRAGM

In the interchangeable pattern of diaphragm, which is now generally fitted, the cross-hairs, instead of being mounted on the diaphragm ring itself, are carried by a cylindrical cell which fits into the ring (Fig 19) The object of the design is to facilitate the rapid replacement of a defective reticule by the withdrawal of the cell and the insertion of a spare one in the manner described on page 23

The cross-hairs, being viewed through the eyepiece, appear magnified, and it is therefore essential that they should be very fine. They should show as clean lines without fringes, and should be opaque Various means are employed for providing suitable lines, and each has its merits and deficiencies

Spider Webs.—These are most commonly used, and furnish a very fine, clean, and sufficiently opaque line. The most suitable webs are the darkest coloured ones of which the garden spider makes its nest Although in moderate climates such webs should last for several years, excessive damp may cause them to develop slackness and sag, particularly if they have not been properly stretched when being mounted on the diaphragm As most webs tend to lose their elasticity with age, extreme dryness may cause them to break by tension Owing to their delicacy, spider webs must never be exposed outdoors by removal of the eyepiece for any purpose

Silk Webs —Single filaments of silk furnish somewhat similar, but coarser, lines They are, however, sufficiently fine for use with low powers, and are rather less troublesome to insert than spider webs

Lines on Glass —Lines ruled upon a thin glass plate afford the least fragile reticule, and have the advantage that the relative position of the lines is permanent The ruling can be performed so that the lines appear little thicker than webs The plate, however, absorbs a small amount of light, and is subject to troublesome dewing with a falling temperature

Points —The desirability of eliminating fragility without the necessity of inserting a glass diaphragm has led to the use of small pointers of platinum-iridium alloy, the fine ends of which fulfil the office of a hair They are very durable, and are not liable to be damaged, but, if they happen to meet with injury, they cannot be repaired in the field Points are quite convenient for ordinary theodolite work, but are useless for astronomical observations In staff reading they involve a considerable waste of time owing to the necessity of bringing the extreme end of the horizontal point exactly to the edge of the graduations of the staff, an operation which demands the provision of a clamp and tangent screw on the instrument

A few typical arrangements of lines and points are illustrated in Fig 18

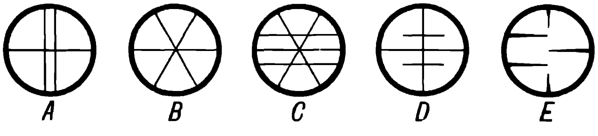


FIG 18

A shows a common system for levels The horizontal hair defines the reading, and the vertical hairs enable the observer to detect lateral deviation of the staff from its required verticality

B is used in theodolites The two inclined hairs are usually preferred to a single vertical hair for ease in bisecting the signal. In a variation of this pattern the diagonal hairs intersect a little above or below the horizontal hair, so that their intersection may be seen more distinctly.

C is the same with the addition of stadia hairs for tacheometry

D is used both in levels and theodolites The stadia lines do not extend right across the diaphragm, indicating that the lines are ruled on glass Two closely spaced vertical lines or hairs are better than a single vertical line if the telescope is to be used for sighting plumb lines, since accurate pointings can be obtained by making the image of the plumb line bisect the narrow space between the lines

E shows the usual arrangement of points for levels and theodolites, stadia points being fitted The end of the horizontal point defines the line of sight, and is in line with the vertical points

Replacing Cross-Hairs.—To replace broken hairs, first remove the diaphragm by loosening the capstan screws, and place it on a flat surface with that side uppermost which is provided with scratches for receiving the hairs Clean off old cementing material adhering to these notches Suitable webs should be selected from a spider's cocoon, failing which, if a spider can be found, procure a V-shaped piece of wire or stick, and get the spider on the end of one of the prongs On shaking him off, he will spin a web, which is to be wound on the fork so that the separate strands are not too close Immerse each web to be mounted for a few minutes in warm water, and then gently remove the superfluous moisture with a colour brush Stretch the web across the ring, using small weights to ensure tightness, and by means of a pin adjust it to lie exactly in the notches. Fix it by letting a drop of shellac fall on one end, and, when this has hardened, see that the web remains taut before similarly fixing the other end. If spider webs cannot be procured, floss silk forms a convenient substitute, as it is readily obtainable in

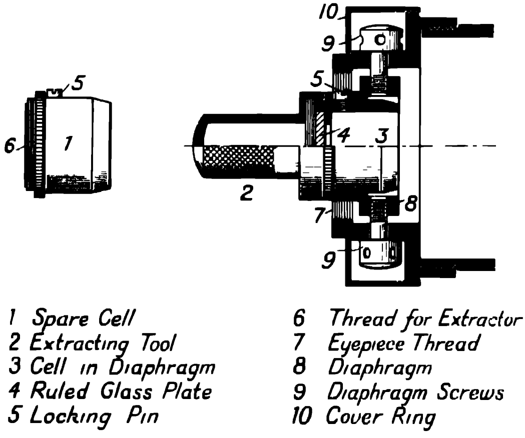


FIG. 19 —INTERCHANGEABLE DIAPHRAGM

reels The operation of mounting is the same, except that the silk need not be moistened.

When the diaphragm is replaced in the telescope tube and screwed up, it is most unlikely that the intersection of the hairs will occupy the same position relatively to the optical axis as it did before The position of the intersection of the hairs controls the position of the line of sight in the telescope, and it is therefore essential to adjust the line of sight by the methods to be described in connection with the adjustment of instruments

The delay occasioned by the breakage of webs in the field becomes quite insignificant if the instrument is provided with an interchangeable diaphragm, a spare cell, fitted either with webs or a ruled glass plate, being carried In order to extract a cell, the eyepiece is removed, and an extracting tool is screwed on to the projecting thread of the cell, which can then be withdrawn from the diaphragm carried by the capstan-headed screws Fig 19 shows the extracting tool in position The spare cell to be inserted is screwed to the extractor, by which it is pushed into position It is prevented by a pin from turning in the diaphragm ring while the tool is being unscrewed

Interchangeable cells, as they leave the maker, are designed to have the intersection of the cross lines correctly centered, so that the position of the line of sight will be unaltered by the substitution of one cell for another It is, however, wise to test the telescope after the operation, and it becomes absolutely necessary to do so if the spare cell has been re-webbed by the surveyor himself

The Ramsden Eyepiece.—The type of eyepiece almost exclusively adopted in this country for surveying telescopes is the Ramsden or positive eyepiece (Fig 20). It consists of two equal plano-convex lenses, called the field lens and eye lens respectively, placed with their curved surfaces facing each other at a distance between optical centres of two-thirds the focal length of either The eyepiece does not invert, so that, on observing the real inverted image formed in the plane of the cross-hairs, the virtual image of the object sighted is also inverted. The Ramsden eyepiece, while not strictly achromatic, is very free from spherical aberration and curvature of field.

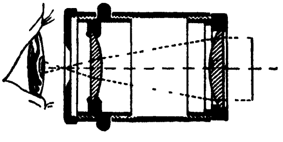


FIG 20

RAMSDEN EYEPIECE.

The eyepiece of Fig 20 is held by friction in a tube projecting from the end of the telescope, and the movement required for focussing the cross-hairs is effected by sliding the eyepiece out or in with a turning motion. Screw focussing eyepieces (Fig 21) are

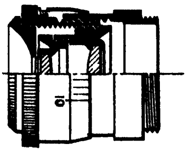


FIG 21

SCREW FOCUSSING RAMSDEN EYEPIECE.

regularly fitted in modern instruments, and give a much smoother movement In this case, focussing is performed by turning the milled ring encircling the eyepiece The ring carries a scale, which is read against a fixed index, so that the observer, having once ascertained the focus which suits his eyesight, can readily set the eyepiece for distinct vision of the hairs

With the object of reducing chromatic aberration, modifications of the Ramsden eyepiece are sometimes employed. In the Kellner eyepiece the eye lens is a compound lens of crown and flint glass similar to the objective, and the field lens is a simple double convex glass In the Steinheil eyepiece both lenses are compound

The Erecting Eyepiece.—An erect image may be obtained by means of an eyepiece consisting of four lenses, which are arranged as in Fig 22 Objections to the erecting eyepiece are the loss of light occasioned by the passage of the rays through the two additional lenses, and that for a given overall length of telescope the focal length of the objective must be smaller with an erecting than with a Ramsden eyepiece Particularly on account of its better illumination, the Ramsden eyepiece is to be preferred, any inconvenience arising from the inversion of the image being negligible.

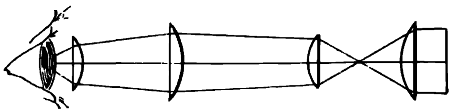
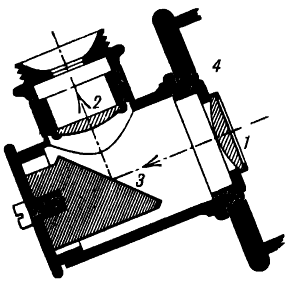


Fig 22 Erecting Eyepiece

The Huygens Eyepiece.—This eyepiece is used only in the Galilean telescope in place of the concave eye lens shown in Fig 13 It consists of two plano-convex lenses with their convex faces towards the objective The focal length of the field lens is three times that of the eye lens, and the distance between them is the difference of their focal lengths The eyepiece must be placed to receive the rays from the objective before a real image is formed, and the virtual image presented to the eye is erect



1 Field Lens 3 Reflector
2 Eye Lens. 4 Telescope Body.

Fig. 23.
Diagonal Eyepiece.

The Diagonal Eyepiece.—When the inclination of the telescope to the horizontal is great, it becomes inconvenient to bring the eye into position for sighting, and a diagonal eyepiece is required. That of Fig 23 is of the Ramsden type. Between the two lenses there is fitted a reflecting prism or a glass or metal mirror placed at 45° to the telescope axis. This reflector turns the line of sight through 90°, so that the observer is enabled to look into the telescope in a direction at right angles to its axis. In consequence of the reflection, the image is presented right side up, but the right- and left-hand sides are still reversed. The diagonal eyepiece is a necessity for astronomical work, and often proves useful for terrestrial observations in precipitous country and in constructional work.

Parallax.—Parallax is a condition arising when the image formed by the objective is not situated in the plane of the cross-hairs. It is evidenced when movement of the observer's eye apparently produces movement of the image relatively to the hairs, the direction of the apparent movement of the image being the same as, or opposite to, that of the eye, according as the image is on the side of the hairs remote from or next the eye. Unless parallax is eliminated, accurate sighting is impossible, as the line of sight may apparently be made to intersect different points according to the position of the eye.

Elimination of Parallax.—The two steps are: (1) to focus the eyepiece for distinct vision of the cross-hairs; (2) to bring the image of the object into their plane.

(1) Turn the focussing screw until no object can be distinguished in the field, or point the telescope to the sky. Adjust the eyepiece until the cross-hairs appear in sharp focus.

(2) Point the telescope towards the object, and, keeping the eye on the cross-hairs, turn the focussing screw until the image appears in sharp focus. The image and hairs should then be in the same plane, but the eye should be moved about to test whether any relative movement between them can be observed. If necessary, adjust the focussing screw until the apparent movement is eliminated.

Notes.—(1) To test the focussing of the eyepiece, turn aside and look at a distant point, or close the eyes for a moment. On again looking into the eyepiece, the hairs should at once appear perfectly distinct without the eye having to accommodate itself.

(2) The position of the eyepiece for distinct vision of the hairs depends only upon the individual observer's eyesight, and, when once found, should be permanently marked.

Qualities of the Telescope.—In addition to its important function of providing a definite line of sight, the telescope is required to present a distinct view of distant objects, so that the observer may be able to direct the line of sight with sufficient accuracy on to a distant mark or read its position on a graduated rod. The unassisted eye is unable to distinguish clearly between separate marks unless the angle subtended between them at the eye is from 1 to 2 minutes. Such capacity for distinguishing detail, or resolving power, as it is called, is quite insufficient for the majority of surveying operations, so that a telescope is required to assist vision.

The resolving power of the eye varies considerably in different individuals, even in the normal eye it is not constant, but depends upon the various circumstances of the observation. Taking the case of viewing a graduated rod, the normal eye should, under average conditions, be able to distinguish clearly marks 0·01 ft apart at a distance of 20 ft, the angle subtended at the eye being $\frac{1}{2,000}$ radian. Now, if by means of a telescope it is desired to distinguish with equal clearness the same marks at a distance of, say, 500 ft, the angle subtended at the instrument is reduced to $\frac{1}{25}$ of its former value The telescope is required to have a resolving power 25 times greater than that of the unassisted normal eye, and this ratio may be regarded as the resolving power or resolution of the telescope

The performance of one surveying telescope may differ from that of another in several important respects, due to differences in manufacture and in the proportions between the parts The most important qualities are (1) Definition, (2) Brightness of Image, or Illumination, (3) Magnification, (4) Size of Field

Definition —The quality of definition in a telescope is its capability of producing a sharp image It depends upon the extent to which spherical, chromatic, and other aberrations have been eliminated in the objective and eyepiece, as well as upon the accuracy with which all the lenses have been centered on one axis Good definition, particularly in the axial region, is obviously a desirable attribute, and the resolving power of the telescope is chiefly dependent upon it

Brightness of Image —The quantity of light available for the formation of the image depends upon the amount admitted at the objective and upon the loss caused by the passage of the rays through the several lenses The first depends upon the area of aperture of the objective, and therefore varies as the square of the diameter Loss amounting to 15 to 25% of the light falling on the objective is occasioned by reflection and diffusion at the surfaces of the lenses and by absorption due to imperfect transparency of the glass

In forming the magnified image, the light rays are spread out to a degree depending upon the magnification, and the intensity of illumination of the image is inversely proportional to the square of the linear magnification The brightness of the image as judged by the observer is largely dependent upon the diameter of the pupil of the eye, and the best results are attained when the diameter of the beam emerging from the eyepiece—the exit pupil—is the same as that of the pupil of the eye

Brightness of image is always desirable in the surveying telescope, especially for work in a bad light. In general, the brighter the image, the greater is the resolution.

Magnification —Magnification is the ratio between the angle subtended at the eye by the virtual image and that subtended by the object Varying position of the object causes a slight variation in this ratio, and the magnifying power of a telescope is measured as the ratio of the focal length of the objective to that of the eyepiece

Since any focal length of eyepiece can be used in conjunction with a given objective, the magnifying power of a telescope can be increased indefinitely Resolving power is promoted by increase of magnification, but the magnification should not be greater than is required to give a resolving power compatible with the sensitiveness of the instrument generally (page 84) The provision of a greater power than is strictly necessary is a disadvantage since it entails (1) reduction of brilliancy of image, (2) waste of time in focussing, (3) reduction of the field of view To obviate the loss of brilliancy accompanying large magnification, a larger object glass should be used for a high than for a low magnification Although, within limits, the same resolving power may be secured by a high magnification, producing a dull image, as by a lower magnification and a bright image, the latter is usually preferable The magnifying power of telescopes on surveying instruments for ordinary work varies from 10 to 30, and reaches 80 in precise instruments Sometimes two eyepieces of different powers are supplied with an instrument

Size of Field —By the field of view is meant the whole extent visible at one time through the telescope It is measured by angle COD = AOB (Fig 24), the apical angle formed at the optical centre

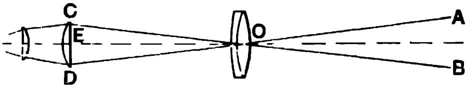


Fig 24

of the objective by the cone of rays refracted to enter the eye, and this angle is evidently that subtended at the optical centre of the objective by a diameter of the field lens of the eyepiece Fig 24 shows that the angle of field (1) increases as the diameter of the field lens increases, (2) decreases as the distance between the objective and eyepiece increases, so that it is smaller for short sights than long ones, (3) is independent of the size of the objective The size of field also decreases as the magnification increases, for if m = the magnification =

$$\frac{\text{focal length of objective}}{\text{focal length of eyepiece}} = \frac{F}{f}, \text{ then}$$

$$\text{angle of field} = 2\tan^{-1}\frac{\mathrm{CE}}{\mathrm{OE}} = 2\tan^{-1}\frac{\mathrm{CE}}{F+f} \text{ practically,}$$

$$= 2\tan^{-1}\frac{\mathrm{CE}}{f\left(\frac{F}{f}+1\right)} = 2\tan^{-1}\frac{\mathrm{CE}}{f(m+1)}$$

A wide field is desirable on account of the facility with which the observer can discover distant objects prior to exact pointing of the telescope. In instruments having magnifying powers up to 30 a field of from 1° to 2° is suitable.

Testing the Telescope.—Looking to the importance of some of the properties of the telescope, it is desirable that the surveyor should be able to ascertain the optical qualities of his instruments. The tests described below are sufficient for this purpose.

Test for Chromatic Aberration.—Point the telescope on a brightly illuminated disc of white paper or the edge of a dark object appearing against the sky. The extent to which residual aberration is present is evidenced by the amount of tinge appearing at the edge.

Test for Spherical Aberration.—Prepare a disc of black paper the size of the aperture of the objective, and cut out a second disc of about half the diameter from its centre, thus forming a disc and a ring. Cover the margin of the objective with the ring, and focus the telescope on a page of print. Without altering the focus, substitute the disc for the ring, affixing it centrally on the objective, and note whether the image produced by the rays now passing through the margin is as sharp as before. If a change of focus is required, the distance through which the focussing slide must be moved is a measure of the defect.

Test for Definition.—Focus on a well-illuminated page of print placed 30 to 40 ft. away. It should be possible to obtain a sharp image. If otherwise, the fault may be due to inaccurate curvature or poor finish of the lenses or bad centering of the system. To test for the latter, cut a small disc of white paper with a sharp knife, and place it against a dark surface at the former distance. Focus sharply, and then, by slightly moving the focussing screw, throw the image out of focus. The disc will appear to be surrounded by a haze, which should have a uniform breadth all round. Defective centering is indicated if the haze is broader on one side than the other.

Test for Aperture.—To ascertain whether part only of the full aperture of the objective is used, prepare a small strip of strong paper about $\frac{1}{16}$th inch wide. Moisten and affix it radially on the objective. On placing the eye a little distance back from the eyepiece, the image of the effective aperture is seen as a small bright circle, and the image of the strip of paper will be seen on it. Draw the strip towards the margin of the objective, and when its image reaches the circumference of the bright circle fix the strip on the lens. The end of the paper marks the edge of the effective aperture, the diameter of which can then be obtained. Instead of viewing the small image directly, a second eyepiece or magnifying glass may be held against the telescope to receive and magnify the image

Test for Illumination.—The only simple test available is that of comparing the telescope being examined with another which is regarded as satisfactory in this respect Set the instruments side by side, and point each telescope towards some well-defined object. Sight through them alternately, and judge the relative brightness of the images Preferably make the comparison towards nightfall, and note through which telescope the object remains visible longest as the light wanes

Test for Magnification.—Focus a boldly graduated staff held about 100 ft from the instrument Sight the staff through the telescope with one eye, and at the same time observe it with the other eye unaided by looking along the outside of the tube Bring the magnified and direct views alongside each other, and count the number of staff divisions as seen directly which appear to occupy the same length as one of the magnified divisions This number represents the magnification

Test for Angle of Field.—Focus the telescope on a levelling staff held vertically or horizontally at a distance of about 250 ft, or such smaller distance that neither end of the staff appears in the field Note the two extreme readings which can be observed The length of staff intercepted within the field of view divided by its distance from the instrument is, with sufficient precision, the average value of the angle of field expressed in circular measure

Test for Distortion.—Bring the image of any straight line towards the edge of the field, and observe whether it appears straight or not Otherwise, draw a diagram of bold lines forming squares, and pin it up at a distance such that on being viewed through the telescope it fills the field Focus, and note whether the squares near the margin appear free from distortion

Care of the Telescope.—Dirty lenses, especially those of the eyepiece, are a source of impaired brilliancy and definition Dust should be removed from the surfaces by means of a small camel-hair brush The presence of the finest film of grease is, however, more objectionable than dust, and, as this may arise from perspiration from the hands, care should be exercised to avoid fingering lenses Very dirty lenses should be wiped with a piece of clean, soft rag or chamois leather moistened with alcohol, using a clean part on every portion of the lens to avoid scratching Lenses should be rubbed as little as possible, hence the importance of protecting them against dirt and moisture (page 124) If the objective is unscrewed for the purpose of dusting the inside surface, it must be returned exactly to its original position Should a film settle between the two component lenses which does not evaporate, it is unwise to separate the lenses, and the defect is best treated by the maker Dust which has settled on the cross-hairs may be removed by taking out the eyepiece and blowing very gently on the webs, but, unless the dust particles are on or near the line of sight, they should preferably be allowed to remain

Both the main focussing and eyepiece slides should work freely, but with sufficient friction to stay in place whatever the inclination of the telescope A screw is sometimes provided for adjusting the friction of the main slide , otherwise, a little tallow smeared on the tube will help to make a loose slide stick When it is detected that slackness gives rise to collimation error (page 86), the telescope should be sent to the maker for repair Grit adhering to the object glass slide may cause fretting of the brass tube, and, when this occurs, it must be attended to as soon as possible, as the defect rapidly gets worse. The injured portion should be smoothed off with the back of a penknife blade The use of oil on slides is a fruitful source of such trouble, and should be avoided

THE LEVEL TUBE

Since nearly all field measurements, angular or linear, are made in either the horizontal or the vertical plane, it is essential to have some ready method of determining the positions of those planes through any point, so that particular lines or planes in an instrument may be made to lie in them The most convenient and accurate means available for this purpose is the level tube, the action of which depends upon the fact that the surface of a still liquid, being at every point normal to the direction of gravity, is a level surface

Construction.—The level consists of a glass tube, carefully ground so that a longitudinal section of the inside surface is a circular arc. The area so ground may be the upper half only or the whole of the interior surface In the latter case, the figure is barrel-shaped, the cross section being circular throughout. This is the better form, since, owing to its symmetry, the level can register correctly although it is turned about its longitudinal axis, and it must be used in instruments in which the level is required to function when turned upside down.

The tube is nearly filled with a liquid, such as alcohol, chloroform, or sulphuric or petroleum ether, the remaining space, forming the bubble, being occupied by air and spirit vapour These liquids are considerably less viscous than water , ether has the lowest coefficient of viscosity, and is the most suitable for sensitive levels All these liquids possess the advantage of having a low freezing point, but their coefficients of expansion exceed that of water, the length of the bubble being considerably reduced in hot weather. For the purpose of estimating when the bubble is central, a number of equally spaced lines is etched on the tube on either side of the centre or sometimes on an attached scale. The length of the divisions is usually either 2 mm or 0·1 in

The level is usually mounted in a brass enclosing tube, which is connected to some part of the instrument in a manner permitting its adjustment relatively to the supporting piece The tube may, however, be entirely unattached, in which case the bottom is made a plane surface It is employed in this form in conjunction with the plane table and in certain artificial horizons

Principle of the Level Tube.—The *axis* of a level tube is the trace in longitudinal section of the plane tangential to the interior upper surface at the mid-point or zero of the graduations In section (Fig 25) the trace AB of the level liquid surface forms a chord of the circular arc representing the surface of the tube, the curvature being exaggerated in the diagram A tangent to this arc at a point midway between the ends of the bubble is parallel to the chord, and is therefore a horizontal line irrespective of the position of the bubble in the tube If the tube is tilted so that this mid-point is coincident with C, the centre of the graduation, the tangent at C coincides with the axis of the tube, which is now horizontal

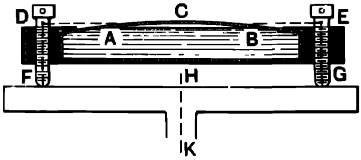


FIG 25

Let the tube be provided with two rigid supports in the same vertical plane with its axis, the lengths of which, measured from the axis, are DF and EG, and let these supports rest without attachment upon a plane surface FG If it is desired that FG should be horizontal when the bubble is central, it is evidently necessary that DF = EG In place of attempting to verify the equality of the supports by direct measurement, a test by reversal may be made by lifting the tube, turning it end for end, and replacing it in the reversed position If the bubble preserves the same position as it occupied before reversal, line FG is horizontal, and DF = EG

If not, let DF be the shorter, and let e denote the resulting angular error of parallelism between DE and FG DE (Fig 26) being levelled, DEFG represents the conditions before reversal, FG making an angle e with the horizontal On reversing, the axis has the position D′E′, and is now inclined at $2e$ to the horizontal. The apparent error is consequently twice the actual, and, in adjusting the supports to equality, it is therefore necessary to eliminate only half the apparent error by manipulating adjusting screws to bring the bubble half-way back to the centre. The adjusted tube can then be used to examine whether points on which it is supported

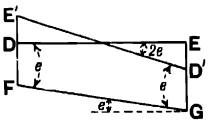


FIG 26.

are at the same level In levelling a plane surface, the test must be made for two non-parallel positions of the tube, preferably at right angles to each other

The bubble tube provides means for securing verticality as well as horizontality Let the supports of the tube of Fig 25 be now screwed into the plate FG, which can be rotated about an axis HK, not necessarily in the plane of the tube axis Reversal is now effected by rotating the plate and attached tube about HK through 180° If the position of the bubble is unaltered by the reversal, HK is truly vertical If not (Fig 27), by the same reasoning as before, only half the apparent error falls to be eliminated by adjusting the lengths of the supports The adjustment of HK to verticality is secured when the bubble maintains one position during rotation

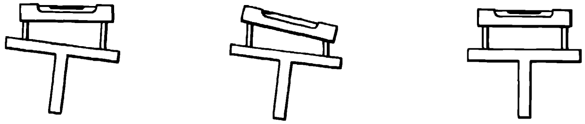

Before Reversal *After Reversal* *After Adjustment*

Fig 27

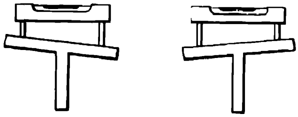

Before Reversal *After Reversal*

Fig 28

It is frequently overlooked that this method of reversal does not test the horizontality of the plate FG, since, as in Fig 28, FG may not be perpendicular to HK. The horizontality of a plate can be ensured only if the bubble is not displaced on removing the tube and replacing it end for end in two non-parallel positions

Sensitiveness.—By sensitiveness in a level tube is meant its capability of exhibiting small deviations from the horizontal This quality depends primarily upon the radius of curvature of the internal surface—the larger the radius, the greater the sensitiveness Sensitiveness is also promoted by increase in the diameter of the tube and in the length of the vapour bubble and by decrease of viscosity and surface tension of the liquid A very smooth finish to the internal surface of the tube also adds appreciably to its sensitiveness.

Sensitiveness is sometimes designated in terms of the radius of curvature of the tube, but the better way is to state the angle through which the axis must be tilted to cause the bubble to travel a specified distance, usually that between adjacent graduations of the engraved scale.

The sensitiveness of a tube should accord with that of the instrument mounting it. If the sensitiveness is greater than is really necessary, time is wasted in levelling the instrument, and the accuracy of the observations is in no way increased

Measurement of Sensitiveness.—*Laboratory Method.*—The apparatus, called a level trier, consists of a beam, 18 to 21 in long, supported on two points at one end and by a micrometer screw at the other Rotation of this screw tilts the beam, the inclination being recorded by the micrometer, which reads to one or two seconds. Having placed the instrument on which the bubble is mounted, or the tube alone, on the beam, the movement of the bubble corresponding to an observed change of inclination is quickly determined

Field Method —If the tube is attached either directly or indirectly to the telescope of an instrument, the tilt corresponding to an observed movement of the bubble may be measured by the resulting motion of the line of sight on a levelling staff held at a known distance from the instrument A base of about 300 ft is measured on level ground by steel tape With the instrument at one end of the base, and the staff held at the other, staff readings are observed (*a*) with the bubble near one end of its run, (*b*) with the bubble moved towards the other end by means of the levelling screws In each case the position of both ends of the bubble is recorded, and that of its centre is deduced in terms of the tube graduations, it being insufficient to observe one end only owing to possible change in length with varying temperature The observations should be repeated several times for different positions of the bubble, and the results are averaged

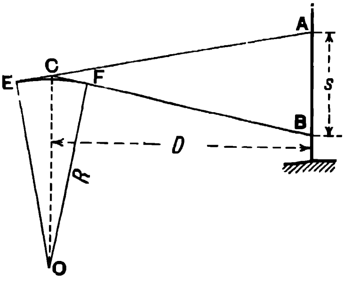


FIG 29.

Expressions for the radius of curvature and the angular value of one division of the tube may be obtained from a consideration of Fig 29, in which the line of sight is made coincident with the tube axis, and O, the centre of curvature of the tube, is shown stationary AB represents the staff, and EF the average run of the centre of the bubble

Let D = the horizontal distance between the instrument and the staff,

s = the average length of staff intercepted between the upper and lower lines of sight,

n = the average number of divisions through which the centre of the bubble is moved,

R = the radius of curvature of the tube,

d = the length of one division on the tube, expressed in the same unit as D and s.

The angle between the lines of sight is practically $\frac{D}{s}$ radians, since its value is small, and this angle equals the tilt of the tube, or

$$\frac{s}{D} = \frac{\text{EF}}{R} = \frac{nd}{R};$$

$$\therefore R = \frac{Dnd}{s}$$

The angular value of one division, expressed in radians, $= \frac{s}{Dn}$, or $\frac{d}{R}$, and, expressed in seconds, $= \frac{s}{Dn \sin 1''} = \frac{206{,}265\, s}{Dn}$.

The Chambered Level Tube.—Since increase of temperature shortens the bubble, and therefore reduces its sensitiveness, tubes used in precise instruments are commonly provided with means for regulating the length of the bubble An air chamber is fitted at one end, so that, by tilting the tube, the required quantity of air may be added to or taken from the bubble

The " Constant " Level Tube.—With a view to eliminating the loss of sensitiveness caused by shortening of the bubble at high temperatures, Messrs E R Watts & Son, of London, have developed an interesting design of level tube, in which the bubble remains of constant length throughout a range of 130° F or more.

Rise of temperature causes not only expansion of the liquid in a level tube but also decrease of its surface tension The effect of the former is to decrease the length of the bubble , that of the latter is to increase the length owing to the cylindrical form of the upper surface of the tube By suitably proportioning the volumes of the liquid and of the air bubble, it is possible to balance the two effects and secure a constant length of bubble Messrs Watts found that accurate results were attained with about 71% of spirit, such as petroleum ether, and 29% of air, but that, to obtain constancy of bubble length with a tube of circular cross section, it was necessary to use a longer tube than would be suitable for attachment to a surveying instrument. This difficulty is overcome by departing from the circular form, either by placing within a level tube of circular section a glass rod or sealed glass tube of smaller circular section, or, more neatly, by using a level tube of non-circular section In the " Constant " bubble tube (Fig. 94) the upper wall is of relatively large radius, the side walls are sharply curved, and the bottom is flat, but may be curved like the top to form an oval section, or it may be convex upwards, so long as the required proportion between the volumes of liquid and air is obtained.

The Circular Level.—This level, Figs. 89 and 100, consists of a circular box, the upper glass surface of which is a portion of a sphere, concave on the lower side The bubble is circular, and, when central, shows that the axial *plane* is horizontal Although it is usually of low sensitiveness, the circular level forms a convenient adjunct to various instruments, as it is equivalent to two tubes at right angles.

THE VERNIER

The vernier is a device for measuring fractional parts of the divisions of a graduated scale Although invented by Vernier as early as 1631, it still remains the most commonly used means for this purpose

In reading the position of an index relative to the divisions of a straight or circular scale, it may for certain purposes be sufficient to estimate by eye the amount by which it is past a graduation of the scale The probable error made in the estimation of the fractional part of a division will depend primarily upon the observer, but also upon the circumstances of the observation for any particular set of conditions it may be expressed as a constant fraction of a scale division If, therefore, the scale divisions are comparatively large, the absolute probable error of a measurement made in this way may become serious, and if a high degree of precision were required, very close division of the scale would apparently be necessitated Close graduation has, however, serious disadvantages The division and engraving present mechanical difficulties Further, the eye is confused by close spacing, and is unable to distinguish clearly one space from another The use of a sufficiently powerful magnifier involves waste of time, and does not altogether eliminate the confusion

These defects are overcome by the use of the vernier, the principle of which is based on the circumstance that the eye can perceive without strain and with considerable precision when two graduations coincide to form one continuous straight line It consists of a small scale arranged to slide in contact with the fixed scale, which latter may be distinguished as the main or primary scale The vernier carries the index point, the position of which is to be recorded, and which forms the zero of the vernier divisions If the graduations of the main scale are numbered in one direction only, the vernier used is of the single vernier type If the graduations are figured in both directions, a double vernier, consisting of two single verniers, is preferable, one being read when the scale is used from left to right, and the other for the reverse direction.

Whether single or double, verniers may also be classed as direct and retrograde.

Direct Vernier.—Let d = the value, linear or angular, of the *smallest* spaces into which the main scale is divided,

v = the value of the smallest spaces on the vernier scale,
n = the number of spaces on the vernier.

In the direct vernier, v is such that the n vernier spaces occupy a length equal to that of $(n-1)$ main scale spaces,

$$\text{i e}\quad nv = (n-1)\,d,$$

$$\text{or}\quad v = \left(\frac{n-1}{n}\right) d\,,$$

and the difference between the value of a main scale division and that of a vernier division

$$= (d-v) = d\left(1-\frac{n-1}{n}\right) = \frac{d}{n}$$

This quantity is termed the least count of the vernier The position of the index can be read by means of this vernier to $1/n$ of a main scale division

The method of using the vernier can be followed most easily from a simple example In Fig 30, S is the main scale, and V is a single vernier provided with 10 spaces of such size that they altogether occupy the same length as 9 main scale divisions The vernier is therefore a direct one with a least count of $\frac{1}{10}$th of a main scale division In Fig 30 the reading is exactly 11, and, since the vernier spaces are $\frac{9}{10}$ths the length of the main scale spaces, graduation 1 of the vernier falls short of a scale graduation by $\frac{1}{10}$th of d, graduation 2 is $\frac{2}{10}$ths of d short of another, and so on If, therefore, the vernier is moved forward a distance equal to $\frac{1}{10}$th of d, graduation 1 of the vernier falls into coincidence with a main scale mark, and, in general, if the pth graduation on the vernier is in the same straight line as one of the main scale marks, the index has passed a main scale graduation by $p/10$ths of d In taking a reading, therefore, when the index does not happen to record an exact number of scale units, first note the main scale graduation beyond which the index lies, in the direction of increase of the numbers on the scale (the approximate reading), and then observe where the coincidence of a vernier mark with a scale mark takes place The number of the coinciding graduation on the vernier scale multiplied by the least count is the fractional part to be added to the approximate reading Thus, in Fig 31 the reading is 11·6 No account is taken of the main scale graduation in coincidence with that of the vernier

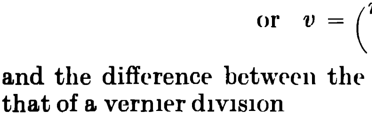


FIG 30 —DIRECT VERNIER

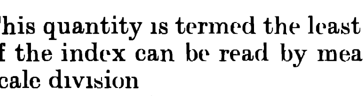


FIG 31 —DIRECT VERNIER

The converse of reading a vernier is setting it to a given reading. Having set the index approximately in place, a small adjustment is made by means of the screw provided for the purpose until the desired coincidence of the vernier graduation representing the given fractional part is obtained It is well to check the approximate reading after the fractional part has been set

Fig 32 shows a double vernier, the single verniers composing it being similar to that just described The part in advance of the index in the direction of measurement is that to be used The readings illustrated are 15 6, from left to right, and 84·4, from right to left The adoption of two different characters of figures on the scales tends to obviate confusion in the diagram the main scale numerals slope the same way as those on the half of the vernier to be used

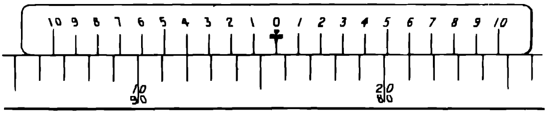


FIG 32 —DOUBLE VERNIER

Retrograde Vernier.—In this type the n vernier divisions occupy a length equal to that of $(n+1)$ main scale divisions, so that, with the previous notation,

$$nv = (n+1)\,d,$$
$$v = \left(\frac{n+1}{n}\right)d,$$

and, as before, the difference between the value of a main scale division and that of a vernier division

$$=(v-d)=d\left(\frac{n+1}{n}-1\right)=\frac{d}{n}, \text{ the least count}$$

In this vernier the numbering of the vernier divisions increases in the opposite direction from that of the main scale Fig 33 illustrates a retrograde vernier reading to tenths, the reading shown being 20 7 This type of vernier may also be doubled for use with scales figured in both directions

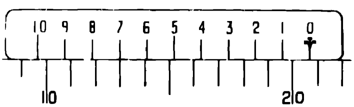


FIG 33 —RETROGRADE VERNIER

Examples of Verniers in Surveying Instruments.—Before using a particular vernier for the first time, it must be examined to ascertain

(1) Whether it is direct or retrograde

This is at once determined by observing whether the number of main scale spaces subtended by the vernier is one less or one more than the number of vernier spaces The great majority of verniers used in surveying instruments are direct

(2) Its least count

The value of the smallest division on the main scale and the number of spaces on the vernier scale must be noted. By the latter is meant the number intercepted between the zero and the last numbered mark, as sometimes one or two divisions are engraved beyond the end of the scale proper. The last numeral may not indicate the number of spaces, as it does in Fig. 34.

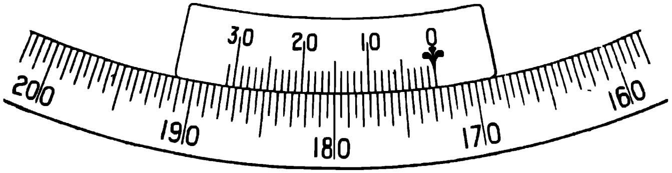


Fig. 34.—Vernier Reading to 1 Minute.

In Fig 34 a theodolite circle is shown divided to half degrees. The vernier contains 30 spaces, and these occupy a length of 29 circle divisions, or 14° 30′. The vernier is therefore direct, and reads to $\frac{1}{30}$th of half a degree, *i.e.* to one minute The index is situated between 172° 30′ and 173°, so that the approximate reading is 172° 30′, and, since the 14th graduation of the vernier is that in coincidence, the complete reading is 172° 44′.

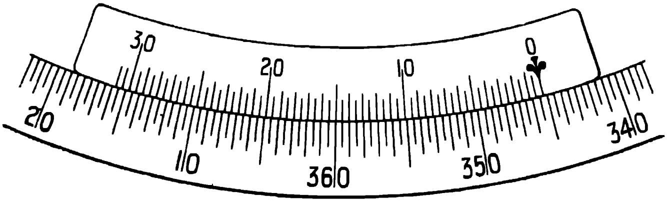


Fig. 35.—Vernier Reading to 30 Seconds

The circle of Fig 35 is similarly divided, but in this case there are 60 vernier spaces with a total length of 59 circle divisions. The vernier is therefore direct, and reads to $\frac{1}{60}$th of half a degree, or 30″. The approximate reading is 345°, and the vernier reading 14′ 30″, giving a total of 345° 14′ 30″. It will be observed that the longer graduations and the figures on the vernier scale represent whole minutes.

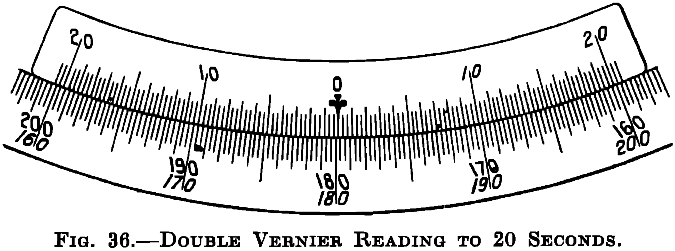


Fig. 36.—Double Vernier Reading to 20 Seconds.

In Fig 36, the circle is figured in both directions, and the accompanying vernier is of the double type. The 60 spaces on either half of the vernier are equivalent to 59 circle divisions, so that the vernier is direct, and consequently that half of it in advance of the index, in the direction of increase of the set of circle figures being used, falls to be read. The least count is 20″. If the measurement is being made clockwise, the figures which slope to the left are read, and the approximate reading is 179° 40′, which with the vernier reading of 11′ 20″ gives a total of 179° 51′ 20″. In the counter-clockwise direction, the reading is 180° 8′ 40″.

Special Forms of Vernier.—An extended vernier is one in which n vernier spaces occupy the same length as $(2n—1)$ main scale spaces. With the previous notation (page 35),

$$nv = (2n - 1)d,$$

$$v = \left(2 - \frac{1}{n}\right)d,$$

and in this case the difference between two main scale spaces and one vernier space

$$= (2d - v) = \frac{d}{n} = \text{the least count.}$$

The extended vernier is therefore equivalent to a simple direct vernier of which only every second graduation is engraved, the object being to afford a small least count without having the vernier graduations too close together for convenience in reading The extended vernier is regularly employed in the astronomical sextant. In the commonest form the vernier has 60 spaces equivalent to 119 arc spaces, each of 10′, and the least count is 10″. This pattern is illustrated in Fig. 37, in which the approximate reading is 29° 50′, and the vernier reading 4′ 10″, giving a total of 29° 54′ 10″.

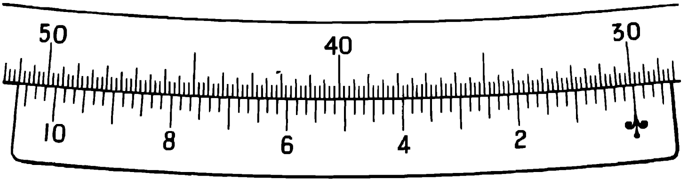


Fig 37 —Extended Vernier

A form of vernier sometimes used in compasses has the zero in the middle of the length. It is known as a folded vernier, and may be either direct or retrograde. In the case of a direct folded vernier dividing half degrees

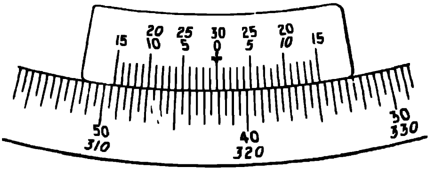


Fig. 38.—Double Folded Vernier.

to minutes and reading from left to right, the values of the fifth divisions are marked 15, 20, 25, 0, 5, 10, and 15 minutes. When the main scale reads both ways a double folded vernier has the advantage over an ordinary double vernier in occupying less room Fig 38 shows such a vernier, the readings being 42° 7′ clockwise and 317° 53′ anti-clockwise

Reading the Vernier.—In obtaining the vernier reading to apply to the approximate reading, there may be a little difficulty in judging where the coincidence takes place It will be found that the time spent in searching for the coincidence may be reduced by making a rough estimate, from the position of the index, of the vernier reading to be expected If the graduation is somewhat open, none of the vernier marks may appear to coincide with a scale graduation, but two successive marks may seem to be about equally off exact coincidence The mean value of those two vernier graduations will be more nearly correct than either On the other hand, with close graduation, there may appear to be more than one coincidence If two or more vernier marks are seemingly coincident with scale marks, their mean is to be taken In any case it is always advisable, before deciding upon the reading, to examine a few marks on either side of that which at the first glance is thought to be in coincidence.

A reading glass is a necessity except when the graduation is open and the lines are bold The accuracy and ease of reading are increased if the silvered surfaces upon which the scales are cut are shaded from brilliant side illumination, which, since it is reflected brightly from one side of the engraved marks, makes accurate reading somewhat troublesome Reflectors of white celluloid or opal glass provide a soft and satisfactory illumination

Mistakes in Reading Verniers.—Of the various mistakes made by beginners in using a vernier, two are much more common than others, and, since they refer to the reading of the main scale itself, lead to serious error

(1) Assuming a wrong value for the approximate reading, particularly by the omission to record one or more subsidiary scale divisions

Thus, in the case already cited of a circle divided to third parts of a degree, when the index lies between a 20′ and a 40′ mark, the approximate reading is $(n^\circ + 20')$, and if it lies beyond the 40′ mark, the reading is $(n^\circ + 40')$, because the vernier subdivides the smallest main scale divisions only.

(2) Reading the main scale in the wrong direction

If the main scale is graduated in one direction only, the reading must be made in that direction irrespective of that in which the vernier has been moved. If the direction of motion of the vernier is opposite to that of the scale graduation, the difference of the readings before and after movement represents the required measurement Thus, if the vernier of Fig. 34 has been brought to the position illustrated by a counter-clockwise rotation, say, from the 180° graduation, the mistake might be made of recording the angle as 7° 14′, instead of (180°—172° 44′) Special care must be exercised in reading double verniers

THE MAGNETIC NEEDLE

The magnetic needle is an essential feature of the various forms of compass used by the surveyor, and is a convenient adjunct of several instruments, notably the theodolite and the plane table It consists of a piece of magnetised cast steel supported at its centre on a sharp, hard steel pivot In obedience to the directive influence of the earth's magnetism, such a magnet floats in a definite position, its longitudinal axis lying in the plane of the magnetic meridian and exhibiting the direction of magnetic north and south This direction is not usually coincident with that of the true or geographical meridian, or true north and south, the difference being explained on page 159.

A second effect of the earth's magnetism is that a symmetrical floating magnet will not usually rest in a horizontal position, but will be inclined towards the nearer of the earth's magnetic poles This deviation from the horizontal is termed the dip of the needle Its amount varies from 0° at the magnetic equator to 90° at the magnetic poles, but does not remain constant at any place At present, 1931, at Greenwich, the north end of the needle dips downwards at 66° 55′ to the horizontal. To balance the needle, it is weighted on one side by the maker This is best done by means of a sliding weight, or rider, which, for use in different parts of the world, can be adjusted to counteract the dip obtaining in the locality

Construction.—The commonest types are (*a*) the broad needle (Fig 87), (*b*) the edge-bar needle (Fig 83); but tubular forms are also used In each case the centre is fitted with an aluminium cell containing a hollow conical bearing of agate or other hard mineral to receive the fine point of the support pivot, which should be in the same horizontal plane as the ends of the needle Extreme fineness of the supporting point is necessary to prevent the possibility of the needle coming to rest other than in the magnetic meridian, and hardness of both bearing surfaces is therefore essential to minimise wear with consequent friction and sluggishness of the needle. To prevent unnecessary wear, the needle should never be allowed to bear on the pivot except when in use Needles of surveying instruments are always provided with means for lifting them from the pivot: the device may take the form of a lever or a cam

To observe directions with reference to magnetic meridian, the needle is either read against the graduations of a compass box or it may itself carry a graduated card or ring to be read against an index. The former arrangement is applied in the surveying compass and the theodolite, an edge-bar needle being used. The box may be circular with a completely graduated circle, so arranged that by reading the position of the north end of the needle, the magnetic direction of any line can be obtained. In the trough compass the needle is mounted in a narrow prismatic box (Fig 59) or is enclosed in a tube The trough compass is a very useful form for the establishment of magnetic meridian, directions with reference to magnetic meridian being measured by the instrument to which the compass is attached The broad needle, usually of uniform breadth, is employed when a graduated card or ring has to be carried, as in the prismatic compass (Fig 87) and the mariner's compass Aluminium or cardboard is used to minimise the weight on the pivot

Requirements of the Magnetic Needle.—The principal requirements of the needle are :

1. It should be sensitive.
2. The magnetic axis should coincide with the geometrical axis
3 The ends should lie in the same horizontal and vertical planes with the pivot point (page 98).
4. The centre of gravity should for stability be as far below the pivot point as possible.
5. The compass box should be entirely non-magnetic.

Sensitiveness.—If a needle, on being lowered on its pivot, comes to rest quickly, the sluggishness may arise from (*a*) loss of polarity, (*b*) wear of pivot The latter is the more common defect.

To re-magnetise a needle, it is best to place it in the magnetic field of a dynamo, taking care not to reverse the poles. If this method is impracticable, procure a bar magnet, and draw the N. end of the magnet several times from the centre of the needle towards its S end, using an axial stroke Repeat with the S end of the magnet on the N. side of the needle

To sharpen the pivot, use a piece of very fine oil-stone, and rub the point all round until it will adhere to the finger nail at the slightest touch This test for sharpness should be frequently repeated during grinding, as over-grinding reduces the height of the pivot appreciably, and lowers the needle relatively to the graduations of the compass box

Magnetic Axis.—The axis of magnetism may not coincide with the geometrical axis of the needle, and, as it is the former which lies in the magnetic meridian, a needle defective in this respect will not record magnetic bearings correctly. The error, however, is of constant amount, and therefore does not influence the values of the included angles of a compass survey.

Non-magnetism of Compass Box.—Cases are not infrequent of the faulty performance of a compass being due to the presence of iron in the alloys used in the manufacture of the instrument. The defect may be tested for in instruments having a horizontal circle and vernier by setting off successive angles of, say, 5° right round the circle and reading the position of the needle on the compass circle each time. Irregularity of the intervals between the needle readings indicates that a source of attraction is present.

THE TRIPOD STAND

The most important requirement of a tripod is rigidity or steadiness, but lightness and portability are also to be desired. Although varying in detail, tripods may be classed as solid leg or split leg. The former is the more convenient to carry, as the legs, when brought together, form a cylinder, and mainly for this reason it is the more common type, particularly for small instruments The split leg or framed tripod possesses the advantages of superior rigidity and lightness, and is to be preferred for precise work.

The wood of which the legs are made is commonly mahogany, but ash, yellow pine, and cedar are also utilised. The lower ends of the legs are fitted with pointed steel shoes so that they may be thrust into the ground Some makers provide projections near the bottom by which the user can press down the legs with his foot. At the top the legs are screwed to a casting which forms the immediate support for the instrument. The connection of the legs to this piece is the weak point in many tripods, the rapid development of looseness in the screws necessitating their frequent tightening If the line of sight of the telescope is directed towards a fixed mark, the quality of the tripod as regards rigidity may be tested by grasping one of the legs and giving it a twist The intersection of the cross-hairs will appear to leave the mark, but, on removal of the twist, should return to the point if the tripod is stable Accurate work is impossible with a tripod which has developed shakiness at the head or in the shoes.

CHAIN SURVEYING INSTRUMENTS

Chain.—The chain (Fig. 39) is formed of 100 pieces of straight wire, the ends of which are bent to connect to each other through small oval rings, which for flexibility should number three at each joint. The wire is of iron or steel of from 8 to 12 gauge (·16 to ·10 in), the use of steel enabling weight to be reduced without sacrificing strength. The ends of the chain consist of brass handles, each with a swivel joint to eliminate twist.

Chains used in English-speaking countries are of two lengths, the 100-ft. chain divided into feet, and the 66-ft. or Gunter's chain, which measures one *chain* and is divided into 100 *links*. The length of a chain is the total length from outside to outside of the handles. At every tenth foot or link is attached a distinctive tag or tally of brass of the patterns shown in Fig 39 As each tag represents its distance from both ends of the chain, either handle can be regarded as the zero, so that a little care is necessary to avoid misreading. In taking readings between tags, one must count the number of feet or links from the previous tag, estimating fractions of a unit if necessary Time is saved if the intermediate fifth divisions are distinguished by a small oval or circular tally, as inserted by some makers

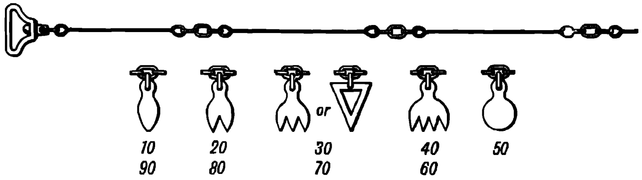


FIG 39 —CHAIN

In countries in which the metre is the standard unit of length, chains of 20 and 25 metres are employed, the former being the more generally used It is divided into 100 parts of 2 decimetres with a tally at every two metres

Steel Band.—The steel band or band chain consists of a ribbon of steel with brass handles attached by swivel joints to the ends (Fig 40) It is obtainable in lengths of from 50 to 1000 ft, with widths ranging from $\frac{1}{16}$ in to $\frac{3}{4}$ in, and thicknesses from ·01 to ·03 in The effective length may include the handles, but greater accuracy is possible if the ends are marked on the tape itself The steel band is wound either upon an open cruciform frame or on a reel in a case as shown.

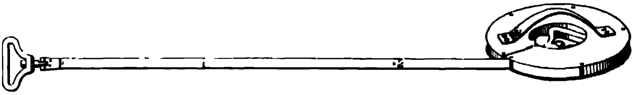


FIG 40 —STEEL BAND

The marking of the graduations is performed in various ways—by etching with acid to leave marks and figures in relief, by brass rivets, by brass sleeves soldered on, or by stamping attached pieces of solder Etching and riveting have a weakening effect, but the former is necessary when each foot or link is figured and subdivided In the other systems only the foot or link at each end is subdivided, decimally in the case of the link, and decimally or duodecimally for the foot

Comparative Utility of the Foot and the Link as a Unit.—The foot is the unit employed in all structural work, so that surveys made in connection with construction are usually executed with the 100-ft chain or band, which in consequence is sometimes referred to as the engineer's chain For purely land surveys, on the other hand, Gunter's chain is preferred owing to the facility with which areas may be computed from the measurements, since the unit of land measurement, the imperial acre, = 10 sq chains = 100,000 sq links Gunter's chain has the further advantage of subdividing the mile exactly On this account it is sometimes used by engineers in this country in the pegging out of railway centre lines By virtue of its shorter length, it can be more rapidly manipulated than the 100-ft chain, but, as it has to be put down so much oftener, the measurement proceeds more slowly

Relative Merits of the Chain and the Steel Band.—The principal comparative features of the chain and band may be summarised as follows

Advantages of the Chain —(1) It can withstand rather rough treatment

(2) If broken, it is easily repaired temporarily with wire or even with string

(3) It is easily read.

Disadvantages —(1) It is very subject to alteration in length. As it has some 800 wearing surfaces, continued use increases the length In iron chains especially, the application of excessive pull may cause stretching of the small rings by flattening and, unless brazed, by opening out Causes tending to shorten it are bending of the links and the adhesion of mud between bearing surfaces.

(2) It is comparatively heavy, and therefore takes some time to lay straight on the ground, while it sags considerably when suspended.

(3) It is liable to become entangled in shrubbery and to collect mud, etc.

Advantages of the Steel Band —(1) It maintains its length very much better than the chain, the stretching with prolonged use being negligible for ordinary work

(2) For the same strength it is lighter than the chain.

(3) Both foot and link graduations may be furnished on opposite sides

Disadvantages —(1) If carelessly used, it is liable to be broken, especially by pulling it when twisted into loops or by stepping upon it.

(2) It can be repaired only by riveting or soldering, and this cannot be done in the field unless a repairing outfit is carried.

(3) It is less easily read than the chain.

(4) It must be kept free from rust. If the graduations are etched, special care must be exercised to preserve their legibility.

The steel band yields so very much better results than the chain that its employment is a necessity in such operations as city surveying and setting out works The chain is, however, still used to a large extent for ordinary small surveys, principally on account of its durability.

Testing and Adjustment of the Chain.—The length of a chain must be checked at frequent intervals It may be compared with : (*a*) an official standard ; (*b*) a good steel band or tape , (*c*) a chain of known length, reserved for the purpose , (*d*) a levelling staff laid down successively Official standards are established in several large towns as the distance between marks on metal plugs, but the surveyor may with a steel tape set out a private standard by marks on a corridor or pavement In extensive surveys a field standard should be established as the distance between tacks driven into two stout pegs fixed in the ground, a similar point being fixed midway

Before making the comparison, the chain should have all bent links straightened and the bearing surfaces freed from mud On stretching it with a moderate pull, it will probably be found too long. To save the trouble of computing corrections from the known error and applying them to measured lengths, the end of the nominal length may be marked on one handle, but it is better to adjust the length by removing one or more connecting rings In doing so, an endeavour should be made to keep the 50 tally midway and the others as nearly right as possible. Some chains are provided with means for adjustment at the handles.

Arrows.—For marking the ends of chain lengths and recording the number of times the chain is laid down in measuring a line, a set of marking pins or arrows, generally ten in number, is used These are formed of iron or steel wire, preferably a little heavier than that of the chain, and are pointed at one end for thrusting into the ground, while the other end is bent to form a ring for facility in carrying The usual lengths are 12 and 15 in , but 18-in. arrows may advantageously be used in long grass. Arrows should have a small strip of red bunting tied to the ring to make them conspicuous on the ground.

Linen Tape.—In ordinary surveying, short measurements are made by means of a linen tape, which consists of a painted and varnished strip of woven linen usually $\frac{5}{8}$ in. wide. It is attached to a spindle in a leather case, into which it is wound when not in use (Fig 41). The usual lengths are 33, 50, 66, and 100 ft., the 66-ft. tape being the most serviceable on the whole. The graduation is in links on one side and feet and inches on the other. In place of figuring each inch, the marking shown in Fig 41 is less liable to lead to mistakes in reading, and is preferred for surveying tapes.

Linen tapes should not be used where precise results are required, as they are subject to serious variations in length. They stretch when pulled, and may easily be permanently elongated, since a considerable pull is required to straighten them in wind. Exposure to wet causes them to shrink. Linen tapes are not very durable, and in time become frayed and illegible, particularly towards the zero end. Short end lengths, complete with ring, are obtainable, and a worn end portion may be replaced by a new length sewed on in the proper position A complete refill tape may be mounted by unscrewing the spindle of the box and attaching the end of the tape to it, either by sewing or by insertion in a slit for the purpose If treated with care, a tape box will serve for several refills

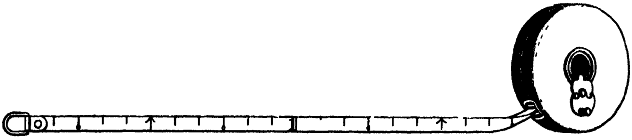


FIG 41 —LINEN TAPE

Metallic Tape.—The so-called metallic tape is a linen tape into which brass or copper wires have been woven for the purpose of promoting constancy of length. The device is only partially successful.

Steel Tape.—Far superior to the others is the steel tape, which is of lighter section than the steel band and is mounted similarly to the linen tape. The graduations, links on one side, and feet, inches, and eighths on the other, are marked by etching, and the tape must be kept well oiled and clean to preserve their legibility. The precautions against breaking apply as in the band

Ranging Poles.—For the conspicuous marking of points and the ranging of lines, wooden poles or pickets are used They are generally made of red pine, but ash, lancewood, and bamboo are also used. They are of circular or octagonal cross section, tapering towards the top, and are fitted with a strong iron or steel shoe. The more commonly used lengths are 6 ft and 10 links. To render them as conspicuous as possible they are painted black and white, or red and white, in alternate bands, or red, white, and black successively. By having the painted divisions 1 ft or 1 link long, a ranging pole can be used as a measuring rod. Poles which have to be observed from a long distance should have a small flag attached.

Offset Rod.—An offset rod is simply a ranging pole, similarly shod and painted, which is primarily intended for the measurement of short distances. The usual length is 10 links. It may be jointed in the middle, and it is fitted with a recessed hook for pulling or pushing the chain through a hedge or other obstruction.

Line Ranger.—The line ranger is a small instrument whereby intermediate points can be established in line with two distant signals without the necessity of sighting from one of them It consists essentially of two reflecting surfaces, either small plane mirrors or square prisms, as in Fig 42, one above the other, and with their reflecting surfaces normal to each other

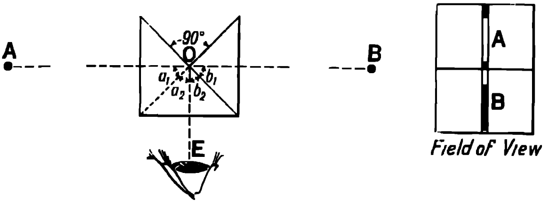


FIG 42 — LINE RANGER

In locating an intermediate point in line with the poles A and B, the observer stands approximately in line and places the instrument on a rod, or holds it at the level of the eye, turning it until the image of one of the poles, A, is seen in the field of view He then moves backwards or forwards at right angles to the line until the image of B appears A point in the line AB is reached when the images of A and B lie in the same vertical line The reflected rays from both A and B are then situated in the same vertical plane OE The direction of OE depends upon the position of the eye, and is not necessarily normal to AB, but, by the laws of reflection, $a_1 = a_2$ and $b_1 = b_2$, and, since $(a_2+b_2) = 90°$, we have $(a_1+a_2+b_1+b_2) = 180°$ AOB is therefore one straight line, and O can be transferred to the ground by pole

One of the mirrors or prisms is commonly made adjustable for securing the necessary perpendicularity between the reflecting surfaces To perform the adjustment, three poles are ranged by theodolite, and the line ranger is held over the middle one If the images of the others do not lie in the same vertical line, they are made to do so by turning the adjusting screw.

Cross Staff.—The cross staff is the simplest instrument used for setting out right angles It is mounted on a pole shod for fixing in the ground, and is arranged in the form of a frame or box furnished with two pairs of vertical slits yielding two lines of sight mutually at right angles Figs 43 and 44 illustrate two common types. The open form has usually a longer base between the slits than the octagonal box pattern. The latter has additional openings, so that angles of 45° may also be set out. In another form of the instrument, the head consists of two cylinders of equal diameter placed one on top of the other. Both are provided with sighting slits. The upper cylinder carries an index or vernier, and can be rotated relatively to the lower, to which is attached a graduated circle. This arrangement enables any angle to be set out or measured roughly.

To set out a perpendicular to a line from a point on it, it is only necessary to fix the staff in the ground at the given point and bring one pair of slits in range with the poles marking the line. The slits at right angles then define a perpendicular line of sight, in which poles or marks may be set To drop a perpendicular to a line from a point outside it, the surveyor, holding the staff on the line, must proceed by trial and error until a point on the line is obtained at which the poles in the line and the point outside it are simultaneously in the two lines of sight of the instrument.

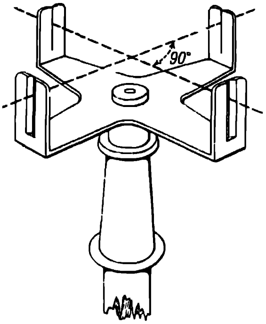


Fig 43 —Cross Staff

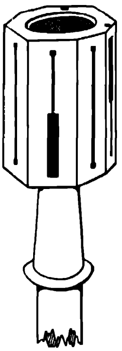


Fig 44.—Cross Staff

The cross staff is non-adjustable, and is not susceptible of high accuracy, but is useful for setting out long offsets

The Optical Square.—This is a compact hand instrument for setting out right angles, and is capable of greater accuracy than the cross staff. It is based on the principle that a ray of light reflected successively from two surfaces undergoes a deviation of twice the angle between the reflecting surfaces In the instrument these surfaces are placed exactly at 45° to each other, and belong to small mirrors mounted in a circular box or on an open frame, or, alternatively, they form two sides of a prism, the instrument in this case being sometimes distinguished as a prism square. The latter is the more modern and better form of the instrument.

Fig. 45 shows a plan of the essential features of the box form, the top cover being removed The periphery is formed of two cylinders, of about 2 in. diameter and about ½ in. deep, the one capable of sliding on the other, so that, when not in use, the eye and object openings can be closed to protect the mirrors from dust. Mirror A is silvered over half the depth only, and the other half is of plain glass. Mirror B is completely silvered. To an eye placed at E objects such as C are visible through the transparent half of A, and at the same time objects in the direction of D are seen in the silvered part after reflection at B. If the object D is so placed that its image appears directly above (or below) C, so that the eye receives the rays from D in the same vertical plane AE as those from C, then C and D subtend a right angle at the instrument. For the angle G formed by the reflecting surfaces is made 45°, and if EAF be denoted by a, then BAG $= a$ by the laws of reflection of light, and the values of the remaining marked angles are readily deduced, giving CHD a constant value of 90°

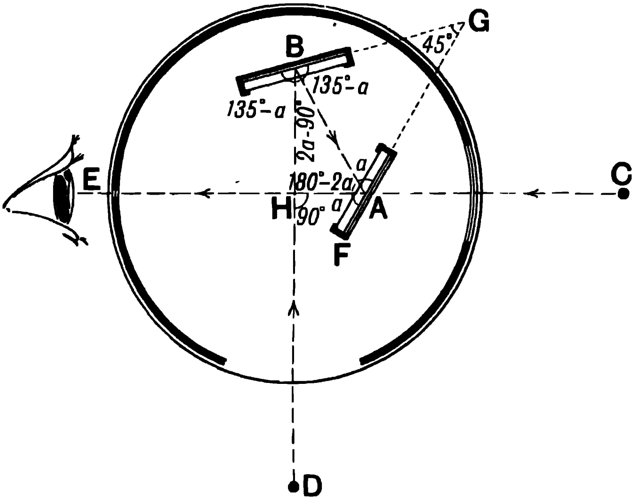


FIG 45 —OPTICAL SQUARE

Use of Optical Square.—To set out a perpendicular to a line EC from a point on it, the observer holds the instrument over the point, preferably resting it against a ranging pole, and turns it until the pole C, marking the line, is seen through the clear glass A chainman, having been sent out with a pole in the direction of D, is then directed to right or left until the reflected image of his pole appears coincident with the line pole.

To find where a perpendicular from a given point D would meet the line EC, the surveyor, holding the instrument to his eye, walks along the line until he obtains the coincidence as before, when the instrument will be over the required point.

Notes —(1) It is always advisable to have two forward poles marking the survey line. By keeping them in range, the surveyor can maintain the instrument in the line without trouble.

(2) When the perpendicular lies to the other side of the survey line, the instrument is held upside down.

Testing and Adjusting of Optical Square.—The instrument is often made with both mirrors permanently fixed, but some makers mount mirror B (Fig. 45) so as to permit adjustment of the angle between the mirrors. An adjusting key is fitted in the box, and is readily available when required. The test and adjustment are as follows.

Object.—To place the mirrors at 45° to each other, so that the angles set out shall be right angles.

Test.—1. Range three poles A, B, and C in line, preferably at least 300 ft. apart

2. With the instrument at B, sight A, and set out a right angle ABD.

3 From the same point sight C, and if the pole at D appears in coincidence with C, the instrument is correct.

Adjustment—1 If not, mark opposite D the point D′ which appears coincident with C, and erect a pole E midway between D and D′ to mark the true perpendicular BE

2. By means of the key, turn the screw controlling the adjustable mirror until the image of E is made coincident with C

3 On again sighting A, E should appear in coincidence · if not, repeat the test and adjustment until the error is eliminated

Note.—The above test is a straightforward case of reversal, and corresponds exactly to the testing of a set-square (page 7)

The Prism Square.—The same principle applies to the prismatic form of the instrument, shown diagrammatically in Fig 46, and it is used in the same manner. The prism square has the merit that no adjustment is required, since the angle between the reflecting surfaces cannot vary. The deviation of this angle from 45°, due to errors of workmanship, may be regarded as negligible for the purposes of the instrument.

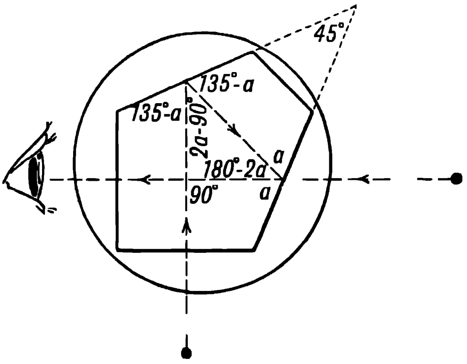


FIG. 46 —PRISM SQUARE.

The Box Sextant.—The sextant, of which the box sextant is the most compact form, is a reflecting instrument capable of measuring angles up to about 120°. Although the box sextant is not strictly a chain surveying instrument, it is commonly included in the equipment in place of an optical square, and may appropriately be dealt with here. The principle that the deviation of a ray of light reflected successively from two mirrors is twice the angle between them is applied as in the optical square, except that in this case one of the mirrors, called the index glass, is mounted on an axis about which it can be rotated. The variable angle between the mirrors is set off on the instrument, which therefore measures that subtended by the objects.

The box has a diameter of about 3 in. and a depth of about 1½ in., and is provided with a cover, which is removed and screwed on underneath to form a handle when the instrument is in use. Fig. 47 represents an interior plan of the instrument. The fixed mirror, or horizon glass, is silvered on the top half only, and the index glass is silvered all over. Attached to the latter is a toothed segment gearing with a small pinion which is actuated by the milled head *3* on the top of the box, shown in the exterior plan (Fig. 48).

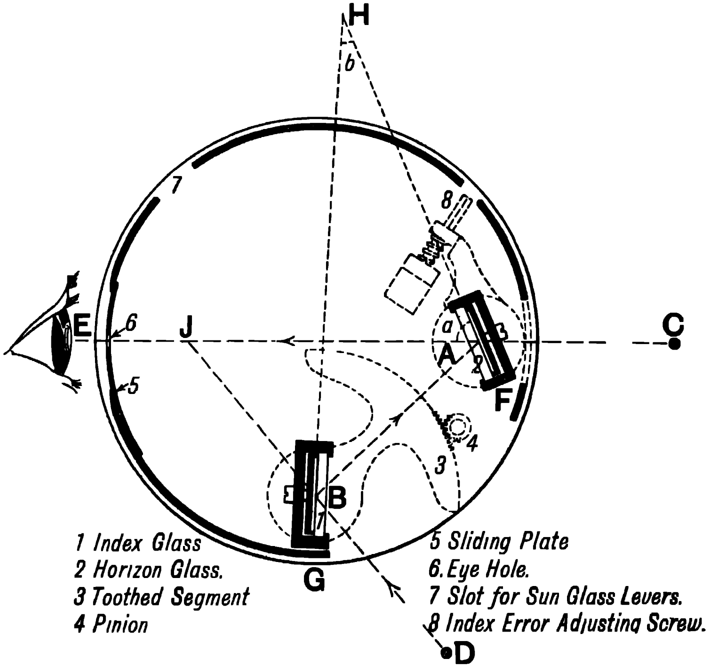
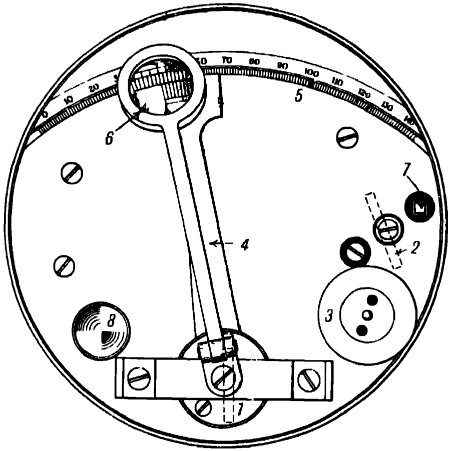


FIG. 47.—INTERIOR PLAN OF BOX SEXTANT

The axis of the index glass carries an index arm with a vernier, which on rotation of the mirror is carried over the graduated arc. The graduation is carried to half degrees, subdivided to single minutes by the vernier, which is viewed through the hinged reading glass.

The observations are usually made by sighting through an eyehole. A small telescope is provided for long-distance sighting, and this is attached either by inserting it in the circular opening exposed on pushing back the sliding plate or by means of an external bracket screwed on when required. The telescope has a dark glass for use in the solar observations required in adjusting the instrument, and, to intercept the sun's rays reflected from the index mirror, dark glasses are mounted in the box on hinges so that they may be lowered out of the way for terrestrial observations.



1 *Index Glass*
2 *Horizon Glass*
3 *Milled Head actuating Index Glass*
4 *Index Arm*
5 *Arc*
6 *Reading Glass*
7 *Adjusting Screw for perpendicularity of Horizon Glass*
8 *Adjusting Key*

FIG 48.—EXTERIOR PLAN OF BOX SEXTANT.

The theory of the sextant is as follows (Fig 47).

Let a = the angle EAH between the horizon glass and the line of sight EC,

b = the angle AHB between the mirrors when the image of a pole D appears in line with pole C

By the laws of reflection, BAF $= a$, and $\therefore$ EAB $= (180° - 2a)$

But BAF is an exterior angle of triangle ABH,

$\therefore$ ABH $= (a - b) =$ GBD, so that ABD $= 180° - 2(a - b)$.

But ABD is an exterior angle of triangle ABJ, whence

AJB, the angle between the signals, $= 180° - 2(a - b) - (180° - 2a)$

$= 2b =$ twice that between the mirrors.

The point J is not fixed in position, and, in general, the angle observed between the objects is not the angle subtended by them at the centre of the box. The difference, however, is negligibly small, unless the objects sighted are very near the instrument.

To avoid the necessity of doubling the angle between the mirrors at each observation, the graduated arc has the angles figured twice their real values, so that the readings give the required measurements.

The sextant differs from other angular instruments in that it measures the actual angle between objects of different elevations instead of its horizontal projection As the latter is required in surveying, it may be necessary, when the levels of the two signals differ considerably from each other, to reduce the observed angle to its horizontal equivalent Thus, in Fig 49, the angle measured at A between the objects B and C is BAC $=\phi$. If B′AC′ is the horizontal plane through A, and BB′ and CC′ are verticals, then B′AC′ $=\theta$ is the required angle.

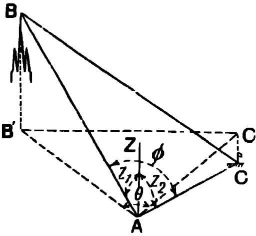


FIG 49

Denoting the inclinations of the lines of sight to the vertical AZ at A by z_1 and z_2 respectively, then

$$\cos\theta = \frac{\cos\phi - \cos z_1 \cos z_2}{\sin z_1 \sin z_2}$$

Observing with the Box Sextant.—The instrument is rested against the station pole, and one hand is left free to turn the milled head *3* (Fig 48). To measure the angle subtended by two distant signals, the instrument is turned in the hand until one of them is visible through the transparent part of the horizon glass, and screw *3* is then slowly rotated until the image of the second signal is brought in line with the first Unless the signals are at the same level as the instrument, the latter must be tilted to lie in their plane. The angle is given by the vernier index

In setting out an angle, the index must first be set to read its given value. Keeping the line of sight, through the clear glass, on the given signal, the surveyor directs a chainman into the position at which a pole held by him appears coincident with the signal

Notes—(1) In measuring an angle, an inexperienced observer may find difficulty in picking up the point to be sighted by reflection This frequently arises through inattention to the holding of the instrument in the correct plane, and it is well to keep tilting it up and down as the index glass is being turned. Time is saved if the value of the angle is first estimated and the index set roughly to this reading

(2) When the signals are not very clear, the more distinct of the two should be that sighted by reflection When the left hand object is thus sighted, it is necessary to hold the instrument upside down.

(3) The reflected image loses in distinctness when the angle between the objects exceeds 110°, and the measurement of angles greater than about 120° should be made in two parts by erecting a temporary pole in an intermediate position or by using a well-defined permanent point in the same way.

Testing and Adjustment of the Box Sextant.—The requirements of the astronomical sextant (Vol. II, Chap. II) are applicable to the box sextant, but in the latter instrument both the index glass and the telescope are mounted without provision for their adjustment. The horizon glass has the two movements required for setting it perpendicular to the plane of the arc and for the elimination of index error.

First Adjustment.—*Object.*—To set the horizon glass perpendicular to the plane of the instrument, so that both mirrors may be perpendicular to the same plane

Test.—1. Set the vernier to zero.

2. Sight the sun or any terrestrial object, and observe whether there is any vertical gap or overlapping of the reflected image relatively to the object (*a*, Fig. 50).

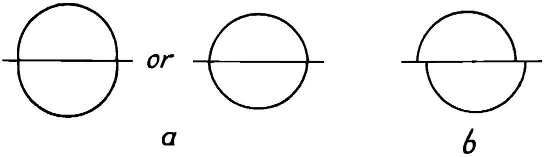


FIG 50.

Adjustment.—If so, by means of the key, kept at *8* (Fig 48) when not in use, turn screw *7* until the vertical displacement is eliminated.

Second Adjustment.—*Object*—To eliminate index error, so that, when the mirrors are parallel, the vernier will read zero

Test—1 Set the vernier accurately to zero

2. Sight the sun or any distant object furnishing a definite line, and observe whether the reflected image appears displaced laterally with respect to the object (*b*, Fig 50)

Adjustment.—If so, by means of the adjusting key turn the screw *8* (Fig. 47) until the lateral displacement is eliminated.

THE THEODOLITE

The theodolite is an instrument designed for the measurement of horizontal and vertical angles. It is the most precise instrument available for such observations, and is of wide applicability in surveying.

The general constructive arrangement will first be considered, beginning with the parts relating to horizontal measurements.

Parts for Horizontal Measurement.—The line of sight is telescopic, and, in observing the horizontal angle subtended at the instrument by two objects, the telescope is directed first to one and then to the other. In order that the angle may be recorded, the instrument must evidently be furnished with :

(1) A horizontal graduated circle.

(2) A vertical axis passing through the centre of the circle and about which the telescope may be turned from the direction of one object to that of the other.

(3) A horizontal axis about which the telescope may be moved in a vertical plane, since the points sighted may be at different elevations.

(4) An index, connected with the telescope, and placed against the graduations of the circle, so that the horizontal angular movements of the telescope may be exhibited by the change of reading of the index on the fixed circle (The possible alternative of attaching the circle to the vertical axis of the telescope and observing readings against a fixed index is not adopted)

Perhaps less obvious at the outset is the necessity for adding the following features .

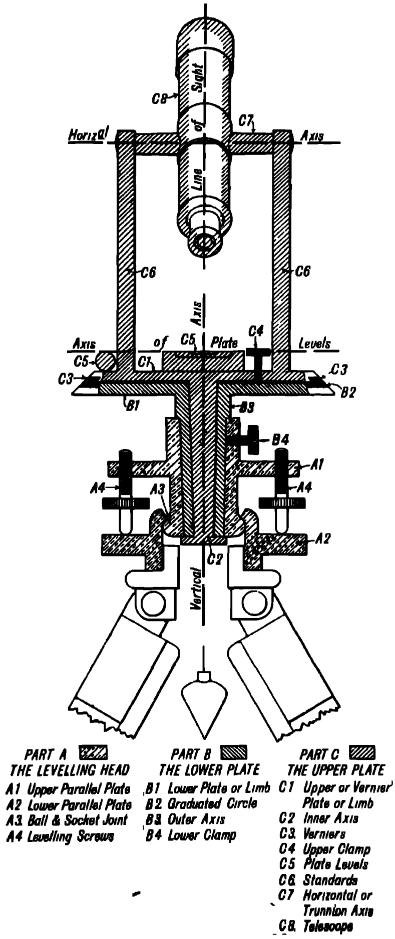


FIG. 51.—DIAGRAMMATIC SECTION OF THEODOLITE.

(5) A clamping device for holding the index at a given reading, and some form of slow motion, or tangent, screw to facilitate adjusting it exactly to that reading

(6) Mounting of the graduated circle so that it may, when required, be rotated about the vertical axis of the instrument, as it is commonly necessary that the graduations should lie in particular directions from the centre As the circle must be stationary during the measurement of an angle, it must be fitted with a clamping arrangement and also with a slow motion screw for its exact orientation

The manner in which these features are embodied in the theodolite is illustrated diagrammatically in Fig 51, which indicates an instrument provided with four levelling screws . the slow motion screws are not shown. The index forms the zero of a vernier, of which there are usually two The provision of an inner and an outer axis, coned to eliminate looseness due to wear, is necessary to meet the requirements of paragraphs 5 and 6 above. Keeping in view the fact that *A* is prevented from rotating by its connection to the tripod and by the grip of the levelling screws, the effects of tightening one or both clamps can be followed from the diagram. When the lower clamp is tightened, *B* is fixed, but, if the upper clamp is slack, *C* can be rotated because the inner axis is free If now the upper clamp is also tightened, no part can be rotated horizontally, but, by releasing the lower clamp, while the upper is tight, *B* and *C* can be rotated as one upon the outer axis By actuating the appropriate tangent screw, however, a piece which is clamped may be moved through a small angle Thus, if both clamps are tight, turning the lower tangent screw will have the effect of moving *B* and *C* together relatively to the levelling head In consequence, the line of sight may be adjusted exactly on an object without altering the vernier reading The upper tangent screw controls part *C* only, and using it to direct the line of sight on to a signal necessarily changes the position of the index on the fixed circle.

Parts for Vertical Measurement.—The observation of vertical angles differs from that of horizontal angles in that the former are always measured from a fixed reference plane, *viz* the horizontal plane through the horizontal axis of the telescope The further essentials of the instrument are therefore

(1) A vertical graduated circle, or, since very large vertical angles have rarely to be measured, an arc or arcs in place of a complete circle

(2) An index

(3) A clamp for fixing the circle and index together, and a slow motion screw for fine adjustment

(4) A sensitive spirit level for the definition of the horizontal plane

One arrangement of these parts is sketched in Fig 52 The relation between the vernier index and the circle is the reverse of that obtaining with the horizontal circle The vertical circle is rigidly connected with the telescope, and the verniers remain stationary during an observation. The two verniers are mounted upon a T-shaped frame attached to one standard by clip screws, by means of which adjustment may be effected when required. The level tube is either attached to the telescope or mounted on the vernier frame as

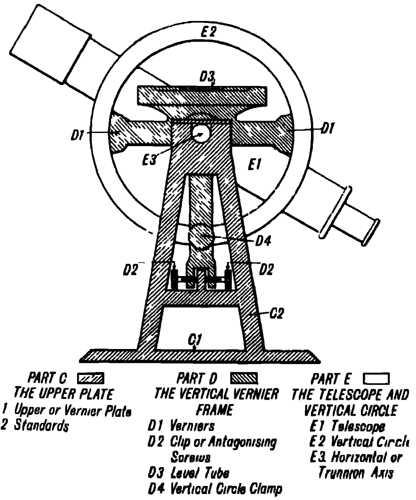


FIG. 52.

shown. some instruments have two tubes, one in each position. It will be seen that the effect of clamping is to fix *E* relatively to *D* and *C*, but a small motion of *E* can be obtained by actuating the tangent screw When the clamp is fixed, slackening one clip screw and tightening the other causes a movement of *D* and *E* together relatively to *C*.

Types of Theodolite.—The arrangement shown in Figs. 51 and 52 indicates that the telescope can transit, *i e.* make a complete rotation about its horizontal axis. The possibility of reversing the line of sight by rotation of the telescope constitutes the necessary and distinguishing feature of the *Transit Theodolite* or *Transit.* It is not essential that the rotation should be complete, and frequently the proportions of the telescope and standards are such that reversal can be effected only with the eyepiece end downward The standard British transit is furnished with a complete vertical circle, but this feature does not constitute the distinctive characteristic of the type A transit without means for measuring vertical angles is known as a *Railway Transit*

Non-transiting patterns comprise the obsolete *Wye* or *Plain* and *Everest* theodolites In these, the manner of mounting the telescope is such that the line of sight cannot be reversed by rotation of the telescope, but the effect of transiting is obtained by removing the telescope from its supports and turning it end for end. Although these instruments possess the merit of compactness, they are inferior in utility to the transit, which has entirely superseded them.

Transit Details.—Fig 53 shows the construction of an ordinary pattern of transit theodolite, of which important fittings are further described

Levelling Screws should be of fine pitch, $\frac{1}{32}$ in being suitable. Delicacy of action is also dependent upon the distance of the screws from the vertical axis, the greater the distance the smaller will be the tilt caused by one turn. Either three or four screws are used, and the construction of the levelling head differs in the two types.

A three-screw head is illustrated in Fig 53, the foot plate *4* having three arms which receive the lower ends of the levelling screws. The foot plate forms a separate part, which, in the assembling of the instrument, has to be screwed to the tripod head. The locking plate is provided with slots of keyhole shape, as in Fig. 54; by sliding it round sufficiently to bring the wider part of the slots opposite the recesses in the foot plate, the levelling screws may be entered. On sliding back the plate, and clamping it, the screws are clipped, but the connection is not very firm if the locking plate or the balls of the levelling screws have become worn. Greater rigidity is obtained by having the ball ends of the screws permanently fitted into the foot plate, as illustrated in Figs. 71 and 91, which also show the encasement of the threads of the screws as a protection against dust The screws operate in bushes fitted into, and extending above and below, the levelling head, and dust caps are screwed on to the upper ends of the bushes.

Four levelling screws necessitate the arrangement of parallel plates and ball and socket joint outlined in Fig. 51. As constructed, the four-screw instrument has the more compact levelling head, but this is its only merit. The use of four points of support, when three are sufficient for stability, leads to uneven distribution of pressure on, and excessive wear of, the screws, and is likely to strain both the axes and the screws. The three-screw instrument is free from these objections, and has the important advantage of being more rapidly levelled (page 78).

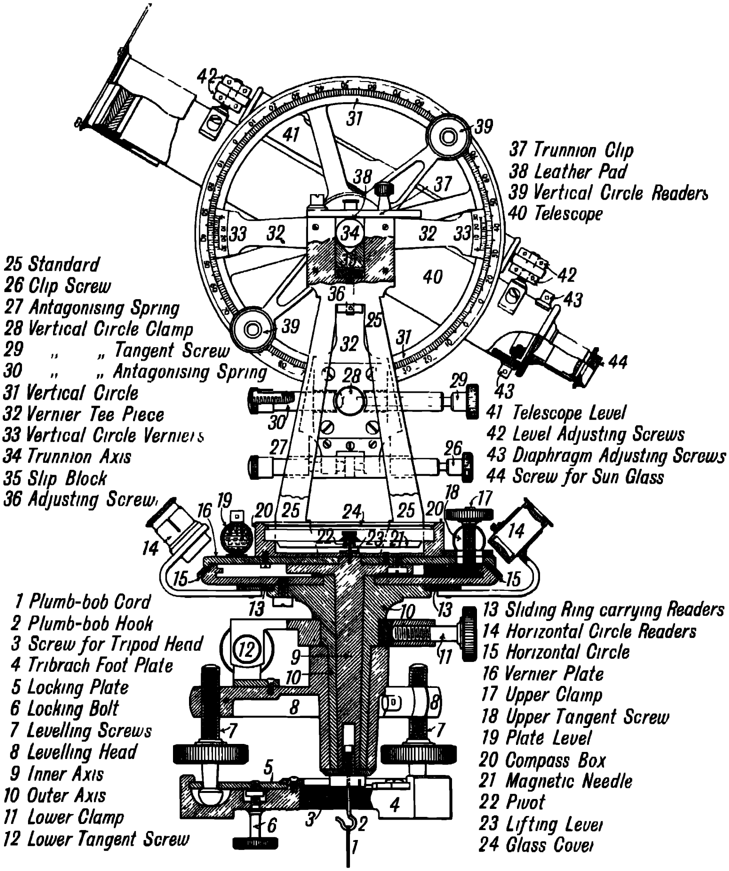


FIG 53.—TRANSIT THEODOLITE.

A *Plumb Bob* is suspended from the vertical axis to enable the instrument to be centered over the ground point from which the measurements are required The cord must have a slip knot or other means to permit the plummet to be adjusted to the proper level The operation of centering the instrument (page 77) is facilitated by the provision of a *Movable Head* or centering arrangement, whereby the instrument and attached plumb line may be moved independently of the tripod and clamped thereto when the plumb bob is over the mark.

Several patterns of movable head are in use Fig 54 shows a simple form The available movement consists of a straight line motion controlled by the pin *7* in the slot *8* formed in the bottom plate, which is screwed to the tripod, and a rotation about the pin limited by the projections on the underside of the arms of the tribrach plate

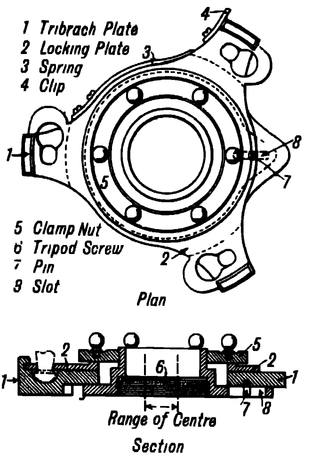


FIG 54 —MOVABLE HEAD.

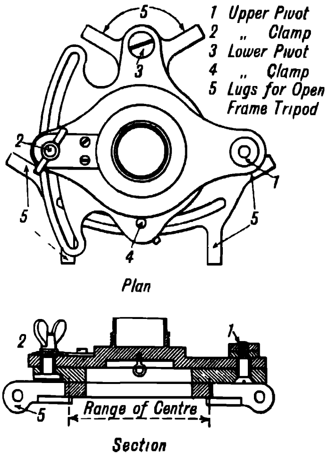


FIG 55 —TROUGHTON AND SIMMS' CENTERING ARRANGEMENT.

Fig 55 illustrates Messrs Troughton and Simms' centering arrangement, which permits a much greater range. It consists of two plates movable relatively to the tripod head and to each other The upper plate, carrying the instrument, can be rotated about pivot *1* and clamped to the lower plate, which is provided with a similar movement at right angles to the first.

An improvement is made in some instruments by introducing the centering arrangement just above the levelling screws instead of on the tripod head.

In circumstances requiring the instrument to be placed at some height above the station point, it is a difficult matter to perform the centering by means of a long plumb line. An optical plumbing device is fitted in some modern theodolites, and consists of a small telescope which can be attached horizontally to the levelling head, an axial, vertical line of sight being obtained by means of a prism Optical plummets may be adapted for sighting upwards, as well as downwards, so that in tunnel work the instrument may be centered below a mark in the roof.

Level Tubes —Two plate levels are provided for the levelling up of the instrument, and sometimes a small circular level is fitted to the foot plate to facilitate the setting up of the instrument Of the plate levels, one is placed at right angles, and the other parallel, to the line of sight, the latter being sometimes mounted on one of the standards The former should be the more sensitive, since it controls the horizontal axis and therefore the verticality of the plane of rotation of the line of sight The sensitiveness of both should depend upon the resolution of the telescope and the delicacy of the instrument generally It ranges from 20 to 60 sec per 2 mm division in instruments for ordinary work The bubble tube attached to the telescope (Fig. 53) or to the vertical vernier frame (Fig 52), being used in vertical angulation and occasionally for ordinary levelling, should be more sensitive, a value of 10 to 20 sec per 2 mm. division being required This altitude level, as it is called, is better placed on the vernier frame, since in that position it exhibits the stability of the instrument while the telescope is tilted for the observation of vertical angles

For observations necessitating levelling of the horizontal axis with greater delicacy than is possible by means of the plate level normal to the telescope, a *Striding Level* (Fig 56) is required This level is mounted on two legs with V-shaped ends, which can be placed upon the trunnion axis The level tube is adjustable so that its axis and the feet of the legs may be parallel By reversing the striding level on the trunnion axis, the latter may be accurately levelled. The sensitiveness of the striding level is usually equal to, or greater than, that of the altitude level

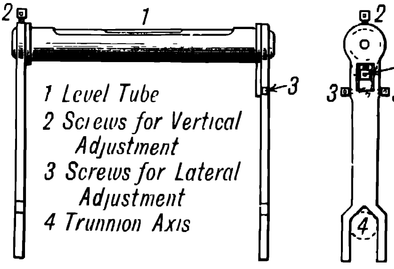
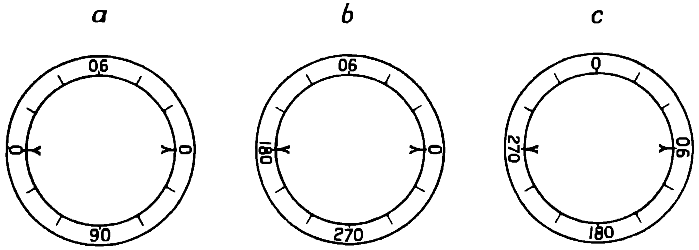

2
1
2
2
3
3
3
1 Level Tube
2 Screws for Vertical Adjustment
3 Screws for Lateral Adjustment
4 Trunnion Axis
4

FIG 56 —STRIDING LEVEL

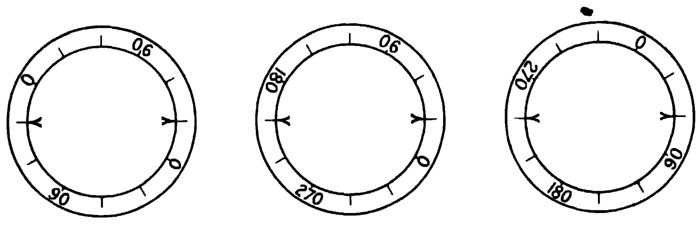
Circles —The diameter of the horizontal circle between the reading edges of the graduations serves to designate the size of an instrument : this ranges from 3 in to 12 in The $4\frac{1}{2}$-in , $4\frac{3}{4}$-in , and 5-in. sizes are those most commonly used in ordinary surveying and engineering work The 6-in instrument is employed when a higher degree of accuracy than usual is required, but 6-in. vernier theodolites have been largely superseded by smaller instruments in which the circles are read by micrometer microscopes (page 66) instead of verniers. Instruments of sizes smaller than 5 in. have increased greatly in popularity of recent years, largely owing to modern improvements in graduating machines. The use of instruments larger than 6-in. is confined principally to large triangulation surveys The vertical circle is commonly of the same diameter as the horizontal circle, but is sometimes made rather smaller. The graduations are cut on a ring of silver, and for instruments read by verniers the division is generally as follows .

Diameter	Circle Division	Verniers read to
3″	30′	1′
4″ to 5″	30′ or 20′	1′, 30″, or 20″
6″	10′ or 20′	10″ or 20″

The figures increase clockwise from 0° to 360° on the horizontal circle In some instruments two sets of figures are engraved, increasing in the clockwise and counter-clockwise directions respectively, and double verniers (page 37) are used This graduation enables angles measured counter-clockwise to be read as directly as those measured clockwise, but the risk of mistakes in reading is greater than with the single figuring, and the latter is to be preferred.



Telescope Horizontal



Telescope at 30° Elevation

FIG. 57.

The vertical circle is divided and figured either in quadrants or continuously from 0° to 360°. The former system is generally preferred in small instruments, the readings at the horizontally opposite verniers both being 0° when the telescope is horizontal (*a*, Fig. 57).

With this arrangement, the same reading is obtained, on both verniers, for an angle of elevation as for an angle of depression of the same magnitude. It is therefore necessary to distinguish between elevations and depressions in booking, and in the observation of small vertical angles care has to be taken to notice whether they are above or below the horizontal, especially if the circle is not marked + and — on either side of the zeros To obviate mistakes from this source, and to afford different readings at the opposite verniers, a continuous graduation round the circle is sometimes preferred The figuring may be that of *b*, Fig 57, giving readings of 0° and 180° for a horizontal line of sight, or as shown at *c*, in which the corresponding readings are 90° and 270°.

When an angle of elevation is being measured with the telescope pointing from right to left, the position of the figures is as shown on the lower half of Fig 57, since the circle rotates with the telescope For an elevation of n°, the readings given by both verniers of circles *a*, *b*, and *c* respectively, taking the vernier next the eyepiece first, are: n°, n°, n°, $(180^\circ+n^\circ)$, $(90^\circ-n^\circ)$, $(270^\circ-n^\circ)$. For the same angle below the horizontal, the readings would be n°, n°; $(360^\circ-n^\circ)$, $(180^\circ-n^\circ)$; $(90^\circ+n^\circ)$, $(270^\circ+n^\circ)$ There is evidently little possibility of confusion between elevations and depressions with the whole circle figurings, of which *b* is preferable to *c* The risk of mistakes in the extraction of trigonometrical ratios is, however, greater than in the quadrantal system, which is the more convenient for ordinary work

To facilitate reading, British instruments have the graduated face of the horizontal circle made in the form of the frustum of a cone . the vertical circle is plane. It is desirable to protect both circles by a solid guard in which glass covered openings are provided at the verniers The horizontal circle of Fig 53 is shown covered in : the vertical circle is more commonly left exposed

The two *Horizontal Verniers* are placed diametrically opposite each other, and do not appear in Fig. 53 as the line joining their zeros is perpendicular to the line of sight This is the usual arrangement, but it necessitates the observer's moving round from his position at the eyepiece in order to read even one of the verniers, which is sometimes all that is required. A position with the zeros at 30° to 40° with the line of sight is most convenient for work in which only one vernier is consulted, and this arrangement is adopted in some patterns. The plates carrying the vernier scales have the same bevel as the circle, and they may be fitted with a weak flat spring to maintain contact with it. In modern designs, however, the verniers are not actually in contact with the circle, so that wear and frictional resistance to rotation are minimised.

The magnifiers used for reading the verniers are mounted either on small brackets hinged to an arm which can turn about the centre of the upper plate or as shown in Fig. 53. The optical arrangement of the ordinary pattern is that of the Ramsden eyepiece (page 23), and, in taking a reading, the eye must be brought close to the eye lens. An alternative pattern of reader, consisting of an achromatic doublet, enables the graduations to be focussed with the eye 6 or 8 inches away, and is more convenient to use

The *Vertical Verniers* are given a slight bevel only, and are rigidly connected to their frame If the vertical circle is divided in quadrants, the readings have to be taken clockwise and counter-clockwise in alternate quadrants. Instead of fitting double verniers, however, a common arrangement is to have single verniers with two sets of figures, reference being made to those which increase in the same direction as the figures on the quadrant being read

Clamp and Tangent Screw fittings may take several forms In Fig. 53, the lower clamp *11* is screwed into a circular casting which embraces the outer axis, and which is connected to the fixed levelling head through the tangent or slow motion screw *12*, the latter being threaded through a lug projecting from the levelling head On applying the clamp, a pad is pressed against the outer axis, which is then secured through the tangent screw to the levelling head On turning the tangent screw, the outer axis, circle, and clamp fitting are rotated together relatively to the levelling head In an alternative, but less satisfactory, pattern, the clamp is placed tangentially, and tightens a split ring round the axis as shown in Fig 58.

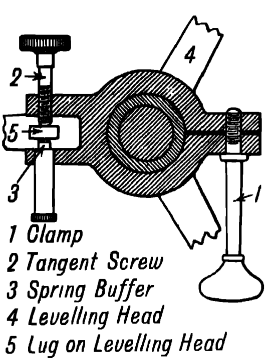


FIG 58 —CLAMP AND TANGENT SCREW

In the upper clamp and tangent screw and those of the vertical circle in Fig 53 the tangent screw operates directly on the clamp The tangent screws are protected from dust, and are fitted with opposing springs to take up wear and give a uniform motion.

The *Telescope*, to permit of transiting, is limited in length by the height of the standards, which must be reasonably low for stability and lightness If the telescope is of the internal focussing type, it will be capable of being transited both ways, *i e.* with the eyepiece or the object glass downwards With an external focussing telescope the design may admit of this, but a common arrangement is to make the telescope to transit only with the eyepiece downwards, and therefore the focussing movement takes place at the object glass end. Two Ramsden eyepieces are generally provided, giving a high and a low power The magnifications range from about 15 to 30 diameters, but should accord with the capabilities of the instrument in other respects (page 84). Stadia hairs should always be fitted, and, if the instrument is being used for astronomical observations, a diagonal eyepiece and a sun cap are necessary. Small open sights are sometimes fixed on the outside of the telescope to enable the surveyor to direct the line of sight rapidly towards a mark, and they prove a convenience for work in wooded country. They should be fitted on both the upper and lower sides, so that they may be equally available in the direct and reversed positions of the telescope.

If the instrument is to be used at night or in underground work, provision must be made for illuminating the cross-hairs, which would not otherwise be visible The simplest method is to attach a reflector in front of the object glass in order to project a small amount of light from a lamp or candle into the telescope. The reflector may take the form of a very small mirror or prism, a silver bead, or simply a piece of white paper with a sufficiently large opening through which to sight Otherwise, the telescope trunnion is made hollow on the side remote from the vertical circle, and a lamp, supported from the standard, projects light along the hollow axis into the telescope The rays are focussed on to a metal reflector of about $\frac{1}{8}$-in diameter placed in the longitudinal axis of the telescope, and are reflected on to hairs Some theodolites are fitted with a complete lighting set with lamps for the illumination of the circles at the verniers or microscopes, as well as of the reticule A cell is carried on the tripod, and the current is transmitted through the vertical axis, the intensity of the light being controlled by a rheostat

The *Compass* needle of Fig 53 floats in a circular box, the circumference of which is divided to degrees. This second graduated circle is somewhat unnecessary, since magnetic bearings may be read upon the larger horizontal circle when oriented by the needle, and a better arrangement is to contain the needle in a narrow rectangular box provided with a scale of only a few degrees on either side of zero (Fig. 59). Such a compass is called a trough compass, and has the advantage that a longer needle can be employed, while weight is reduced. It is attached when required to either the upper limb or the lower, according to the design of the theodolite. In the former case, the compass is screwed on to one of the standards so that its axis is parallel to the line of sight of the telescope. In the latter, it is fitted to the underside of the circle plate, the projecting flanges shown in Fig. 59 being slid into grooves on the plate. These grooves must be so situated that, when the needle floats centrally in the box, the circle is oriented for the reading of magnetic bearings, *i.e.* the line of sight lies in the magnetic meridian when the circle verniers read 0° and 180°. Screws are provided for adjusting the position of the scales.

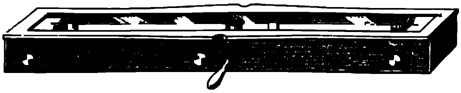


FIG 59.—TROUGH COMPASS.

Although they are sufficiently accurate for most purposes, neither the circular compass nor the ordinary trough pattern lends itself to very precise setting, owing to parallax arising from the difficulty of ensuring that the eye is in the vertical plane of the needle A greater degree of accuracy is attained by the use of a so-called telescopic compass. In this case, the needle is contained in a tube at one end of which there is fitted an eyepiece and a diaphragm carrying a glass plate with vertical rulings, which is nearly in the same plane as one end of the needle. The reticule being suitably illuminated by a reflector, the observer, on looking through the eyepiece, sees the end of the needle, which may be formed with a sharp edge on top or may be fitted with a light frame carrying a reference line.

A *Quick Levelling Head* may be fitted to facilitate levelling up, but these are not so common in theodolites as in levels (page 112), principally on account of the increase of weight—an important consideration in the theodolite.

Wall Tripod.—It may be necessary to set the instrument over a point where the use of the ordinary tripod is impossible. In the case of the three-screw levelling head, the tribrach plate is usually provided with three points which can be used as supports, but the four-screw head may require the attachment of a separate piece, called a wall tripod (Fig. 60). The instrument is centered when the three supporting points are equidistant from the station point.

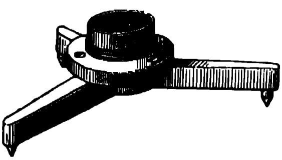
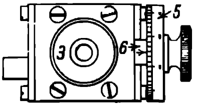


FIG. 60.—WALL TRIPOD

The Micrometer Microscope.—As the required fineness of reading increases, the vernier decreases in utility as a means of measuring fractional parts of the circle because of the difficulty of estimating the position of coincidence. Recourse may be had to the repetition method of angle measurement (Vol. II, Chap. III), but in geodetic and astronomical instruments the vernier has long been superseded by the micrometer or measuring microscope, which is superior not only in accuracy but also in the rapidity with which readings may be taken. Of recent years the micrometer has been increasingly used in ordinary work as a fitting for small theodolites.

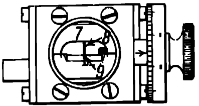
Principle.—The principle of the micrometer screw is widely used in physical and workshop instruments for the accurate determination of length, the required measurement being primarily obtained as a certain number of times the pitch of a fine screw working in a fixed nut, plus a fraction of a pitch or turn. By dividing the circumference of the head of the screw into a number of equal spaces, a convenient and accurate means is afforded for estimating the fractional part by reference to a fixed index mark. As applied in the theodolite, the micrometer screw is used in conjunction with a microscope, and is arranged to measure parts of the real image of the circle divisions which is formed by the objective of the microscope.

Details of Micrometer.—Figs. 61 to 64 illustrate the micrometer as used in theodolites for ordinary work. The microscope consists of an achromatic objective *1* and a Ramsden eyepiece *3*. In front of the latter the microscope tube is enlarged to form a rectangular box, through one side of which the micrometer screw passes. The screw is provided with a milled head and a graduated drum *5*, the rotation of which can be read relatively to the fixed index arrow *6*. By rotating the head a lateral motion is imparted to the slide *7*, across the opening in which are stretched either two closely spaced parallel webs *8*, cross-hairs, or a single hair. The slide is controlled by two light spiral springs (not shown), with the object of eliminating backlash. By suitably arranging the position of the entire microscope relatively to the circle and the distance of the objective from the hairs, (*a*) the real image of the circle graduations may be formed in the plane of hairs, and (*b*) the size of the image of a circle division may be made such that an exact number of turns of the screw is necessary to carry the hairs from the image of one graduation to that of the next. When these two requirements are met, fractional parts of the scale divisions can be measured on the image, the circumstance that they are measured on a chord instead of an arc leading to no appreciable error.



Elevation

FIG 61.



Elevation - Eyepiece Removed

FIG 62.

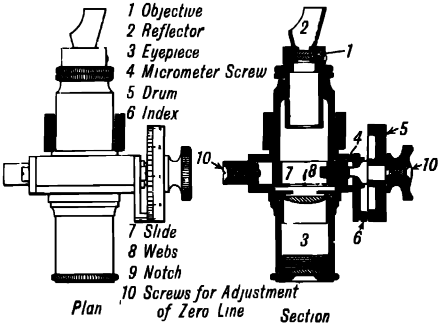


FIG 63. FIG. 64.

MICROMETER MICROSCOPE.

As in the vernier, there must be provided a point or line of reference from which to obtain both the approximate and fractional readings. This zero is defined, for the reticule illustrated, by the imaginary line midway between the parallel hairs when these occupy the middle of the field of view, the drum reading zero. For convenience, this point is approximately marked by a notch *9*, cut in a small fixed plate immediately in front of the hairs, it being impracticable to place it in their plane. When the eyepiece is focussed for the hairs and the image, the notch is not therefore in precise focus, but *the whole function of the notch is to facilitate the determination of the approximate reading, and it is not referred to for the measurement of the fractional part.*

Least Count of Micrometer.—To determine the least count, the nature of the circle division must first be ascertained by examination through the microscope In the case of ordinary patterns of instruments up to 5-in diameter, the circle spaces represent 10 min. The 6-in. theodolite is usually divided to 10 min., but the division may be carried to 5 min. The latter graduation is adopted in instruments larger than 6 in. Since the Ramsden eyepiece is non-erecting, the figures are engraved in the inverted position, so that they appear right side up when viewed through the microscope. The inversion of the image makes the circle appear to be graduated counterclockwise instead of clockwise.

By rotating the milled head, the number of turns of the screw necessary to carry the hairs from the image of one graduation to that of the next should then be found If t turns are required, and the circumference of the drum is divided into n parts, the value of a drum division is $\frac{1}{nt}$ of a circle division In theodolites with circle spaces of 10 min , t is made 1, and the drum has altogether 60 divisions, so that the value of each is 10 sec In more precise instruments t is 5, and the drum divisions represent 1 sec or 2 sec

To Read the Micrometer.—As an example, let the circle be divided to 10 min., and let one turn of the screw carry the hairs across a circle division, the drum having 60 parts The eyepiece should, if necessary, first be adjusted until the hairs are very sharply focussed. The appearance presented through the microscope may then be as in Fig 65. The hairs may be in any position, but if they were brought to the middle of the field of view, so that the drum reads zero, they would appear to lie in the notch. The approximate reading, given by the position of the notch, is 144° 30′, and, to obtain the additional part, all that is necessary is to move the hairs until they are centered over the graduation next the notch, as shown in Fig. 66. The drum now records 4′ 10″, so that the complete reading is 144° 34′ 10″.

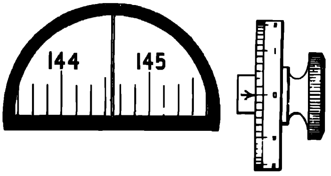


FIG. 65.

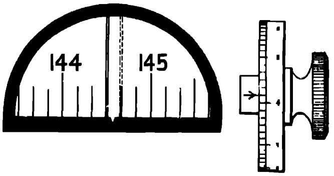


FIG. 66.

Notes.—(1) The object of using two parallel hairs, instead of cross-hairs or a single hair, is to facilitate centering over a graduation. The distance between the hairs is very little more than the width of the image of an engraved line, and it is thus possible to estimate the bisection by the latter of the space between the hairs with considerable precision.

(2) Since one turn of the screw corresponds to a motion of the hairs across a division, it is immaterial on which graduation they are centered, as the drum will register the same reading, if the micrometer is in adjustment. Thus, in Fig. 66, in taking the reading, the hairs might have been moved to the position shown dotted, with the same result as before.

(3) For the same reason, it is altogether unnecessary to put the hairs over the notch before taking a reading They have simply to be moved from the position in which they were left from the previous observation For the avoidance of lost motion, the hairs should, however, always be brought on to a graduation by screwing against the spring. It may therefore be necessary to run the hairs past a graduation in order to bring them on to it in the constant direction

(4) Care should be exercised not to press upon the head of the screw. It should be fingered as lightly as possible

(5) The caution, mentioned on page 40, against side illumination of the circle, applies with special force to micrometer reading. The illumination, whether daylight or artificial, must be thrown upon the circle in a radial direction by turning the reflector (Fig 64) into the proper position The reflector is usually formed of plaster of Paris, and its matt surface affords a soft uniform illumination

(6) Apparently the reading could be carried to the nearest second by estimation of parts of the drum divisions With small instruments, however, the readings cannot in general be relied upon to less than 10 sec , as the accuracy of an observation depends primarily upon the theodolite itself. The magnifying power of the micrometer microscopes should accord with the sensitiveness of the instrument to which they are fitted, and for instruments of standard pattern smaller than 8 in this is insufficient to permit of reading to a single second with accuracy.

To Set the Circle to a Given Reading, the angle must be considered as consisting of a number of circle divisions (the approximate reading) and a fractional part (the micrometer reading) The hairs have first to be placed to the left of the centre point by a distance corresponding to the latter reading, and then the graduation representing the approximate reading must be brought to bisect the space between the hairs, the final adjustment being made by the upper tangent screw

Errors and Adjustments of the Micrometer. See Vol. II, Chap. III.

The Estimating Microscope.—A simple method of reading the circles by microscope without using a micrometer screw and drum has developed widely on the Continent. The microscope is fitted with a diaphragm carrying a glass plate on which is ruled a scale of equal parts, the total length of the scale being equal to that of the real image of a circle space produced by the microscope objective. The scale, being placed at the common focal plane of the objective and eyepiece, enables estimates to be made of the fractional parts, the zero of the scale forming the reference point.

Thus in Fig 67, which represents the view presented by the microscope in the case of a circle divided to 20 min , the scale is divided into 10 parts, each representing 2 min. The approximate reading given by the position of the zero is 144° 20′, and, if the scale reading is estimated to be 7·1, the complete reading is $144° 20' + 14'{\cdot}2 = 144° 34' 10''$, to the nearest 10″. If the readings of opposite microscopes are taken for averaging, the reading is simplified by regarding the scale divisions as single minutes and adding the sum of the two scale readings to the approximate reading. Thus, if the scale readings are 7·1 and 7·2 respectively, giving a sum of 14′·3, the mean reading is 144° 34′·3

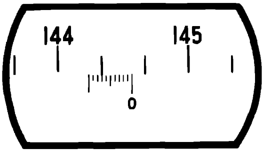


FIG 67

In the case where the circle is divided to 10 min, a scale of ten parts enables estimates to be made at each microscope to 0·1 min Apparently by increasing the power of the microscope, so that a more finely divided scale can be used, any degree of refinement of reading is possible, provided that the microscope is adjusted so that the length of the image of a circle space equals that of the scale. Messrs. Cooke, Troughton and Simms, of London, have patented a design of estimating microscope in which the scale can be moved parallel to the circle graduations and is of the diagonal pattern, affording intersections at 10 sec intervals from which to estimate. Actually, however, high magnification makes the engraved lines of the circle appear too coarse for refined estimations, unless the lines are very accurately tapered at the end or are ruled on a glass circle instead of a silver ring, as is done in some modern theodolites. The estimating microscope is therefore inferior to the micrometer microscope for precise readings, but it forms a very simple substitute for the vernier.

Double Reading Theodolites.—Important angle measurements, especially such as warrant the use of a micrometer theodolite, always require that readings at diametrically opposite points on the circle should be taken for averaging (page 83). This necessitates the observer's moving round the instrument, and introduces the possibility of his disturbing it, particularly when working in the dark. To minimise this risk, and to save time in reading the circles, various designs of theodolite have been introduced in which it is possible to observe the readings of the opposite points of both circles from one position.

Casella's Double Reading Theodolite.—In the instrument designed by Messrs. C. F. Casella & Co., of London, the images of the opposite parts of the horizontal circle are brought into the field of a single microscope by means of prisms placed in the vertical axis. These images are presented side by side to the observer along with an image of the micrometer drum, as shown in Fig. 68. The two readings are obtained by adjusting the micrometer hairs on the graduations of both images successively in the usual manner. In the case illustrated,

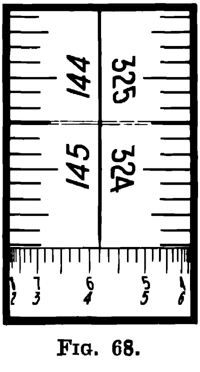


FIG. 68.

the micrometer hairs are centered on a graduation of the left-hand image, and the reading is 144° 34′ 10″. The vertical circle microscope, which is of the same pattern, is placed with its axis in that of the telescope trunnion, and is just above the horizontal circle microscope. With this arrangement the observer can obtain all the angle readings from one position.

The Wild Universal Theodolite.—This instrument, designed by Dr. H. Wild, the inventor of the type, and made by his firm at Heerburg, Switzerland, embodies a number of novel features tending to precision of reading and saving of time in observing. The horizontal circle is only $3\frac{3}{4}$ in , and the vertical circle 2 in., in diameter. Both circles are read through a single microscope placed alongside the telescope, the movement of a prism determining which is viewed. A noteworthy feature is that the same micrometer

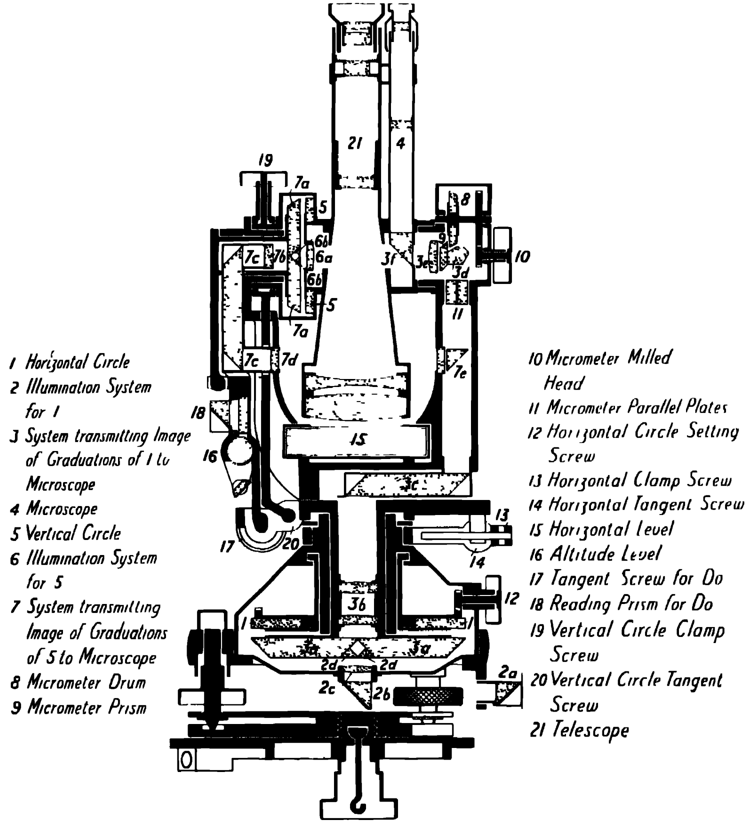

19
7a
5
21
4
8
7c
7b
6b
6a
6b
5
3f
3e
9
3d
10
11
7a
7c
7d
7e
18
15
16
3c
13
14
17
20
12
3b
1
3a
3g
2d
2d
2c
2b
2a
0
1 Horizontal Circle
2 Illumination System for 1
3 System transmitting Image of Graduations of 1 to Microscope
4 Microscope
5 Vertical Circle
6 Illumination System for 5
7 System transmitting Image of Graduations of 5 to Microscope
8 Micrometer Drum
9 Micrometer Prism
10 Micrometer Milled Head
11 Micrometer Parallel Plates
12 Horizontal Circle Setting Screw
13 Horizontal Clamp Screw
14 Horizontal Tangent Screw
15 Horizontal Level
16 Altitude Level
17 Tangent Screw for Do
18 Reading Prism for Do
19 Vertical Circle Clamp Screw
20 Vertical Circle Tangent Screw
21 Telescope

FIG. 69.—WILD UNIVERSAL THEODOLITE.

system serves for both circles, their different sizes being compensated for by magnifying the graduations different amounts.

The optical arrangement may be followed from Fig 69. External light is received by the adjustable prism *2a*, and is transmitted through the system *2 b, c, d* to opposite points of the horizontal circle *1*, being reflected to right and left by the prisms *2d*. The glass ring forming the horizontal circle is graduated on the upper side, and it is silvered on that face The rays from the illuminated graduations at diametrically opposite points are reflected downwards into the prisms *3a*, which transmit them upwards side by side through the lenses *3b* and the prism *3c* Prism *7e* forms part of the vertical circle system, and is withdrawn while the horizontal circle is being read. The rays traverse plates of parallel glass *11*, which form part of the micrometer device, and are reflected in prism *3d* They are projected through lens *3e* on to prism *3f*, which transmits them along the reading microscope *4* to the eye

A similar scheme enables the images of opposite points of the vertical circle *5* to be viewed in the microscope when the prism *7e* is placed, by the movement of a button, in the position illustrated Light is introduced by a prism, not shown, and travels by way of lens *6a* and prisms *6b* to the circle The rays reflected from the illuminated graduations proceed through *7a, b, c, d* to the prism *7e*, and thence through the parallel plates to the microscope as before

The micrometer drum *8*, consisting of an unsilvered glass ring, is suitably illuminated Light passing through it is reflected by prism *9*, which carries the index mark, proceeds through the object lens *3e*, and is transmitted along the microscope by prism *3f* together with the circle images. The circles are divided to 20 min., and the approximate reading is estimated to 10 min. The micrometer drum is divided into 10 single minute spaces, each of which is subdivided to single seconds. The turning of the micrometer screw *10* rotates the micrometer drum, and simultaneously tilts the plates *11*, moving the images of the two parts of the circle in opposite directions

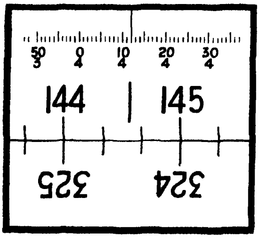


Fig 70.

To take a reading, the position of the fixed index mark relatively to the circle graduations is observed, and the approximate reading is taken. The micrometer screw is then adjusted until the images of the adjacent graduations of the two parts of the circle are brought into exact coincidence, as shown in Fig. 70. The mean value of the minutes and seconds of the fractional parts of the opposite readings is exhibited on the image of the micrometer drum scale against the fine index line Before the coincidence was effected in the case illustrated, the index mark appeared more than half-way between 144° 20′ and 144° 40′, so that the approximate reading was 144° 30′, and the complete reading is 144° 34′ 12″.

The Tavistock Theodolite.—This instrument (Fig. 71), made by Messrs Cooke, Troughton and Simms, Ltd , incorporates the method of reading circles patented in 1927 by Instructor Captain T Y. Baker and Mr. R W Cheshire whereby opposite parts of the circle are viewed side by side with the reference mark between them. A feature of this system is that the fine reading device simultaneously moves the circle images in the same direction Readings are obtained by displacing the images until the graduations on either side of the index mark are equidistant from it The fractional part of the mean reading is obtained on the image of the micrometer scale, which appears in the field.

The horizontal and vertical circles are divided on glass to 20 min , and are of 3½ in and 2¾ in diameter respectively The former is carried on an independent axis, as in the ordinary pattern, and the optical system for the illumination and reading of the circle rotates with the telescope as the latter is wheeled horizontally. The circles are read by means of independent micrometers direct to single seconds. The reading microscopes are pivoted from the standards, and can be adjusted to suit the observer whether he is using the telescope in the direct or reversed position

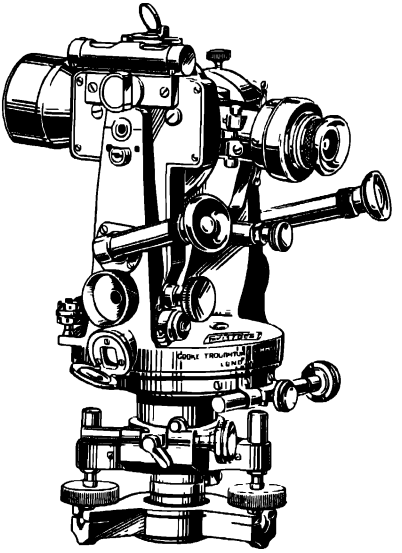
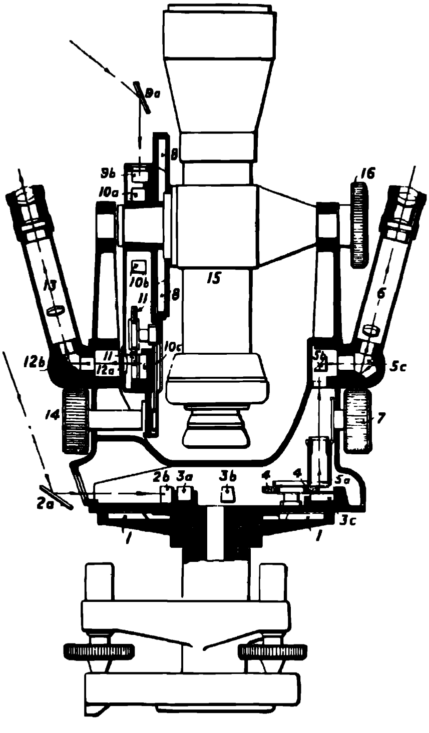


FIG 71 —THE TAVISTOCK THEODOLITE

Fig 72 illustrates the optical arrangement as far as it can be shown on one view In the case of the horizontal circle, light entering by the reflector *2a* is transmitted to diametrically opposite points of the horizontal circle by a series of prisms of which *2b* is the first The rays reflected from the silvered surface of the graduated circle are, in the course of their path, reflected by prisms *3a*, and transmitted through movable wedges, or travelling prisms, *3b*, to the prisms *3c*. The latter lie just below the glass disc *4* on which the fine reading scale is engraved, and the rays from the circle are brought to a focus in the plane of the micrometer scale. By means of the system *5 a*, *b*, *c*, rays from the fine reading scale and from the images of the opposite points of the circle are transmitted side by side into the horizontal circle microscope *6*. The vertical circle reading system is designed on similar lines.

The manner in which the readings are shown in the microscopes is illustrated in Fig. 73, which shows the field of the horizontal circle microscope Of the three apertures, the bottom one shows the approximate reading, that at the top presents an image of part of the micrometer scale, and in the middle one are brought side by side images of the diametrically opposite points of the circle separated by a relatively broad line forming the index The three index marks and the mask defining the apertures are placed on a thin glass plate between *3c* and *4* (Fig. 72). To minimise the possibility of the wrong microscope being read, the relative position of the apertures is made different in the case of the vertical circle, and the letter V is shown



1 Horizontal Circle
2 Illumination System for 1
3 System transmitting Image of Graduations of 1 to Plane of 4
4 Horizontal Fine Reading Circle
5 System transmitting Images into 6
6 Horizontal Circle Microscope
7 Horizontal Circle Micrometer Milled Head.
8 Vertical Circle
9 Illumination System for 8
10 System transmitting Image of Graduations of 8 to Plane of 11
11 Vertical Fine Reading Circle
12 System transmitting Images into 13
13 Vertical Circle Microscope
14 Vertical Circle Micrometer Milled Head
15 Telescope
16 Telescope Focussing Screw

FIG. 72.—TAVISTOCK THEODOLITE—PART SECTION.

To read, say, the horizontal circle, the milled head *7* is first adjusted until graduations from opposite points of the circle are placed symmetrically about the reference line. This is effected by equalising the narrow bands of light between the index and the graduations on either side. The rotation of *7* required to do so displaces the images by moving the travelling prisms *3b*, turns the fine reading scale to the mean of the opposite readings, and at the same time places the graduation representing the coarse reading exactly opposite the index point in the bottom aperture. In the example illustrated the reading is 144° 20′ + 14′ 12″ = 144° 34′ 12″. To set the circle to a given reading, the micrometer scale is set to the fractional reading by means of the micrometer milled head, and the circle is wheeled until the approximate reading is brought roughly to its index mark The final setting is effected by turning the upper tangent screw until the opposite graduations appear equidistant from the reference line in the small aperture.

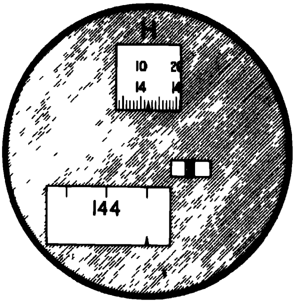


FIG. 73.

The American Transit.—The ordinary American transit is commonly lighter than, and differs in several respects from, the British model The trunnion axis is not usually removable from the standard supports, and there is no fitting corresponding to *26*, *27* (Fig 53) The four-screw levelling head is employed to a far greater extent than in this country, but three screws are favoured for precise instruments The graduated faces of the horizontal circle and vernier are placed in a horizontal plane (Fig 74), this arrangement having the merit of minimising wear between the circle and verniers The circle is commonly figured in both directions, the verniers being of the double type ; and a somewhat open graduation, usually with verniers reading to 1 min , is preferred, so that reading microscopes are commonly dispensed with. A much used pattern is fitted with a vertical semicircular arc, read by a single vernier attached to the standard. The American type of transit is manufactured in England, principally for export to Canada.

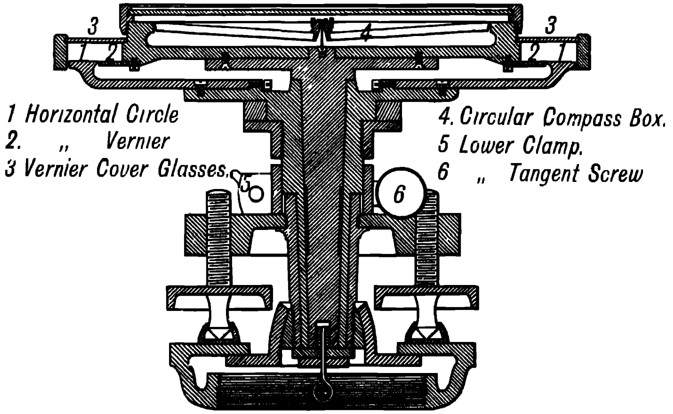
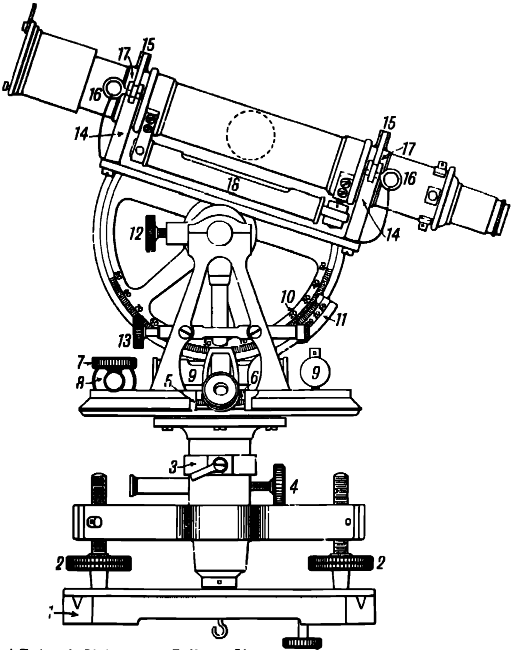


FIG. 74.—LOWER PART OF AMERICAN TRANSIT.

Non-transiting Theodolites.—In these the general constructive arrangement of the parts relating to horizontal measurements is the same as in the transit. The manner of mounting the telescope creates differences in the method of adjustment. The Wye pattern is still in use to a limited extent, and its adjustments are given on page 92 The Everest theodolite is now rarely encountered

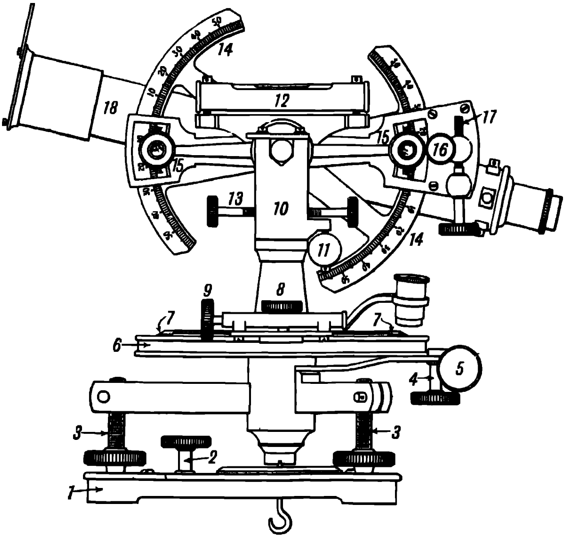


1 Tribrach Plate
2 Levelling Screws
3 Lower Clamp
4 „ Tangent Screw
5 Horizontal Circle
6 „ Vernier
7 Upper Clamp
8 „ Tangent Screw
9 Plate Levels
10 Vertical Arc
11 „ Vernier
12 „ Clamp
13 Vertical Tangent Screw
14 Wyes
15 Wye Clips
16 „ Pins
17 Telescope Collars
18 „ Level Tube

Fig 75 —Wye Theodolite

The Wye Theodolite.—The essential difference between this instrument (Fig. 75) and the transit is that the telescope is not directly mounted on the trunnion axis, but is supported on two U- or Y-shaped forks, called wyes. Although held by clips, the telescope is capable of rotation about its longitudinal axis, and, on opening the clips, it can be removed and placed end for end to reverse the line of sight. The collars which rest in the wyes must be truly cylindrical and equal, so that these movements of the telescope may not alter the direction of the axis. The stage carrying the wyes is attached to the trunnion axis, and carries a graduated semicircle for the measurement of vertical angles. Only one vertical vernier is fitted, and that is attached directly to the horizontal vernier plate.

The Everest Theodolite.—This instrument (Fig. 76) was at one time largely used in India. The trunnion axis of the telescope is placed low, and is supported on two short arms projected from a central pillar. In place of a vertical circle, two arcs, graduated up to 50°, are fixed to the trunnion axis. For purposes of adjustment, reversal of the line of sight is effected by removing the telescope from the supports and turning it end for end.



1	Tribrach Plate	7	Horizontal Verniers	13	Clip Screws
2	Locking Screw	8	Upper Clamp	14	Vertical Arcs
3	Levelling Screws	9	„ Tangent Screw	15	„ Verniers
4	Lower Clamp	10	Standards	16	„ Clamp
5	„ Tangent Screw	11	Plate Level Tube	17	„ Tangent Screw
6.	Horizontal Circle	12	Vernier Arm „	18.	Telescope

FIG 76.—EVEREST THEODOLITE

TEMPORARY ADJUSTMENTS OF THE THEODOLITE.

The temporary adjustments are :

(1) Setting over the station.
(2) Levelling up.
(3) Elimination of parallax.

Setting Up.—This includes both centering of the instrument over the station by the plumb bob and its approximate levelling by manipulation of the tripod legs only. The accuracy of centering should depend upon the required precision of the observations.

In the majority of ordinary small surveys refined centering is not required unless the sights are short. Centering is facilitated if the instrument has a shifting head, but, even without one, rapid setting up should present no difficulty if performed systematically.

Notes.—(1) The legs of the tripod can be moved not only radially but also circumferentially or sideways. The first movement shifts the plumb bob, and tilts the instrument. The second causes a considerable change of inclination without disturbing the plumb bob, and this movement is therefore effective in the approximate levelling of the instrument

(2) First hold the instrument off the ground with the legs spread out and the plumb bob hanging approximately over the station On being lowered to the ground, it will not be seriously out of centre, and the exact centering and approximate levelling may be completed by small radial and tangential motions of two legs and the pushing of all three into the ground.

(3) On a hillside, place one leg uphill

(4) A heavy bob is preferable to a light one, especially when working in wind. In windy weather spread the legs well out, and push them firmly into the ground.

(5) If chaining is to be conducted from the station, avoid setting a leg in the chain line

Levelling Up.—The manner of levelling the instrument by the plate levels depends upon whether there are three or four levelling screws.

Four-Screw Head—(1) Turn the upper plate until one level tube is parallel to a pair of diagonally opposite screws The other tube will be parallel to the remaining pair.

(2) By manipulating one pair of screws, as in Fig. 77, bring the bubble to the centre of its run in the tube to which they are parallel. In turning the screws, the thumbs move towards or away from each other, and the left thumb must be moved in the direction in which the bubble is required to travel

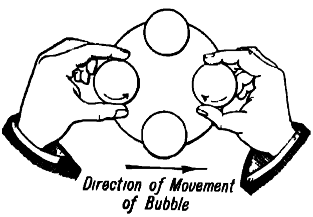

Direction of Movement of Bubble

FIG. 77—MANIPULATION OF LEVELLING SCREWS.

(3) Level the other bubble tube with the remaining two screws, and then relevel the first tube if necessary.

Notes.—(1) If there is only one plate level, it will be necessary to rotate the plate through 90° after step (2) to bring the level parallel to the second pair of screws.

(2) If the instrument has been set up much out of level, jamming of a pair of screws may occur, and this may necessitate resetting the instrument. If one screw jams, turn the other only, or slacken the other pair. Do not force the screws.

(3) Screws should be left bearing firmly upon the lower parallel plate so that there is no tendency to rock. To ensure that all screws are gripping, it is well to finish the working of a pair by turning both clockwise for a small fraction of a turn.

Three-Screw Head.—(1) Turn the plate until one level tube is parallel to any pair of screws. The other tube will be parallel to the line joining the third screw and the point midway between the first pair.

(2) Manipulate the pair of screws as before until the tube parallel to them is levelled.

(3) Level the other tube with the remaining screw by using one hand only.

Note —With the plate in the above position, both bubbles may be centered coincidently by turning one screw of the pair simultaneously with the third screw, the plate tilting about the second screw of the pair.

Elimination of Parallax.—See page 25.

ANGLE MEASUREMENT BY THEODOLITE

To Measure a Horizontal Angle.—The temporary adjustments having been made, the usual procedure in measuring the horizontal angle subtended at the instrument by two signals is as follows:

(1) With the upper and lower clamps slack, bring the index of one of the verniers approximately to the zero (commonly marked 360°) of the circle Fix the vernier plate to the circle by the upper clamp, and, by turning the upper tangent screw, set the zero of the vernier exactly to 360°

(2) The circle, verniers, and telescope can now revolve as one upon the outer axis Turn them by hand until the left-hand signal appears in the field of the telescope. Fix the outer axis by the lower clamp, and direct the line of sight exactly upon the signal by turning the lower tangent screw.

(3) Release the upper clamp, and swing the telescope horizontally until the second object appears in the field Fix the upper clamp, and centre the intersection of the hairs on the signal by the upper tangent screw.

(4) Read the angle on the vernier which was originally set at 360°.

Notes.—(1) Instead of setting the vernier initially at the zero of the circle, it may be clamped in any position, and the reading noted. The required angle is obtained by subtracting the initial from the final reading. If the 360° graduation lies between the two readings, subtract their difference from 360°.

(2) By sighting the left object first, the line of sight has to be moved clockwise, *i e* in the direction of graduation of the circle, and, if the vernier was first at zero, the final reading gives the required angle When the right-hand object is taken first, the final reading must be subtracted from 360°. If the circle is figured in both directions, it is equally convenient to start from the right or the left point. Measuring an angle from left to right does not necessarily involve turning the telescope in that direction. The same final reading is obtained by wheeling the telescope in either direction.

(3) To avoid improper use of the clamps and tangent screws, their functions should be clearly understood. The lower clamp and tangent screw are used only for the orientation of the circle, *i.e.* the setting and fixing of it in position

for the measurement Turning the wrong tangent screw necessarily introduces error. Thus, if the exact pointing to the first signal is effected by the upper tangent screw instead of the lower, the vernier is moved, and the measurement is made from the new position of its index instead of from the zero of the circle, as supposed. Erroneously using the lower tangent screw in the second pointing disturbs the necessary fixity of the circle during the measurement.

(4) Sight as far down the poles or signals as practicable, to reduce errors due to their possible non-verticality The vertical circle clamp and tangent screw may be used to increase the delicacy of vertical movement of the line of sight

(5) The accuracy of the measurement may be greatly increased by a system of repeated observations designed to minimise the effects of instrumental and other errors (see page 170).

To Set Out a Horizontal Angle.—This operation is the converse of the above. Being given one point, it is required to locate the direction in which a second should lie from the instrument so that a given angle is subtended between them. Let the given angle be a measured clockwise

(1) Set the index of one of the verniers to 360° by the upper clamp and tangent screw.

(2) Direct the line of sight to the given signal, fix the lower clamp, and bisect the signal exactly by using the lower tangent screw

(3) Release the upper clamp, and turn the vernier to read a approximately. Fix the upper clamp, and obtain the exact reading by turning the upper tangent screw

(4) The telescope now points in the required direction. Mark a point in the line of sight.

Notes.—(1) If the angle has to be set out counter-clockwise, the second pointing must be obtained by setting the vernier to $(360^\circ - a)$

(2) In guiding the staffman into the line of sight in step (4), it will be observed that, with the usual inverting eyepiece, it is necessary to order him to move apparently away from the line of sight.

To Measure a Vertical Angle.—Vertical angles of elevation or depression are measured from the horizontal plane through the trunnion axis. This plane must be established by careful levelling up of the instrument by means of the level tube mounted either on the telescope or the vertical vernier arm.

If the level tube is attached to the telescope, the procedure is as follows :

(1) Having levelled the instrument by the plate levels, bring the zero of the vertical circle approximately to the index of one of the verniers by moving the telescope by hand. Apply the vertical circle clamp, and by means of the vertical circle tangent screw adjust the zero to exact coincidence with the vernier index.

(2) The bubble of the telescope level should now be nearly, if not quite, central. Complete the levelling in two positions at right angles by means of the levelling screws.

(3) Wheel the instrument through 180°. If the bubble deviates

from the central position, bring it half-way back by means of the clip screw *26* (Fig. 53) and the remainder by the levelling screws. Repeat until the bubble remains central while the telescope is rotated horizontally.

(4) Release the vertical circle clamp, and tilt the telescope to bring the image of the distant point approximately on to the horizontal hair. Clamp, and by means of the tangent screw adjust the horizontal hair exactly upon the point, and read the required angle on the circle

If the altitude level is mounted on the index arm, the procedure is the same, except that it is unnecessary to set the vertical circle to zero.

Notes—(1) It is assumed in the foregoing that the altitude level is in adjustment, and in the case where this level is mounted on the vernier arm it is further supposed that index error has been eliminated These adjustments (page 90) have to be attended to before proceeding to measure vertical angles.

(2) Some instruments do not possess the clip screw fitting whereby the telescope bubble may be levelled without altering the reading of the vertical circle In this case the levelling is performed by the vertical circle tangent screw, and note is made of the reading when the telescope is horizontal This index error must be applied positively or negatively to each reading obtained to give the required angles

TESTING AND ADJUSTMENT OF THE THEODOLITE

Requirements of the Theodolite.—Through imperfections in workmanship and the development of defects by continued use, the ideal requirements of a theodolite are only partially fulfilled, and its successful operation is very largely dependent upon a knowledge of the nature and relative importance of the effects produced by instrumental errors Provision is made for the elimination of the more serious errors by adjustment, while the influence of certain defects in the non-adjustable parts can be reduced or completely eliminated by adopting a particular routine in observing

The more important requirements in the non-adjustable parts are :

1. The whole instrument should be stable, *i.e.* devoid of slackness.
2. The inner and outer axes should have the same geometrical axis of rotation, *viz* the vertical axis of the instrument.
3. All movements should be truly circular.
4. The centre of graduation of the horizontal circle should lie in the vertical axis, and that of the vertical circle in the horizontal axis
5. The zeros of the verniers should be diametrically opposite each other.
6. The division of the circles should be accurate.

7. The plane of the reading edge of the horizontal circle should be at right angles to the vertical axis, and that of the vertical circle at right angles to the horizontal axis.

8. The resolving power of the telescope should be so related to the least count of the verniers or microscopes that a just discernible movement of a vernier or microscope produces a perceptible motion of the cross-hairs on a signal, and *vice versa*

9. The resolving power should also be such that a perceptible change of position of any of the bubbles produces a visible movement of the line of sight.

The following further conditions may be established by adjustment:

1. When the plate level bubbles are in the centres of their runs, the vertical axis should be truly vertical.

2. The line of sight should coincide with the optical axis of the telescope, and should be perpendicular to the horizontal axis

3. The horizontal axis should be perpendicular to the vertical axis.

4. The line of sight should be parallel to the axis of the telescope level.

5. When the line of sight is horizontal, the vertical circle reading should be zero.

Testing of Non-adjustable Parts.—1. **Stability** increases in importance with increase of sensitiveness in the various parts. Slackness may be present in the tripod (see page 43) and in various parts of the instrument, but particularly at the trunnion axis supports

Test.—Mount the instrument on a rigid tripod, and sight a point with all clamps fixed. Apply a gentle, but firm, lateral pressure on the eyepiece with a finger. The intersection of the hairs will probably leave the point, but should return exactly to it on removal of the pressure.

Elimination of Error.—The error can be eliminated only by repair, but its effect may be minimised by avoiding pressing unduly on any part. Clamps should be applied gently, and the tripod should not be touched unnecessarily.

2. **Incoincidence of the Inner and Outer Axes of Rotation** may be of two kinds: (*a*) the axes may be parallel (Fig. 78*a*), this being one of the causes of eccentricity between the circle and vernier plates (4, *infra*); (*b*) the axes may be non-parallel (Fig. 78*b*), and not only is there eccentricity, but, if one of the axes is made truly vertical by the plate bubbles, rotation about the other is oblique. For test and elimination of error, see *Note* 2, page 85.

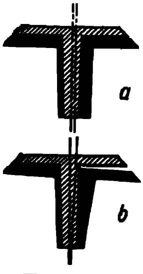


FIG. 78.

3. **Circularity of the Movements** is dependent upon the cross sections of the axes being circular. A simple test is not available.

Elimination of Error —The effect of unequal wear of the axes is reduced, and in certain cases completely eliminated, by reading both verniers. For horizontal angles, repetition of the measurement on different parts of the circle is an additional precaution.

4. With **Eccentricity of the Horizontal Circle,** the line of sight and the verniers have an eccentric motion relatively to the graduations, and an angle read on either vernier is incorrect. In eccentricity of the vertical circle, the circle and line of sight have an eccentric motion with respect to the verniers, and the effect is the same.

Test —Read both verniers in several positions on the circle, and note the differences between the readings If the difference remains constant, there is no eccentricity.

Thus, in Fig 79, let A be the centre of graduation of the circle, and B that of the verniers, the distance AB being greatly exaggerated. When the line of sight occupies the position CBD, the vernier readings at E and F differ by 180°. On turning the line of sight through a into the position GBH, the verniers are actually moved through a to K and L, but they do not record a The reading at K is EAK, say $(a-e)$, and that at L is $(180°+\text{FAL})$ $=(180° + a + e)$, since $\text{AKB} = \text{ALB} = e$ The readings therefore now differ by $(180°+2e)$

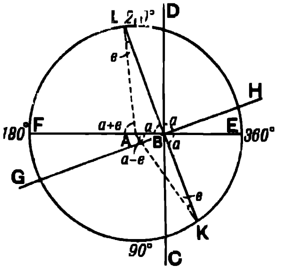


FIG 79

Elimination of Error may evidently be completely effected by averaging the values of the angle given by each vernier.

5. In **Eccentricity of the Verniers,** their zeros are not situated at the ends of the same diameter, but as K and L (Fig 79) The readings in any position therefore differ by a constant angle (KAL), other than 180°, provided B coincides with A.

Test —Read both verniers in several positions on the circle, and note the differences between the readings A constant difference, other than 180°, indicates eccentricity of the vernier zeros.

Elimination of Error —Error from this source can arise only in orienting by one vernier and reading the angle on the other. Measurements are given correctly by either vernier. By drawing a diagram similar to Fig. 79, it will be seen that reading both verniers eliminates errors due to eccentricity of the circle although the verniers are eccentric.

6. **Errors of Graduation** of a circle can be analysed as (1) periodic errors, which recur at regular intervals according to some law, (2) accidental errors, which are quite irregular. Although a vital

matter, accuracy of division cannot readily be tested by the surveyor, but the refinement of modern dividing engines is such that the errors are unlikely to influence his results, particularly in vernier, as distinct from micrometer, instruments

Elimination of Error.—The effect of graduation errors may be reduced to any required extent by repeating the measurement on various parts of the horizontal circle and averaging. For vertical angles, measurement by both verniers is the only available precaution in ordinary instruments

7. **Perpendicularity of the Planes of the Circles to their Respective Axes** is a condition easily fulfilled by the maker within limits of error which will have no appreciable influence on observations If, for instance, the condition does not obtain in the case of the horizontal circle, then, when the vertical axis is placed truly vertical, the circle is oblique, and an observed horizontal angle is not measured on the horizontal plane, but the reading will differ inappreciably from the true value unless the obliquity is very serious.

8 **The Relationship between Resolution and Vernier Least Count** should obtain for convenience in observing A magnification capable of showing movements of the line of sight too small to be recorded by the verniers entails loss of time in sighting, and has the further disadvantage of reducing the brilliancy of the image On the other hand, if the resolving power is too low, advantage cannot be taken of fine subdivision of the circles, and time is wasted in reading the vernier

Test —Centre the hairs exactly on a well-defined distant point with all clamps fixed, and read the vernier Make as small a deviation of the line of sight as possible, and observe if the vernier reading has changed. Alternatively, move the vernier through its least count or a fraction of it, if discernible, and note whether the line of sight has moved perceptibly.

9. **Relationship between Resolution and Sensitiveness of Levels.**—The level attached to the telescope or to the vertical vernier frame, being used in vertical measurements, should be capable of indicating the horizontal direction with as much accuracy as setting of the line of sight can be performed.

Test is similar to the foregoing, a just perceptible movement of the bubble being made.

The plate levels are less sensitive, but they should be able to pass the following :

Test.—Sight a well-defined point with both bubbles central and all clamps fixed. By turning the levelling screws, throw the bubbles out of centre. On relevelling, the line of sight should be brought back to the point.

Adjustment of the Transit Theodolite

In the examination and elimination of errors in the adjustable parts of the theodolite, considerable use is made of the principle of reversal (page 7). An important reversal is that produced by transiting the telescope and then turning it horizontally (termed "wheeling") through 180°, so that, if the vertical circle is initially on the right-hand side of the telescope, it will lie to the left after reversal. These positions are distinguished as "face right" and "face left," the operation being termed "changing face."

1. Adjustment of the Plate Levels.—*Object.*—To set the axes of the plate level tubes perpendicular to the vertical axis.

Necessity —If this requirement is met, levelling the plate bubble tubes levels the horizontal circle and also the horizontal axis of the telescope, provided these are respectively perpendicular to the vertical axis Horizontality of the trunnion axis is an important requirement in all work involving vertical movement of the telescope, as in the measurement of the horizontal angle between points at considerably different elevations (see page 171).

Test —1 Set up the instrument on firm ground. Fix the lower clamp, and level the plate bubbles carefully

2 Wheel the upper plate through 180°. If the bubbles remain central, the adjustment is correct.

Adjustment —1. If not, bring the altitude level, attached to the telescope or the index arm, parallel to a pair of levelling screws. If this tube is mounted on the telescope, set it approximately level by hand, clamp the vertical circle, and complete the levelling by the vertical tangent screw or the levelling screws . if it is mounted on the vernier frame, use the levelling screws. Turn through 90°, and centre the bubble by the levelling screws. Repeat until the bubble is central in these two positions.

2 Wheel through 180° · the bubble will leave the centre of its run Bring it half-way back by the levelling screws and the remainder by the vertical circle tangent screw or the clip screws. Repeat until the bubble remains central in any position.

3 The vertical axis is now truly vertical. By means of the adjusting screws of the plate levels bring the bubble of each to the centre of its run.

Notes —(1) The long bubble is used merely to secure greater delicacy in setting the vertical axis truly vertical The adjustment is commonly performed by levelling the plate bubbles and correcting the error shown by reversal, half by their adjusting screws and half by the levelling screws.

(2) On completion of the adjustment, the test may be repeated with the lower clamp slack and the upper fixed If the bubbles do not remain central on reversal, the outer axis of rotation is not vertical, and is not therefore parallel to the inner axis (Fig. 78*b*). If the error is serious, the instrument should be sent for repair.

(3) The adjustment is sometimes referred to as the setting of the plates

truly horizontal when the plate bubbles are central, but in reality this condition is secured only if the plates are perpendicular to the vertical axis. If either plate is oblique, its diameter generates a cone about the vertical axis on being wheeled, and the bubbles remain throughout in the centres of their runs. The only method of testing the horizontality of a plate is by examining whether a level tube can be reversed on it without deviation of the bubble, and this cannot be satisfactorily performed for the plane of the reading edge of a bevelled limb. Practically it may safely be assumed that the plates are horizontal when the axis has been set vertical, and that the plate levels are parallel to the plates.

2. Adjustment of the Line of Sight, or the Collimation Adjustment. —*Object*—To make the line of sight coincide with the optical axis of the telescope, *i.e.* to place the intersection of the hairs in that axis. This involves the adjustment of both the horizontal and the vertical hair If two inclined hairs take the place of a single vertical hair, their intersection is adjusted as for a vertical hair.

Necessity.—Horizontal Hair: If, in an external focussing telescope, the horizontal hair does not intersect the optical axis, the direction of the line of sight will vary slightly with varying position of the objective due to focussing, the movement of the objective being assumed along the optical axis. In the case of an internal focussing telescope in which the horizontal hair is not in the axis, the tilting of the line of sight caused by movement of the internal lens is much less, and the necessity for the horizontal hair passing through the optical axis is correspondingly less urgent

The requirement with respect to the horizontal hair is quite immaterial in horizontal measurements, but affects vertical measurements and levelling when focussing is necessary.

Vertical Hair: If the maker has set the optical axis of the telescope perpendicular to the trunnion axis, adjustment of the vertical hair amounts to the placing of the line of sight perpendicular to the horizontal axis, and *this constitutes the primary object of the test and adjustment.* If the focussing movement is not in the adjusted line of sight, this defect may be discovered by test, and can be eliminated in certain instruments It is likely to be inappreciable if focussing is effected by an internal lens.

When the line of sight is perpendicular to the horizontal axis, rotation of the telescope about the latter must cause the line of sight to sweep out a plane, but if not, on transiting the telescope, the line of sight will generate a cone, the axis of which is the horizontal axis of the telescope This defect introduces error in the measurement of the horizontal angle between points at different elevations, and is very important in work involving transiting of the telescope, as in the prolongation of a straight line through the instrument. It also occasions errors in setting out a straight line from one end owing to the change of inclination of the telescope necessary to sight the several pegs.

Horizontality and Verticality of Hairs.—Since the adjustment is to be made by means of the capstan screws controlling the diaphragm, it may be desirable before proceeding to the test proper to examine whether the hairs are respectively horizontal and vertical when the instrument is levelled up, as it is convenient to have them so. To test the horizontal hair, level the plate bubbles carefully, clamp the vertical motion, and sight a levelling staff or any definite point on one side of the field of view. Turn the telescope about the vertical axis, and observe whether the same point appears intersected by the other end of the horizontal hair. If not, slacken the four capstan screws, and rotate the diaphragm until the condition is secured. The hairs being fixed relatively to each other, the vertical hair should now appear truly vertical on comparing it with the image of a plumb line suspended in front of the instrument.

In making the adjustment proper, the horizontal hair is examined first, as it is of less importance for ordinary work than the vertical hair. This part of the adjustment is usually omitted if the instrument has an internal focussing telescope.

Horizontal Hair.—*Test.*—1. Set up and level the instrument carefully With all the clamps fixed, take a reading on a levelling staff held upon a firm point a few hundred feet away (Fig. 80). Note the reading and the vertical angle.

2. Unclamp, transit the telescope, and wheel through 180°. Set the vertical circle to the same angle as before

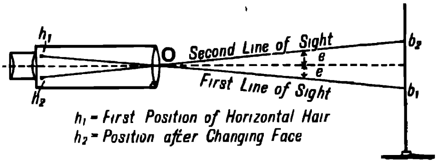

h_1
h_2
O
Second Line of Sight
e
e
First Line of Sight
b_2
b_1
h_1 - First Position of Horizontal Hair
h_2 - Position after Changing Face

FIG 80.

3. Again read the staff. If the previous reading b_1 is obtained, the adjustment is correct.

Adjustment —1. If not, move the diaphragm by the top and bottom capstans until the staff reading is the mean of those previously obtained.

2. Repeat until no error is discernible on changing face.

Notes —(1) An alternative method of reversing the line of sight consists in placing the telescope in improvised wyes and rotating it through 180° about its longitudinal axis as in adjustment 2 of the wye theodolite (page 93).

(2) Having completed the adjustment, a test may be performed to discover whether there is droop of the slide in focussing. Proceed as before, but use short sights, so that the object glass must be well racked out. If the instrument does not now pass the test, the error must be due to droop, which in certain instruments can be corrected by adjusting screws bearing upon the slide. Generally, however, the defect can be remedied only by a maker.

(3) To adjust the horizontal hair in an internal focussing telescope after a replacement, it is adequate for ordinary work to sight a staff sufficiently near that it fills the field of view. The readings at the extreme top and bottom of the field are noted, and the horizontal hair is adjusted to intersect the mean reading.

(4) If, through the horizontal hair being out of position, the line of sight makes an angle *e* with the optical axis, as shown in an exaggerated manner in Fig. 80, it must not be thought that vertical angle measurements will be that amount in error. The altitude level will be adjusted so that its axis is parallel to the line of sight, and it is only the variation of *e*, due to the focussing movement, which constitutes the error.

Vertical Hair.—*Test*—1 Set the instrument on a fairly level stretch of ground and in such a position that a sight of about 300 ft. may be obtained on either side Level up.

2 Establish a point A at about 300 ft from the instrument by thrusting a chaining arrow into the ground, or otherwise Sight A, and clamp the horizontal movement.

3. Transit, and mark B at about the same level as A and so that OB = OA approximately (Fig 81*a*)

4. Unclamp, wheel through 180°, and again sight A and clamp.

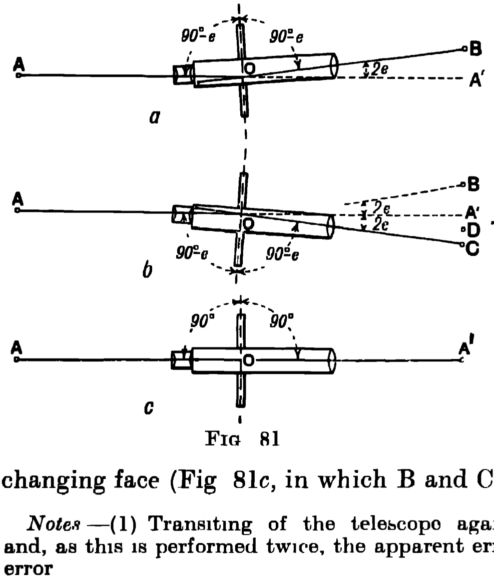


FIG 81

5. If, on again transiting, the line of sight intersects B, it is perpendicular to the horizontal axis

Adjustment.—1 If not, mark C in the line of sight (Fig 81*b*) Measure out from C towards B a length CD $=\frac{1}{4}$CB, and mark D.

2 By means of the diaphragm side capstans bring the vertical hair to D.

3 Repeat until no error is perceptible on changing face (Fig 81*c*, in which B and C coincide at A′)

Notes—(1) Transiting of the telescope again constitutes the reversal, and, as this is performed twice, the apparent error BC is four times the real error

(2) If the adjustment is out, the line of sight will not intersect the vertical axis, as assumed in Fig 81, but the small discrepancy is gradually eliminated in making the adjustment

(3) Selecting A and B at the same level is a precaution against the possibility of adjustment 3 being out

(4) By making the two sights of equal length the necessity for focussing is eliminated, and the adjustment is made correct for an average length of sight. If, however, there is slackness or lateral deviation in the focussing movement, the instrument will exhibit collimation error for sights of different length from those of the test Examination should therefore be made for this defect by repeating the test as above, but with one of the sights very short. If the instrument no longer passes the test, the error lies in the focussing movement In some instruments an adjusting screw is threaded through the side of the telescope and bears on the focussing slide, so that the defect can be remedied. In instruments having interior focussing this error may be regarded as negligible.

3. Adjustment of the Horizontal Axis.—*Object.*—To make the horizontal axis perpendicular to the vertical axis, so that it is truly horizontal when the instrument is levelled up.

Necessity.—Adjustment 2 having secured that rotation of the line of sight shall be in a plane, this adjustment ensures that such rotation shall lie in a *vertical* plane when the instrument is levelled. This condition is essential in all work necessitating motion of the telescope in altitude.

Test —1. Set the instrument in such a position that a highly inclined sight may be obtained to a well-defined point A, *e g.* the finial of a steeple Level up very carefully, and sight A.

2 With both clamps fixed, depress the telescope, and mark a point B on the ground in the line of sight (Fig 82*a*)

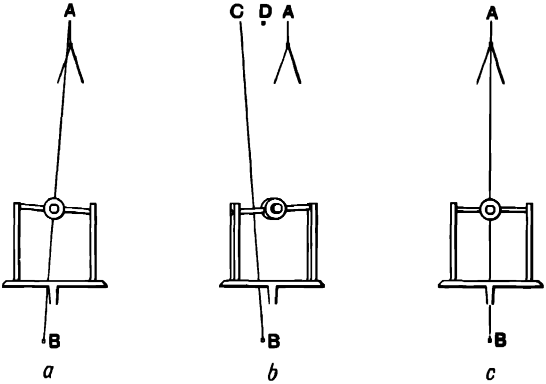


Fig 82.

3. Unclamp, transit the telescope, and wheel through 180°. Sight on B, and, with the horizontal movements clamped, elevate the telescope If the line of sight again cuts A, the horizontal axis is truly horizontal, and is therefore perpendicular to the vertical axis

Adjustment —1. If not, let the hairs appear at C, opposite A (Fig. 82*b*). By means of the adjusting screw at the trunnion support on one standard bring the line of sight from C to D, estimated midway towards A

2 Again perform the test, marking a new point B′ on the ground, and repeat test and adjustment until no error is detected on changing face (Fig 82*c*).

Notes —(1) An alternative method after the change of face consists in re-sighting A On depressing the telescope, an error will be evidenced if B is not intersected The line of sight is brought half-way towards B by means of the adjusting screws.

(2) The larger the angle between A and B the better , the instrument should therefore be placed as near the high object as possible.

(3) Owing to the short length of the sights to B, that point should be marked carefully, a pencil mark or a pin being used.

(4) Since in sighting B, after changing face, the horizontal axis will not lie exactly in the same plane as it did for the first sight if the adjustment is out, the point D, in the same vertical plane as B, is not precisely midway between

A and C. The conditions are greatly exaggerated in the diagram, and the divergence is of no account since the adjustment must be effected gradually by repeated trial.

(5) If a striding level is provided with the instrument, the horizontal axis may be adjusted by the method described in Vol II, Chap III.

(6) Some makers omit means for adjusting the axis, on the ground that it will thereby better maintain its adjustment. Should such an instrument require adjustment, the axis should be removed from the standards and the higher pivot wiped with fine emery until, on replacing the axis, the test is passed.

4. Adjustment of the Telescope Level.—*Object*—To place the axis of the level tube attached to the telescope parallel to the line of sight.

Necessity.—This adjustment secures that the line of sight shall be horizontal when the bubble of the telescope level is central, and is of prime importance in the measurement of vertical angles and when the instrument is used as a level.

Test and *Adjustment*—The procedure is exactly the same as in the "two-peg" adjustment of the dumpy level (page 116), the adjustment being made by means of the screws attaching the level tube to the telescope.

Note—When the long level tube is mounted on the vertical index arm, its adjustment is performed along with that of index error as given below.

5. Adjustment of the Vertical Index Frame.—*Object*—To make the vertical circle read zero when the line of sight is horizontal.

Necessity—If, when the line of sight is horizontal, the vertical circle reading is not zero, such reading is called an index error. On eliminating index error from an instrument otherwise in adjustment, readings of the vertical circle will represent without correction the values of observed vertical angles. No error of measurement need, however, be caused by the existence of index error if its amount is determined and applied as a positive or negative correction to the observed values of vertical angles.

In certain instruments having no means for the elimination of index error, all that can be done in testing the instrument is to determine the value of the error. To do so, set the vertical axis truly vertical by the method given in adjustment 1 (page 85), using the vertical circle tangent screw for correcting half the movement of the bubble on reversal. The reading of the vertical circle is the required index error.

If, as in the British transit, adjustment can be made between the vertical index frame and the standards, index error can be eliminated. The procedure depends upon whether the altitude level tube is mounted on the telescope or on the index frame, and to a greater extent upon the slow motion design adopted for the vertical vernier frame and the circle. In Fig. 53 the vertical circle tangent screw *29* and the clip screw *26* are on the same side of the telescope, and, if the vertical circle clamp is applied, screw *26* imparts a slow motion to the verniers, circle, and telescope without altering the reading. In more recent designs, with the object of securing a better balance, the vertical circle clamp and tangent screw are usually placed on one side of the telescope, and the clip screw fitting on the other, the tangent screw and clip screw operating on different vertical arms With this arrangement, movement of the clip screw moves the vernier along the circle and tilts the bubble tube attached to the vernier arm, but does not move the circle and telescope.

For an instrument having the clip and tangent screws on one arm, as in Fig 53, the elimination of index error is performed as follows, considering first the case in which the altitude level is mounted on the telescope:

1 By means of the vertical circle clamp and tangent screw set the vertical circle to read zero.

2. Place the bubble of the telescope level tube central in two positions at right angles by means of the levelling screws.

3 Wheel the telescope through 180°. If the bubble does not remain central, bring it half-way back by the levelling screws and half-way by the clip screw attaching the index frame to the standard, repeating until the bubble remains central in all positions.

The instrument is now accurately levelled for vertical measurements without index error.

If the altitude level tube is mounted on the index arm, the usual procedure is to combine the adjustment of the level with that of index error, so that the line of sight may be horizontal when the bubble is central and the vertical circle reading is zero

Test —1. Level the instrument by the plate levels, and by means of the vertical circle clamp and tangent screw set the vertical circle to read zero.

2. Bring the bubble on the index arm exactly central by means of the clip screw at the standard Observe a levelling staff held 200 or 300 ft. away, and note the reading

3. Release the vertical circle clamp, transit the telescope, and again set the vertical circle to read zero. Wheel through 180°, relevel if necessary, and again read the staff held on the same point If the reading is unchanged, the adjustment is correct.

Adjustment —1 If the readings differ, bring the line of sight on to the mean reading by turning the clip screw.

2 Return the bubble of the altitude level to the central position by means of the adjusting screws attaching it to the index arm.

3. Repeat until no error is discovered in the test.

Note —In the case of instruments in which the upper portion, consisting of the telescope, vertical circle, and index frame, is disengaged from the standards in packing, the elimination of index error partakes of the nature of a temporary adjustment since it requires attention on each occasion of assembling the instrument for vertical angle measurement. In any case, it is advisable to examine the instrument with reference to index error at least once a day.

Considering now the case of an instrument in which the clip and tangent screws operate on separate arms, the elimination of index error and the adjustment of the altitude level are performed as follows, whether the level is mounted on the vernier arm or the telescope.

Test —As in the last case if the level is on the vernier arm, but if a telescope level only is fitted, set the bubble central by the levelling screws.

Adjustment.—1. If the readings differ, bring the line of sight on to the mean reading by turning the vertical circle tangent screw.

2. Return the vernier index to zero by means of the clip screw.

3. Bring the bubble of the level central by means of the adjusting screws attaching it either to the vernier arm or the telescope.

4. Repeat until no error is discovered in the test

Notes.—(1) If the instrument is fitted with levels on both the vernier arm and the telescope, only one of them need be referred to in making the adjustment When one level is adjusted, and the bubble is placed central, it is only necessary to centre the bubble of the other by means of its adjusting screws

(2) Instruments having the tangent screw and clip screw on opposite sides are usually packed as one piece, so that the clip screw fitting is used only for the purpose of adjustment It is provided with a lock nut, which must be released before the screw can be operated In performing the adjustment, the lock nut should be applied before the bubble is finally centered

Elimination of Residual Errors of Adjustment.—To obtain the best possible results in angle measurement, it must be recognised that the relationships which are aimed at in the various adjustments may not have been precisely established, and that errors may have developed since the adjustments were made.

In horizontal angle work, only adjustments 1 and 3 and that part of 2 which is concerned with the vertical hair can have any bearing on the result If a horizontal angle is measured as the mean of two observations, one face right and one face left, the effects of errors of adjustments 2 and 3 are eliminated. Changing face does not, however, get rid of the error caused by the vertical axis not being truly vertical, and it is therefore advisable to test the adjustment of the plate levels frequently.

In the measurement of vertical angles, adjustments 4 and 5 are of special importance, and residual errors of the others produce small effects Change of face eliminates the principal errors in this case, but not the others, which, however, are of little consequence in vertical angulation

If, as in highly refined horizontal or vertical measurements, a number of observations is taken for averaging, it is always essential that as many measurements should be made face right as face left.

Adjustment of the Wye Theodolite

The special features of service in the testing and adjustment of this instrument are : (*a*) the telescope can be removed from the wyes and replaced end for end ; (*b*) it can be rotated in the wyes about its longitudinal axis.

1. **Adjustment of the Plate Levels.**—As for the transit theodolite.

2. **Adjustment of the Line of Sight.**—*Object*.—To make the line of sight coincide with the axis of the telescope collars. The construction of the telescope should be such that the axis of the collars which rest in the wyes coincides with the optical axis, and the wyes, which are rigidly connected to the horizontal axis, should be so placed with respect to it that the optical axis of the telescope is perpendicular to the horizontal axis.

Necessity.—As in adjustment 2 of the transit theodolite. Further, rotation of the telescope in its supports causes an unadjusted line of sight to generate a cone, and, unless the telescope can be clipped in one position, small rotations are likely to occur in observing.

Test.—1. Plant the instrument firmly, and carefully sight a definite small mark. Fix all clamps.

2. Rotate the telescope in the wyes through 180°. If the intersection of the hairs remains on the mark, the adjustment is correct.

Adjustment.—1. If not, by means of the four screws controlling the diaphragm move the intersection of the hairs half-way towards the point.

2. Set the line of sight again on the mark by the tangent screws, and repeat the test and adjustment until correct.

Notes.—(1) It is unnecessary to level the instrument.

(2) Unlike the same adjustment of the transit, both hairs are adjusted in the one operation. The effect of the reversal is that illustrated in Fig. 80.

(3) The adjustment has secured perpendicularity between the line of sight and the horizontal axis only if the optical axis and that of the collars are at right angles to the horizontal axis. If difficulty is experienced in effecting adjustment 3, there will be reason to suspect that those relations are not fulfilled, in which case the correction cannot be undertaken by the surveyor.

(4) Having performed the adjustment, a test for deviation of the object glass slide in focussing should be made by repeating the procedure with a new length of sight necessitating a considerable change of focus.

3. **Adjustment of the Horizontal Axis.**—As in the transit theodolite, but in place of transiting the telescope in the test, it is removed from the wyes and replaced end for end.

Notes.—(1) An alternative method is to sight a plumb line suspended in front of the instrument and observe whether the intersection of the hairs remains on the line while moving the telescope through as large an angle as possible about the horizontal axis.

(2) In wye theodolites the horizontal axis is not readily removable from the standards, and few are provided with capstans for making this adjustment. The caps at the top of the standards must be unscrewed, and the high end of the axis reduced with emery.

4. **Adjustment of the Telescope Level.**—The axis of the telescope level is placed parallel to the line of sight by the method of adjustment 2 (*A* and *B*) of the wye level (page 119). Alternatively, but

less conveniently, *B* may be performed by the peg method used for the transit and the dumpy level.

5. Determination of Index Error.—The usual construction of the instrument does not enable the surveyor to eliminate index error, and its value must be determined by the method on page 90.

THE COMPASS

The compass is an instrument designed for the measurement of directions with reference to the magnetic meridian The essential parts are a magnetic needle, a graduated circle, and a line of sight. The various types may be classified as ·

1. The surveying compass, circumferentor, or miner's dial
2. The prismatic compass

Instruments in the first category are commonly used in mine surveying, but for general surveying are now employed to a much smaller extent than formerly Some of these instruments are adapted for the measurement of horizontal and vertical angles, but they are inferior to the small theodolite

The prismatic compass is a lighter and simpler instrument, and is of the greatest service to the engineer or surveyor for rapid traversing

Plain Compass.—Fig 83 illustrates a simple form of surveyor's compass with plain sights The compass box, of about 5 in diameter, has the circle graduated to degrees, and mounts an edge bar needle The sight vanes are screwed to a plate attached to the underside of the compass box, and are hinged to fold down upon a metal cover which protects the compass box when not in use. The sight vanes are sometimes adapted for measuring slopes by having the object vane engraved with a scale of tangents and the eye vane provided with a pin-hole sight.

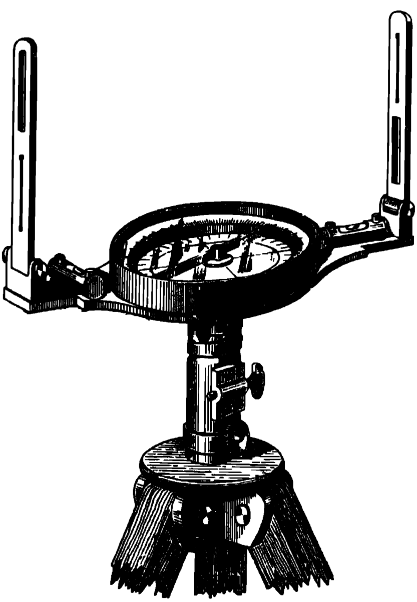


Fig 83.—Plain Compass.

The compass is graduated either on the whole circle or the quadrantal system (page 160). The graduation increases counter-clockwise, so that the 90° or E. mark is to the left, and 270° or W. to the right, of zero or N., this reversal being required because the graduated circle moves with the line of sight, while the needle remains stationary. On wheeling the line of sight from north towards east, for example, the increasing graduations on the left of zero are successively brought opposite the north end of the needle. Some patterns are furnished with means whereby the graduated circle may be turned, independently of the line of sight, through an angle corresponding to the magnetic declination, so that the needle may record true bearings.

The level tubes are sometimes omitted, and the instrument is levelled by adjusting it until the ends of the floating needle lie in the plane of the graduations Levelling is performed by means of a levelling screw head or, alternatively, by a ball and socket joint, as in Fig 83

The instrument is mounted on a single rod, called a Jacob staff, or on a light tripod A form of tripod common in mining instruments has the legs jointed in the middle of their length For observations where headroom is limited, the lower sections are unscrewed, and points are screwed to the upper lengths to form a low tripod. The low tripod sometimes proves very convenient in surface work for sighting through woods by enabling the surveyor to keep his line of sight below the foliage.

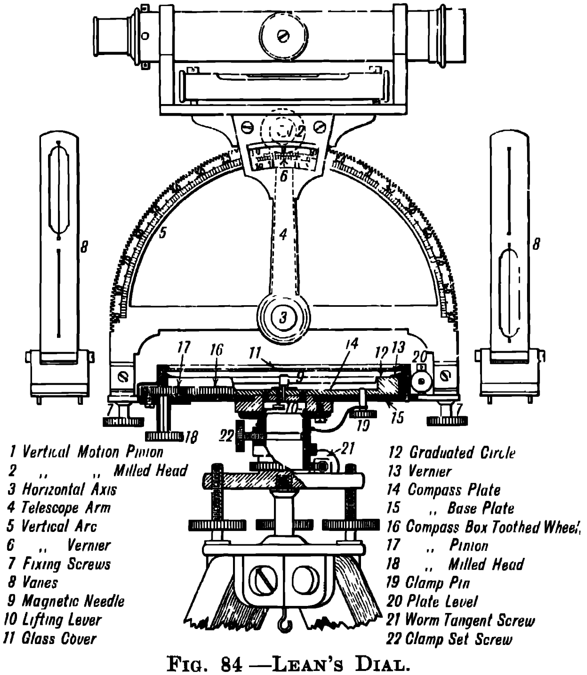


FIG. 84 —LEAN'S DIAL.

Other Forms of Compass.—The more elaborate forms of compass are better adapted for taking inclined sights. Usually both a telescope and vanes are provided, one or the other being used according to circumstances. These instruments can be employed in the manner of a theodolite for the measurement of horizontal and vertical angles. Although varying in detail, many of the patterns available are modifications of the two type instruments, Lean's dial and Hedley's dial.

In Lean's dial (Fig. 84), the compass base plate carries either folding vanes or a telescope and vertical arc, the same screws *7* being used for the attachment of either By means of a pinion *1*, gearing with the toothed vertical arc, the telescope can be moved tangentially round the arc about a short horizontal axis or hinge *3* The proportions are such that a vertical sight can be taken, as required in transferring a line down a shaft in mine surveying.

The needle is of edge-bar form, and the bottom plate of the compass box is graduated counter-clockwise to degrees, usually on the quadrantal system On the step, at the same level as the upper edge of the needle, there is a circle *12*, graduated clockwise on the whole circle system The compass plate *14* is provided with a toothed wheel in gear with a pinion *17* actuated by the milled head *18*. With this arrangement, when the clamp pin *19* is withdrawn, the base plate, with the outer portion of the compass box and the telescope, can be rotated about the fixed compass plate and graduated circle The compass box carries a vernier *13*, so that bearings and horizontal angles may be measured as with the theodolite Otherwise, with the pin *19* locking the compass plate to the base plate, the instrument is used as a plain compass, and bearings are read directly from the needle

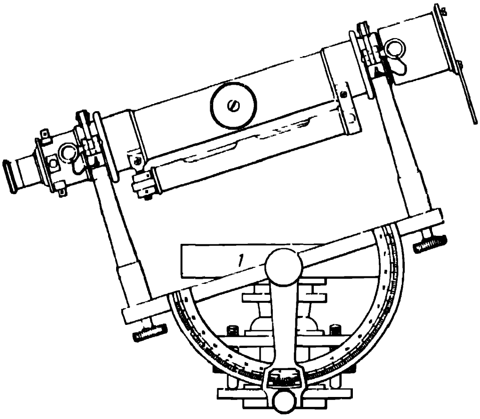


FIG. 85.—HEDLEY'S DIAL.

In Hedley's dial (Fig. 85), the characteristic feature is the method of supporting the frame carrying the telescope or sight vanes It is mounted on two diametrically opposite pivots which project from the underside of the compass box *1*, and about the horizontal axis so formed the line of sight may be moved in the vertical plane. At one side there is fitted a vertical arc, which moves with the line of sight and is read against a vernier, the arm of which is rigidly fixed to the pivot. In other respects this instrument does not differ essentially from Lean's dial.

Testing and Adjustment of the Compass.—In addition to the properties which should be fulfilled by the needle (page 42), the following requirements of the compass can be established by adjustment.

1 When the plate bubbles are in the centres of their runs, the vertical axis should be truly vertical

2. Sight vanes should be vertical when the instrument is levelled.

3 The telescopic line of sight should be perpendicular to the horizontal axis, which should itself be perpendicular to the vertical axis

4 The extremities of the needle should lie in the same vertical plane with the pivot.

5. The pivot point should be vertically over the centre of the graduated circle.

1. Adjustment of the Plate Levels.—*Object*—To set the axes of the plate level tubes perpendicular to the vertical axis

Necessity—Non-adjustment in this respect introduces error whenever the line of sight is moved in altitude Further, since the compass box and plate are placed by the maker perpendicular to the vertical axis, the adjustment practically ensures their horizontality when the bubbles of the plate levels are centered.

Test.—1. Set up the instrument on firm ground, and level carefully

2. Wheel through 180°. If the bubbles remain central, the adjustment is correct.

Adjustment.—1. If not, correct half the errors by the level tube capstans and half by the levelling screws or ball and socket joint.

2. Wheel through 180°, and, if necessary, repeat the adjustment until the bubbles remain central throughout a rotation.

2. Adjustment of the Sight Vanes.—*Object*—To make the sight vanes vertical when the instrument is levelled

Necessity.—The error caused by non-verticality of the vanes will generally be negligible so long as the slit and sighting line are in the same plane. Otherwise, the direction of the line of sight from a particular point on the eye vane will vary according to which point on the object vane is referred to in sighting.

Test.—1. Level the instrument carefully, and look through the sights at a plumb line or the corner of a building. Note whether the line on the object vane coincides with the vertical line observed.

2. Wheel through 180°, and test the other vane in the same way.

Adjustment.—If necessary, either file one side of the bottom of the vane where it rests on the plate or insert a paper packing. Repeat the test and adjustment until the error is eliminated.

3. Adjustment of the Telescope.—In compasses having a telescope, the line of sight must be perpendicular to the horizontal axis, which should be perpendicular to the vertical axis. The necessity for these is the same as in the theodolite (pages 86 and 88), and the method of testing and adjustment follows that of the transit or wye theodolite according to the manner in which the telescope is mounted. Usually no provision is made for adjusting the horizontal axis.

4. Adjustment of the Needle.—*Object.*—To straighten the needle.

Necessity.—A needle bent horizontally will not have the magnetic axis coincident with the geometrical axis, and in consequence the readings are in error by an amount which remains constant if the N. end only is read (*cf* 5, page 83). If the reading ends are not in the same horizontal plane with the pivot point through vertical bending of the needle or faulty construction, they will be found to quiver when the needle swings, and this proves inconvenient in reading. This defect should be eliminated before proceeding to remedy horizontal bending.

Test for Vertical Bending—Lower the needle on the pivot, and observe whether trembling of the needle is accompanied by rocking of its upper surface at the ends.

Adjustment—If so, remove the needle, and bend it in a vertical direction until no rocking is noticeable on replacing it.

Test for Horizontal Bending.—1. Read both ends of the needle in any position

2. Rotate the instrument until the graduation originally opposite the N. end of the needle comes opposite the S end.

3. If the reading at the N. end is now that originally at the S end, the needle is straight. If not, half the discrepancy represents the deviation from straightness

Adjustment.—1. Remove the needle, and bend the N. end horizontally through this angle in the direction which would carry it from the N. end reading after reversal towards the new position of the original S end reading.

2. Repeat test and adjustment as often as necessary.

Note.—The above method of testing for horizontal bending is designed to be independent of whether the pivot is in the centre of the graduated circle or not, and should therefore be applied in place of the more obvious method of noting whether the end readings differ by 180°.

5. Adjustment of the Pivot.—*Object.*—To set the pivot point in the centre of the graduated circle.

Necessity.—If the pivot is not central, the true reading will be obtained only when the needle happens to lie in the same vertical plane as the line joining the centre and the pivot (*cf.* 4, page 83).

Test.—Read both ends of the needle for various positions of the circle, and note whether the readings differ by a constant amount, which should be 180° when the needle is straight If so, the pivot is central.

Adjustment.—1. If the difference between the readings is variable, ascertain as nearly as possible the position of the needle at which the discrepancy is greatest.

2. Remove the needle, and with a small pair of pliers gently bend the pivot at right angles to this position of the needle and towards the greater segment of the circle formed by the needle

3. Repeat test and adjustment until the end readings agree in all positions

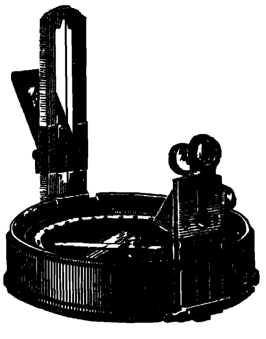


FIG 86 —PRISMATIC COMPASS.

The Prismatic Compass.—This extremely useful instrument (Fig 86) is the most portable form of surveyor's compass, and is specially designed for use as a hand instrument. The construction is illustrated in Fig. 87. The compass needle is of broad form, and carries an aluminium ring of from 2½in to 6 in diameter, graduated to half degrees The special feature of the instrument lies in the construction of the eye vane, which carries a reflecting prism whereby a view of the compass ring is presented to an eye placed opposite the sighting slit. The observer, while sighting through the slit past the object vane wire or bar, sees the latter cutting the image at the required bearing, which is therefore read simultaneously with sighting (Fig. 88). The compass ring is graduated from the S. end of the needle because the

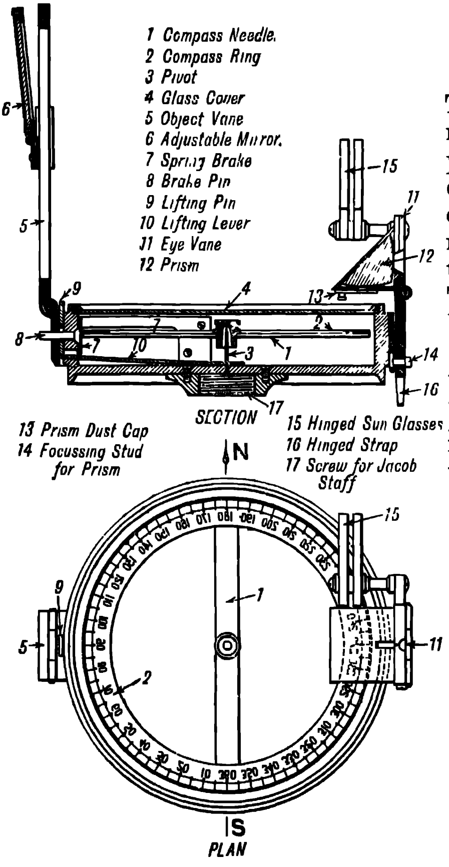


FIG. 87.—PRISMATIC COMPASS.

readings are taken at the end of the diameter remote from the object. The prism has both the horizontal and vertical faces convex, so that a magnified image of the graduation is formed, and focussing to suit different observers is effected by moving the prism vertically by means of the stud *14*. To reduce excessive oscillation of the compass ring caused by unsteadiness of the hand, a light spring *7*, carrying a pin *8*, is fitted inside the compass box. On gently pressing the pin inwards, the spring is made to touch the ring and act as a brake

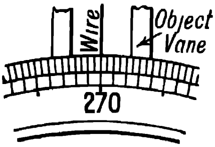


FIG. 88.—APPEARANCE ON SIGHTING

When the instrument is not in use, the object vane is folded down on the face of the glass cover, and presses against the pin *9*, which lifts the needle off the pivot The eye vane folds outwards, and is held by the hinged strap *16*, and a metal lid is placed over the glass cover and object vane In an alternative arrangement, the brass cover is a fixture, and is fitted with a window under the prism The standard military pattern has the cover lid designed to serve as the object vane. It is hinged, and is fitted with a glass disc on which is ruled a sighting line In what is called a liquid compass, the card dial carrying the graduations is floated in liquid with the object of reducing pivot friction

Some accessory parts are usually added. The dark glasses *15*, are so placed that they can be swung into the line of sight when sun observations are required The object vane is commonly provided with a hinged mirror *6*, by means of which points considerably above or below the instrument can be sighted by reflection. The mirror is hinged to a slide which can be placed upwards or downwards on the vane, along which it may be moved as required. No provision is made for adjusting the vanes.

Although commonly used as a hand instrument, the compass may be fitted with a screwed socket for attachment to a Jacob staff or light tripod with ball and socket head The facility thus afforded for levelling the instrument makes for increased accuracy, particularly when the line of sight is considerably inclined to the horizontal.

Observing with Prismatic Compass.—The instrument is held with the forefinger over the brake pin, and is turned until the distant signal appears in the line of sight Care must be exercised to hold it as nearly level as can be judged : if the ring does not oscillate, it is probably in contact with the glass cover through the instrument being out of level. A light touch is applied once or twice to the brake pin until the needle is nearly steady. The readings of the two limits of the swing are estimated to the nearest ¼°, and their mean is taken as the result.

THE LEVEL

The surveyor's level is an instrument designed to furnish a horizontal line of sight. Neglecting for the present various forms of rough levelling instruments, the level consists essentially of a bubble tube attached to a telescope, the axis of the bubble tube and the line of sight being parallel to each other. The instrument is provided with levelling screws by which the bubble is centered and the line of sight brought into a horizontal plane.

Types of Levels.—According to their general arrangement, levels may be classified as · (1) the dumpy or Gravatt level ; (2) the wye level ; (3) levels possessing features of both the dumpy and wye types , (4) the tilting level. The essential feature of the first is the rigid connection of the telescope to the vertical spindle. In the wye level the telescope is mounted as in the wye theodolite. The instruments in the third category are designed to combine the advantages of the solid construction of the dumpy with the ease of adjustment of the wye. The distinguishing feature of the tilting level is that the telescope and attached bubble tube may be levelled without the necessity for setting the rotation axis truly vertical.

General Features.—The several patterns do not differ materially as regards the levelling head, which should be of the three-screw type in preference to one of four screws for the reasons stated on page 59. A quick levelling head (page 112) may be fitted.

The special requirements of the telescope are good definition and illumination, and to promote the latter a positive eyepiece only should be used, and the effective aperture of the objective should be not less than $1\frac{3}{8}$ in The telescope is not limited in length as in the transit. Focussing is effected either by an internal lens or by movement of the tube mounting the eyepiece and diaphragm. In the former case the length of the telescope, and in the latter the focal length of the objective, serves to designate the size of the level, which varies from 7 in. to 16 in. in instruments for ordinary levelling. The reticule in the dumpy level is usually of the form shown at *A* (Fig. 18), but instruments with removable telescopes must have the line of sight defined by the intersection point of two hairs Stadia hairs are commonly inserted.

The sensitiveness of the bubble tube should accord with the delicacy of the levelling screws and the resolving power of the telescope (page 84) In instruments for ordinary levelling the sensitiveness ranges from 15″ to 25″ per $\frac{1}{10}$ in. division.

The Dumpy Level, Original Pattern.—The dumpy level, designed by Gravatt, is of the general construction illustrated in Fig. 89, which shows an obsolescent form of four-screw instrument. The telescope tube is firmly secured in two collars fixed by adjusting screws to the stage carried by the vertical spindle. The upper ends of the collars support the bubble tube by a hinge at one end and a screw and locking nuts at the other to permit of the adjustment of the tube relatively to the line of sight. A circular spirit level, or, in its place, a short bubble tube at right angles to the long tube, is provided as an aid to planting the instrument approximately level.

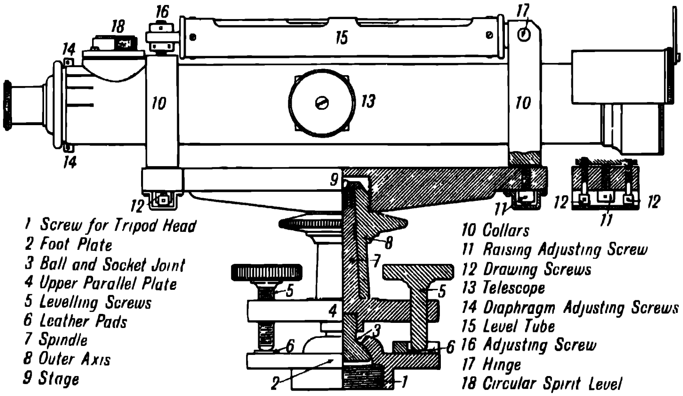


FIG 89 —DUMPY LEVEL

Other Types of Dumpy Level.—The modern form of dumpy level, usually termed the solid dumpy (Fig 90), has the vertical spindle and the telescope barrel cast in one piece. This design affords a maximum of rigidity, while the reduction in height adds to the stability of the instrument. In Messrs. Watts' standard dumpy

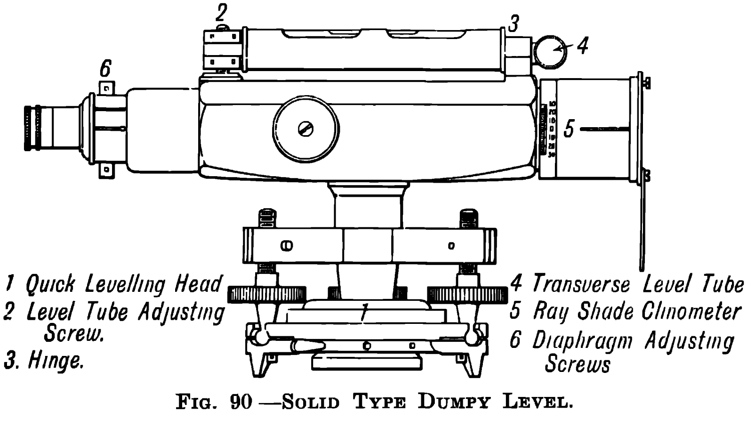


FIG. 90 —SOLID TYPE DUMPY LEVEL.

(Fig. 91), the instrument is of the solid type, but the level tube is mounted on the left-hand side of the telescope. Accessory features are internal focussing of the telescope, screw focussing of the eye piece, bubble reflector, and dust-proof levelling screws.

In a pattern of dumpy level formerly made by Messrs. Troughton and Simms (Fig. 92), the characteristic feature is the rigid attachment of the level tube to the body of the telescope without the usual provision for adjustment Adjustment between the level tube axis and the line of sight must therefore be effected by movement of the latter

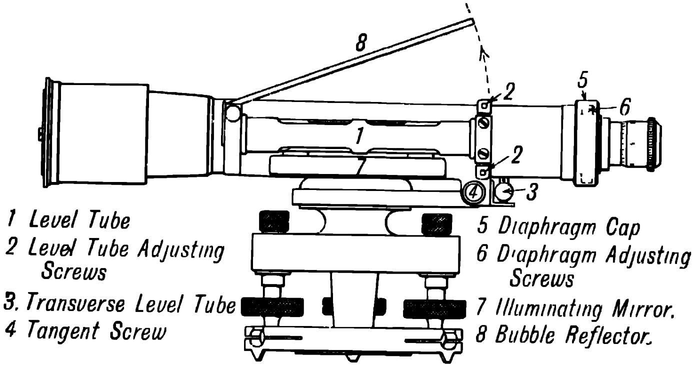


FIG 91 —WATTS' STANDARD DUMPY LEVEL.

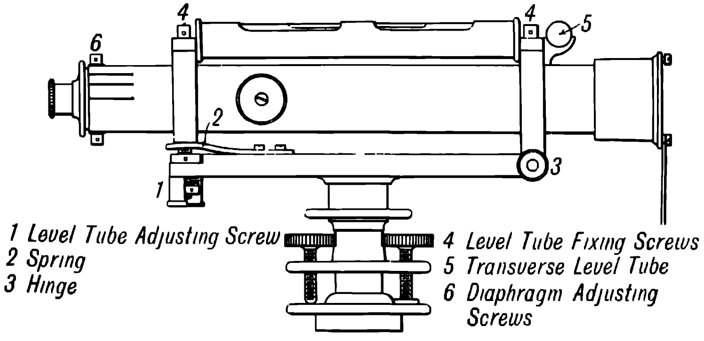
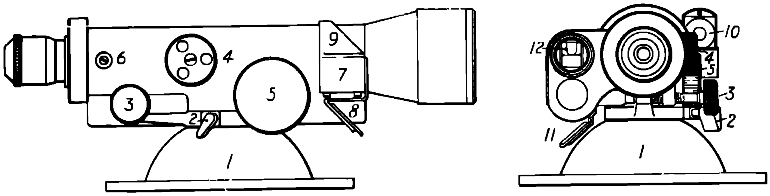

4
4
6
5
2
3
1
1 Level Tube Adjusting Screw
2 Spring
3 Hinge
4 Level Tube Fixing Screws
5 Transverse Level Tube
6 Diaphragm Adjusting Screws

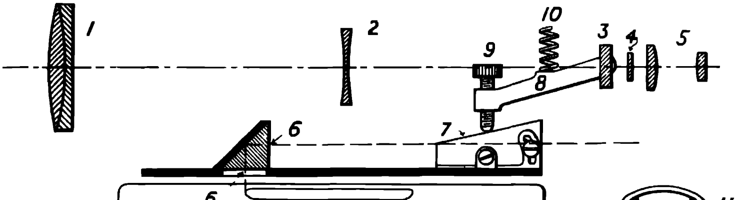
FIG. 92.—TROUGHTON AND SIMMS' DUMPY LEVEL.

Watts' "Autoset" Level.—This interesting pattern of dumpy level (Fig. 93) was designed by Messrs Watts to afford a maximum of rapidity in operation. The novel feature is that the bubble need not be exactly central while an observation is being made. The instrument is levelled by a quick levelling head and handle with reference to a small circular level, and errors of levelling, of as much as 4 min., are neutralised by displacement of the image of the staff relatively to the horizontal hair by the appropriate amount. This is accomplished by means of the device shown in Fig. 94. The prism carries a reference line on the horizontal face and marks for the avoidance of parallax error on its vertical face, and is mounted on a slide. By turning screw *5* (Fig 93) the prism can be placed over one end of the bubble, which is of constant length (page 34) A view of the end part of the bubble is presented to the observer, and the setting is effected when the prism line appears to touch the end of the bubble The slide mounting the prism also carries a wedge-shaped cam *7*, which actuates lever *8* On the lever is mounted a parallel glass plate, through which pass the rays forming the image in the plane of the ruled surface of the diaphragm plate Tilting of the lever caused by adjusting the reference line on to the end of the bubble tilts the parallel refracting plate, and produces a displacement of the image such that the staff graduation actually read against the horizontal hair is that which would be observed with the bubble accurately central. As shown in Fig. 94, the cam is mounted so that



1 Quick Levelling Head
2 Clamp
3 Tangent Screw
4 Focussing Screw
5 Auto levelling Screw
6 Parallel Plate Pivot
7 Circular Level Box
8 Circular Level Illuminating Mirror
9 Circular Level Prism
10 Image of Circular Level
11 Main Level Illuminating Mirror
12 Image of Main Bubble

Fig 93.—Watts' "Autoset" Level



1 Object Glass
2 Internal Focussing Lens
3 Parallel Glass Refractor
4 Diaphragm Plate
5 Eyepiece
6 Prism Index Marks
7 Cam
8 Tilting Lever
9 Adjusting Screw
10 Spring maintaining Contact of 9
11 Section of "Constant" Level Tube

Fig 94.

it can be set to correspond with the sensitiveness of the particular level tube fitted.

The Wye Level.—In this instrument (Fig. 95), the stage carries two wye supports in which the telescope is clipped. The body of the telescope is fitted with two cylindrical flanged collars of equal diameter, which rest in the wyes, and the telescope is capable of rotation about the axis of the collars. For purposes of adjustment, the telescope can be lifted from the wyes and replaced end for end A clamp and tangent screw are provided to facilitate pointing of the telescope In the example illustrated, the bubble tube is suspended from the telescope, but it may be fixed on top of the latter.

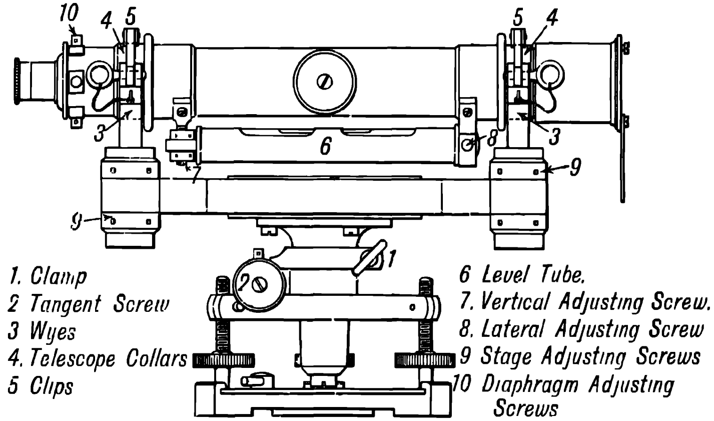


Fig 95.—Wye Level

In a pattern of wye level formerly made by Messrs Troughton and Simms (Fig 96), the bubble tube is mounted on the stage instead of on the telescope

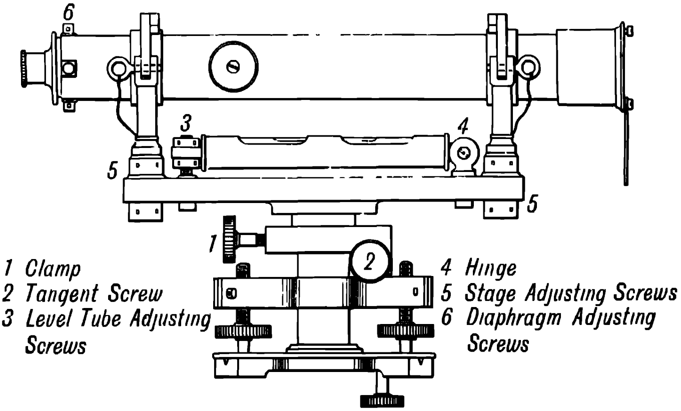


Fig 96 —Troughton and Simms' Wye Level.

Comparative Merits of the Dumpy and Wye Levels.—The dumpy level was introduced because of the objections offered by surveyors to the defective performance of the wye level arising from its non-rigid construction; and, by virtue of its superior solidity and compactness, the dumpy has established itself as the favourite level in this country. The wye instrument has an advantage in the rapidity and ease with which the adjustments can be tested, but the dumpy level retains its adjustments better.

Cooke's Reversible Level.—This instrument (Fig 97) belongs to the class of levels combining features of both the dumpy and wye instruments. The telescope is similar to that of the wye level, having two equal collars, one of which is formed with a stop flange *3*. The telescope supports are connected by a rigid socket into which the telescope can be introduced from either end and pushed home until the flange bears. The telescope can then be fixed in position by screw *4*, and the instrument used as a dumpy level. To reverse the line of sight in the course of testing and adjustment, screw *4* is slackened, and the telescope is withdrawn

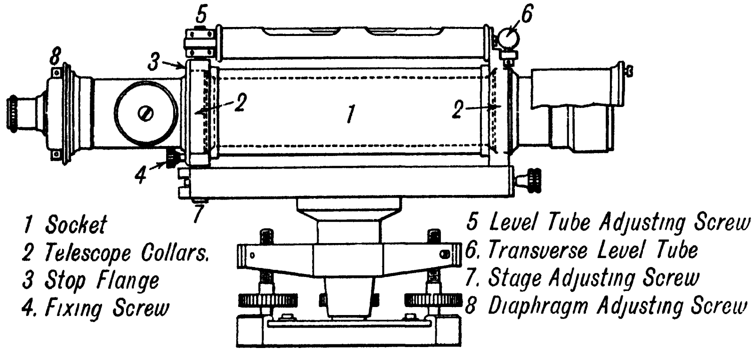


FIG. 97 —COOKE'S REVERSIBLE LEVEL.

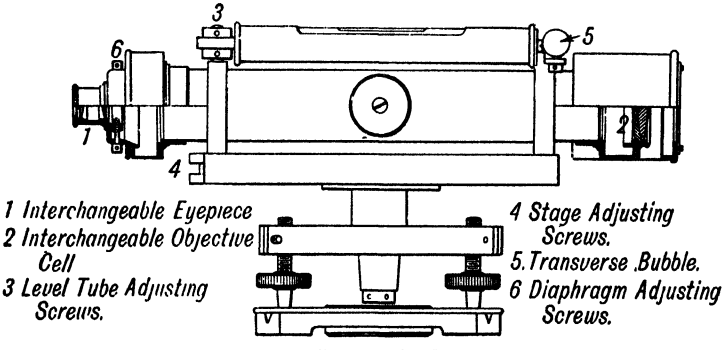

3
6
5
2
1
4
1 Interchangeable Eyepiece
2 Interchangeable Objective Cell
3 Level Tube Adjusting Screws.
4 Stage Adjusting Screws.
5. Transverse Bubble.
6 Diaphragm Adjusting Screws.

FIG. 98.—CUSHING'S LEVEL.

from the socket and replaced end for end as with the wye level. The telescope can also be rotated within the socket about the axis of the collars.

Cushing's Level.—The telescope tube of this instrument (Fig. 98) is secured in collars as in the dumpy level, and is enlarged at the ends to form two exactly equal sockets, ground to receive either the objective cell or the eyepiece and diaphragm. Reversal of the line of sight is obtained by interchanging these parts. The end fittings are normally held in the sockets by bayonet notches and screws, but can be released and rotated in their sockets. To permit of interchanging the ends in the field, the reticule is formed of lines on glass

The Charlton Level.—The bubble tube in this instrument (Fig. 99) is screwed on the stage which carries the telescope supports and which can be adjusted relatively to a lower stage fixed to the vertical spindle The telescope is secured to the collars by the clip screws 7, and, when these are slackened, it can be reversed end for end, or rotated as in the wye level

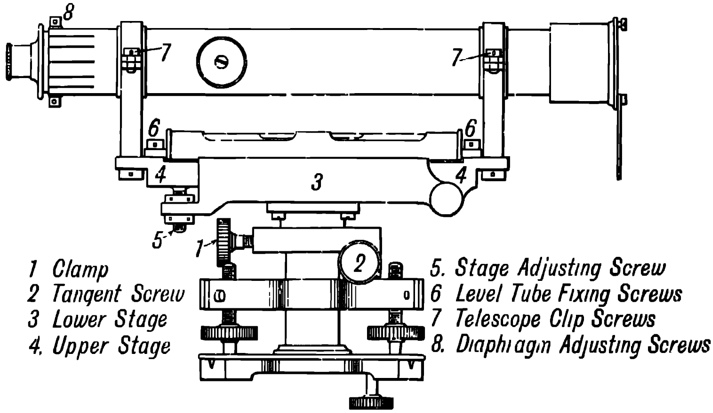


FIG. 99 —CHARLTON LEVEL.

The Tilting Level.—In both the dumpy and the wye level the line of sight is at right angles to the vertical axis if the instrument is in adjustment, so that levelling up of the instrument not only makes the line of sight a horizontal one, but also places the vertical axis truly vertical. In the tilting level the telescope with its attached level tube can be levelled by a finely pitched screw independently of the vertical axis, and, in consequence, the line of sight is not in general at right angles to the vertical axis. This type of level was first designed for precise work, but it has grown greatly in popularity for ordinary levelling, and several patterns are available under different names.

In using these instruments, the axis is set only approximately vertical by the levelling screws, or simply by a quick levelling ball and socket head, with reference to a circular level. When a staff reading is about to be taken, the bubble in the main level tube is centered exactly by means of the fine levelling screw, which also tilts the telescope. It is necessary that the observer should have a view of the bubble while sighting the staff, and a bubble reflector is fitted The circumstance that the level tube axis is not generally at right angles to the vertical axis leads to the necessity for centering the bubble by the fine levelling screw before every observation.

Tilting levels possess the merit that the use of a fine levelling screw facilitates the exact centering of the bubble at the moment of observation They also save time in the levelling of the instrument, since it is only roughly levelled by the foot screws, and the final levelling can be performed rapidly after a little practice This advantage is not so apparent in work involving numerous observations from each instrument station With instruments having the line of sight and the bubble tube axis at right angles to the vertical axis, a series of intermediate sights may be obtained without the necessity for adjusting the levelling screws.

The Reversible Tilting Level.—Although it does not constitute an essential feature of the tilting type of level, the great majority of these instruments have the telescope mounted so that it can be rotated about its longitudinal axis The bubble tube is connected to one side of the telescope, and a rotation of the latter through 180° moves the bubble tube from, say, the left-hand to the right-hand side of the instrument, and turns it upside down The names, reversible level and self-adjusting level, are sometimes used to indicate the inclusion of this feature. A bubble tube capable of being used in the reversed position must be barrel-shaped, but, even although a longitudinal section shows the upper and lower walls to have the same curvature, it does not follow that the axes (page 31) of the tube in the direct and reversed positions will be parallel to each other. Nevertheless, the accuracy of the instrument can be tested from a single station, since a true staff reading can be obtained as the mean of four readings, obtained by rotating the telescope and bubble tube through 180° about the longitudinal axis, and also by reversing the telescope end for end.

The ideal reversible level tube is one which in longitudinal section has perfect symmetry both about a horizontal centre line and about the vertical passing through the centres of curvature of the upper and lower surfaces These conditions are approached very nearly in certain makes of tube. A notable advance in the application of reversible level tubes, however, was made by Mr. T. F. Connolly,* who has patented a type of level in which the bubble tube is so mounted as to be capable of adjustment to secure that it may be rotated about its longitudinal axis or an axis parallel to it without movement of the bubble along the tube Although the bubble tube may not be perfectly symmetrical, the reversal of the bubble without movement defines the position of two parallel axes, which can be recorded by calibration marks on the tube By means of such an adjusted bubble tube, the horizontality of the line of sight of the telescope may be tested by means of two observations, with the telescope in the direct and reversed positions respectively

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* "New Types of Levelling Instruments using Reversible Bubbles," *Trans. Optical Society*, Vol. XXV, and "Theory of the Adjustment of Levels," *Journal of Scientific Instruments*, Vol. I.

The Zeiss Level.—The instrument introduced by the firm of Zeiss is typical of reversible tilting levels Different patterns of the instrument are made ; those designed for ordinary levelling are considered here, the precise instrument being treated in Vol. II, Chap. V

Fig 100 shows an early pattern in sectional elevation Three

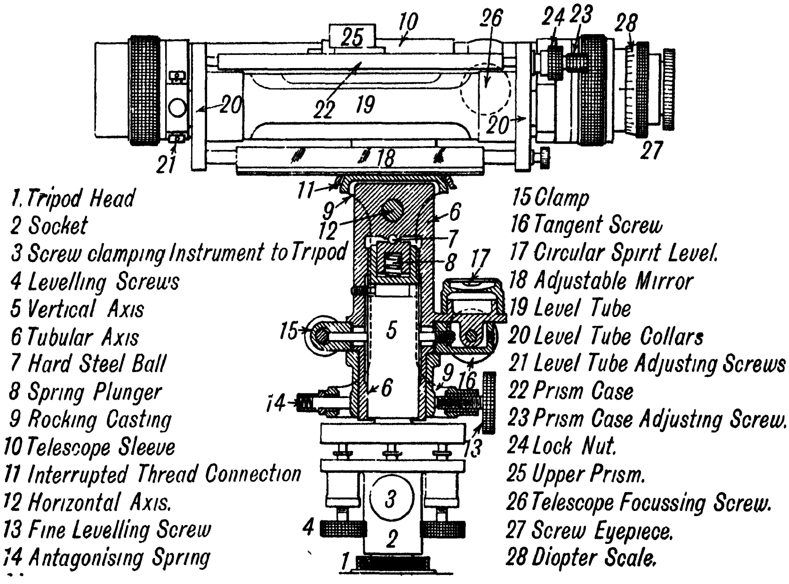


FIG. 100 —ZEISS LEVEL.

levelling screws and a circular spirit level are provided for the rough levelling of the instrument. The vertical axis is long and is of cylindrical form, the weight of the upper portion of the instrument being carried on the hard steel ball *7*. The vertical axis is surrounded by a tubular axis *6*, through the upper portion of which passes the horizontal axis *12*, about which the telescope can be rotated through a small vertical angle. This motion is obtained by mounting the sleeve carrying the telescope on the top of a casting *9*, pivoted at *12* This casting is in the form of a hollow cylinder with the periphery cut away on both the objective and eyepiece sides, and is shown dotted for the greater part of its length The final levelling of the instrument is effected by means of the screw *13* and antagonising spring *14*, which tilt the pivoted casting, telescope, and level tube about the horizontal axis.

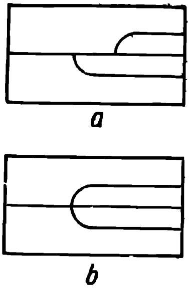
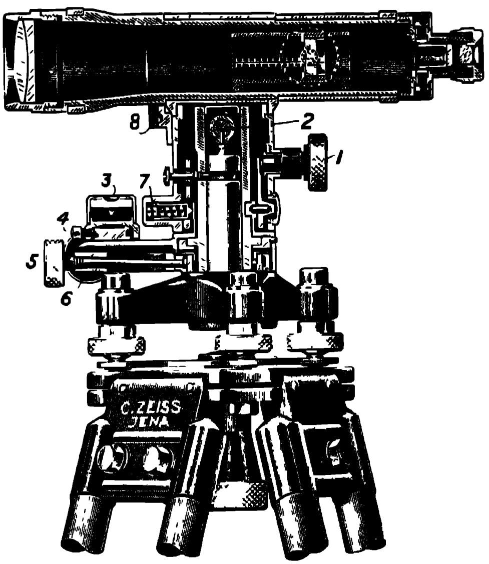


Fig. 101.

The level tube is ground to the appropriate curvature on both the upper and lower sides, and its metal case is cut away on top and bottom. The tube lies on one side of the telescope, and is mounted between two collars which are rigidly fixed to the telescope. An important feature of the instrument is the arrangement whereby a view of the bubble is presented to the observer. The bubble is illuminated by means of the mirror *18*, and the ends of the bubble are reflected in an arrangement of prisms in the case *22*, from which the images are reflected at the prism *25* into the eye. The level tube is not graduated, but when the bubble is not quite central the appearance is as in *a* (Fig. 101). By manipulating the screw *13*, the central position is attained when the halves of the two ends appear coincident as at *b*.

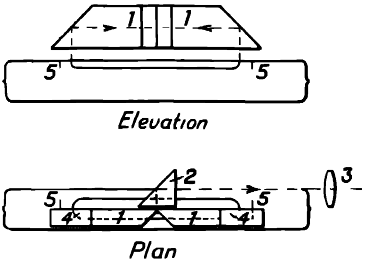


1 Tilting Screw
2 Horizontal Axis
3 Circular Spirit Level
4 Adjusting Screw for Do
5 Clamp
6 Tangent Screw
7 Counter Spring for Tilting Movement.
8 Stop for Telescope Reversal.

Fig. 102.—Zeiss Level II.

For the purpose of testing the instrument, the level tube can be rotated along with the telescope through 180° about the telescope axis, so that, if the level tube is originally on the observer's left, it may be swung over to the right-hand side. For the same purpose, the line of sight may be reversed end for end in an unusual manner. The telescope, which has an internal focussing lens, is also fitted with a lens exactly the same as the objective, and placed in front of the eyepiece. Reversal is effected by removing the eyepiece and fixing it in front of the objective, the additional lens then acting as the objective. Prism *25* can be turned through 180° to face the observer.

Fig. 102 shows the most recent model. The principal difference from former patterns consists in the introduction of Mr. Connolly's type of bubble tube. By means of two reference lines on the tube the bubble reading attachment can be set accurately to define the two parallel bubble tube axes. The correct setting of the prism case is secured when the images of the reference lines are seen to coincide. The testing of the instrument can be effected at a single station by taking two readings on a fixed staff, bubble tube left and right, the mean being the true reading. The necessity for detaching and replacing the eyepiece does not therefore arise, and an ordinary internal focussing telescope, 8¼ in. long, is fitted. The accuracy with which the bubble can be centered is increased in this pattern by adding a lens to the prismatic reading system (Fig. 103) so that the bubble ends are seen with a two-fold magnification. It is worthy of notice that the Zeiss prism reader enables the bubble to be centered with a high degree of accuracy. Tests have shown that with a tube having a sensitiveness of 15 to 16 sec. per 2 mm. the bubble can be centered with a mean error of less than 0·5 sec.



1 Coincidence Prism
2 Reader Prism
3 Magnifier
4 Parts of Bubble Reflected
5 Reference Marks

FIG 103 —BUBBLE READER.

Other Forms of Tilting Level.—The patterns designed by different makers vary in detail. Commonly the fine levelling screw is placed vertically and operates directly upon the telescope barrel. It may be provided with a scale so that it is available for measuring or setting out gradients. In the Casella tilting level use is made of the Zeiss bubble reflector, but the image of the bubble ends is brought into the field of the telescope, the plane separating the halves of the bubble coinciding with that of the horizontal hair. A " Constant " level tube is employed in Watts' self-adjusting level, and a view of one end of the bubble is presented by a prism when the tube is on the observer's left, the other end being shown when it is on his right.

Accessories to Levels.—*Quick Levelling Head.*—This proves a very useful fitting for work in rough country, where it is often troublesome to set up the instrument approximately level Various forms differ in detail, but the majority are constructed as a ball and socket joint, with the addition of a clamping device (Fig. 90) The tripod having been planted, the clamp is released, and the instrument is turned on the ball until approximately level It is then clamped in that position, and the levelling is completed by means of the foot screws

Compass—Some levels, particularly older patterns, have a circular compass mounted on the stage. In general the extra weight is not justified, as this compass is seldom used

Bubble Reflector.—In the majority of old patterns of level no provision was made for presenting a view of the bubble to the observer at the eyepiece This is an essential feature of the tilting level, and it is now generally recognised as a very useful aid to accurate work with all kinds of level In its simplest form, the reflector is simply a plane mirror, hinged as shown in Fig 91

Clamp and Tangent Screw—These must be fitted to the vertical axis in levels with non-rigid telescopes and in instruments provided with points in the diaphragm instead of webs

Ray Shade Clinometer—See page 123

Levelling Staffs.—The levelling staff or rod, the reading of which gives the vertical distance between the instrument line of sight and the point on which the staff is erected, is a wooden rod, 10 to 16 ft. in length Patterns may be classed as (*a*) Self-reading, (*b*) Target. In the former, the graduation is such as to enable the leveller to observe the reading at which his line of sight intersects the staff. The latter form is furnished with a sliding vane or target, the centre of which is moved into the line of sight by the staffman in response to signals The position of the target is then recorded by the staffman.

The target rod is now very rarely used in Britain, but is still largely employed in America in ordinary levelling.

Self-reading Staffs.—The greatest accuracy is achieved by means of a staff in one length, not exceeding about 10 ft., but progress is expedited by using longer staffs, which for convenience of transportation consist of two or three jointed pieces, in the following forms :

(*a*) The Sopwith or telescopic staff,
(*b*) The separate-piece or Scotch staff,
(*c*) The hinged staff.

The *Sopwith Staff* is illustrated in Fig. 104, which shows a 14-ft. staff consisting of two hollow slides and a solid top piece. When extended, the lengths are held by brass spring catches.

The *Scotch Staff* (Fig. 105) is formed of three separate lengths, which are fitted together by means of brass socketed joints and set-screws. It is commonly graduated on both front and back faces, which are hollowed to protect the marking from abrasion This staff, although solid, is usually lighter than the last because of its smaller section

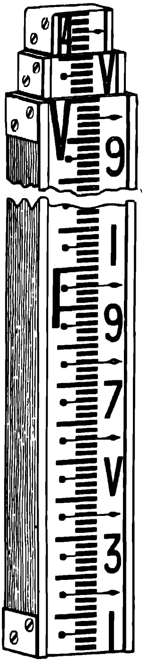


FIG 104.
SOPWITH STAFF

The *Hinged Staff* is usually 10 ft long, and folds to 5 ft with the graduations inward When open, it is kept straight by a strong hook across the hinge

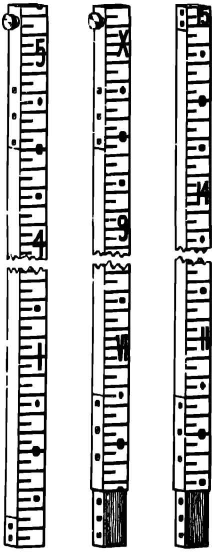


FIG 105.
SCOTCH STAFF.

Relative Merits of Different Forms.—The Sopwith staff has to be carefully made so that the telescopic work will fit properly and not exhibit shakiness when extended Trouble is often experienced in drawing out the slides when swollen with rain An objection sometimes urged is that the top piece has only about half the breadth of the lowest piece, and, unless specially graduated, proves more troublesome to read at a distance It is, however, the most commonly used form in this country, but is also the most expensive

The Scotch staff may have the breadth maintained constant to the top, but some makers give a small taper The smallness of the base compared with that of the Sopwith type is favourable to accurate holding An objection lies in the possibility of a careless staffman losing a set-screw or, when the staff is not fully extended, leaving the top piece behind.

The hinged staff is less used because of its shorter length and its liability to develop shakiness

Graduation.—The advantages of a decimal over a duodecimal subdivision of the foot are recognised in levelling, and staffs are graduated accordingly. The smallest division ranges from ·01 to ·10 ft By means of the former, readings for special work can be observed to ·005 ft. with short sights, but at long range close division is confusing. Numerous patterns have been proposed in which ·01 ft graduations are accompanied by bold marking of the tenths for facility of observation at long distances. Equally good results are obtained with a division to ·05 ft., the position of the hair being read by estimation to ·01 ft., or even ·005 ft , and this graduation remains clear at considerable distances.

Typical graduation patterns will be found illustrated in makers' catalogues. Before using a particular pattern for the first time, it should be carefully examined, and the significance of the markings understood.

Target Staffs.—Details differ in the several types of these. Fig 106 shows a 2-ply New York pattern rod, 6·8 ft. long when closed, but extending to 12 ft. The graduation is carried to ·01 ft., and the target is provided with a vernier whereby its position can be read to ·001 ft In this example, if the reading does not exceed 6·5 ft., the staffman slides the target along the rod until signalled that its centre is in the line of sight, clamps it, and reads the vernier If, however, the reading exceeds that amount, the target is clamped at 6·5 ft , and the upper part of the rod, carrying the target, is slid upwards until the target is bisected by the line of sight The reading is then obtained on a vernier at the back of the rod

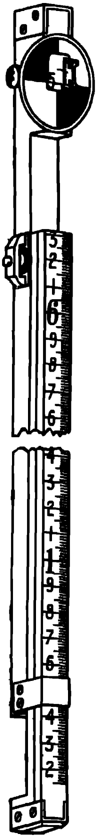


Fig 106 Target Staff.

Some forms (notably the Philadelphia rod) have sufficiently bold graduation that they can also be used as self-reading rods The pattern illustrated can be so used only for short sights.

Relative Merits of Self-reading and Target Staffs.—A comparison is greatly in favour of the self-reading form, which has a decided advantage in the rapidity with which observations may be taken The process of adjusting a target is tedious, and the time occupied in setting and reading largely depends upon the capability of the staffman. Again, in using a self-reading rod, the responsibility for correct reading lies with the leveller, as it should do, and the staffman can concentrate his attention on holding the staff plumb. The duties of a target staffman are nearly as important as those of the leveller, and demand the services of a trained man.

The fineness of reading is greater in the target rod than in the self-reading, but the refinement is usually more apparent than real, as it largely depends upon the accuracy with which the bisection of the target by the line of sight can be estimated. The ultimate degree of accuracy attained is not materially different in the two types.

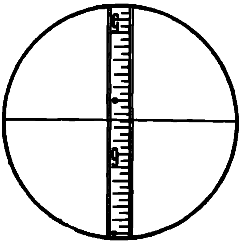


Fig 107.

Reading the Staff.—The graduation of the staff being understood, the only difficulty experienced by the beginner in observing a

self-reading staff is due to its appearing inverted, as this necessitates reckoning downwards. Thus, in Fig 107, the reading is 5·64.

Notes.—(1) When the staff is sighted at a short distance, it may happen that no foot graduations appear in the field of view. In such a case the staffman should be instructed to raise the staff slowly. The figure which appears at the top of the field represents the whole feet to be booked. This trouble is avoided if the staff exhibits the value of each foot at intermediate points on it.

(2) Readings may be taken against any part of the horizontal hair of a dumpy level, but it is safest to point the telescope so that the staff appears midway between the vertical hairs In the wye level, however, the perpendicular cross-hairs may lie in any direction from their intersection, and observations can be made only with respect to the intersection

Temporary Adjustments of the Level.—These are :

(1) Levelling up

(2) Elimination of parallax (see page 25).

The procedure in levelling up is similar to that for the theodolite (page 78) With a four-screw head, the bubble tube must be brought over each pair of diagonally opposite foot screws alternately. With three screws, it should be placed parallel to two of them and levelled by that pair The telescope is then wheeled through 90°, and the bubble is adjusted by the third screw. The levelling is completed by successive trials with the telescope in its alternate positions at right angles to each other

Note —To save time, set up the instrument as nearly level as can be judged before proceeding to use the levelling screws A circular spirit level or cross bubble tube greatly facilitates the preliminary levelling At the first manipulation of the screws do not be over particular with the centering of the bubble, but perfect it gradually at the successive wheelings of the telescope through 90°.

TESTING AND ADJUSTMENT OF THE LEVEL

In the examination of the level with reference to the quality of the non-adjustable parts, the important requirements are stability and a proper relationship between the sensitiveness of the level tube and the resolving power of the telescope. The tests are the same as for the theodolite (pages 82 and 84).

The number and nature of the adjustments of the level, the order in which they should be made, and the routine to be followed depend upon the type of instrument It will suffice to describe here the procedure in the case of the commonest patterns of dumpy, wye, and tilting levels. This will serve as a sufficient guide to the methods to be followed for modified types of the standard instruments, the adjustments of which are usually published by the makers.

In instruments having a fixed telescope fitted with four adjusting screws at the diaphragm, the horizontality of the horizontal hair may be tested and, if necessary, adjusted, as on page 86, before proceeding to the other adjustments.

Adjustment of the Dumpy Level

The requirements of the dumpy level which can be established by adjustment are .

1. The line of sight of the telescope should be parallel to the bubble tube axis.
2. The bubble tube axis should be perpendicular to the vertical axis.

The first of these is very much more important than the second, and, in adjusting an instrument, the latter should be attended to first. This order is essential in the case of instruments of the solid type (Figs. 90 and 91).

It is frequently regarded as essential that the line of sight should coincide with the optical axis of the telescope, but it may be shown* that this condition is not essential to accurate work provided the objective does not move in focussing

1. Adjustment of Perpendicularity between Vertical Axis and Level Tube Axis.—*Object.*—To ensure that, once levelled up, the bubble will remain central for all directions of pointing of the telescope. This will occur when the vertical axis is truly vertical.

Necessity.—This adjustment is made merely for convenience in using the level. Assuming that the instrument is otherwise correct, the essential requirement for accurate observation is that the bubble should be central at the instant of sighting. If the adjustment is out, so that the bubble moves towards one end on wheeling the telescope to point towards the staff, it must be brought back to its central position by the levelling screws Any change in the elevation of the line of sight thus produced is negligible.

Test —1. Set up the instrument on firm ground, and level carefully in two positions at right angles to each other in the usual manner.

2. Wheel the telescope through 180°. If the bubble remains central, the adjustment is correct

Adjustment.—1 If not, bring the bubble half-way back by the adjusting screws *2*, connecting the bubble tube to the telescope in the case of the solid dumpy (Figs 90 and 91), or by the screws *11*, *12* in the case of the stage type (Fig 89)

2. Level up, and repeat the test and adjustment until correct.

2. Adjustment of the Line of Sight, or the Collimation Adjustment. —*Object.*—To place the line of sight and the bubble tube axis parallel.

Necessity.—If this condition obtains, the line of sight is horizontal when the bubble is in the centre of its run. Since the whole function of a level is to furnish a horizontal line of sight, this requirement is of prime importance, and merits frequent examination.

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* See Rankine, *Civil Engineering*, Chap. IV.

Test.—1. Peg two points, A and B (Fig. 108), 300 to 400 ft. apart, on fairly level ground. Set the level between them at C, so that the eyepiece will almost touch the face of a levelling staff held on A.

2. Level up carefully, and note the reading *b* of a staff held on B : then, by looking through the object glass, observe the reading *a* of the staff on A.

3 Transfer the instrument to D near B, level up, and read the staff on both pegs as before, obtaining the readings *c* and *d*. If the difference of level between A and B, as deduced from readings *a* and *b*, equals that given by *c* and *d*, the result is their true difference of level, and the adjustment must be correct, since such equality could be secured only by a horizontal line of sight

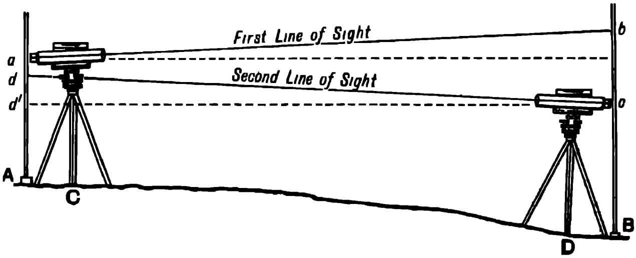


FIG 108

Adjustment—1. If not, compute the true difference of level between A and B. This is given by the mean of the two erroneous differences, for if D be the true difference of level, and e the error introduced in reading the far staff, then, for the case illustrated,

$$b-a = D+e,$$
$$\text{and } c-d = D-e,$$
$$\text{whence } D = \frac{(b-a)+(c-d)}{2}$$

2. By applying D to the reading c, compute d', the reading on A at the same level as c. The instrument being still at D, turn the top and bottom diaphragm adjusting screws *6* (Figs. 90 and 91) until the line of sight is directed to d', taking care that the bubble is in the centre of its run. The line of sight is now horizontal. In the case of the stage dumpy (Fig 89) it is more convenient to direct the line of sight to d' by turning the levelling screws. The bubble is then returned to the centre of its run by means of the adjusting screws *16* connecting the level tube to the telescope, but in this method the present adjustment must be the first made.

Notes—(1) It is presumed that altering the line of sight to read d' instead of d will not appreciably affect the reading c. It is unlikely that the adjustment will be so much in error that change in c could be observed, but, if so, the error is made negligible by repeating the adjustment.

(2) If A and B are nearly at the same level, the results of the observations

from C and D may show a difference of sign, i e. one may indicate a rise, and the other a fall, from one peg to the other The true difference of level is always half the algebraic sum of the two erroneous differences, and therefore has the sign of the greater difference.

(3) The method given is designed to eliminate possible error in focussing due to droop of the slide carrying the diaphragm and eyepiece in the case of an external focussing telescope The hairs are not visible on looking through the object glass, but the reading can be estimated with considerable precision.

(4) On completion of the adjustment, a test may be made for droop by removing the instrument a little farther from B so that the staff on B may be read in the ordinary manner although the sight is short If the difference between the readings is not now the correct difference of level between A and B, the error must be occasioned by the change of focus required The defect may be remedied in certain instruments by means of screws passing through the outer tube and bearing upon the slide, but more commonly these are omitted, in which case the correction is best undertaken by the maker.

Alternative Method.—The possible effect of droop is neglected in the following very commonly used method, which is entirely suitable for the case of internal focussing instruments.

Test —1. Drive two pegs, A and B, as before Set up the instrument at C, exactly midway between them (Fig 109), level up carefully, and read a staff on A and on B. The difference of the readings *a* and *b* is the true difference of level, since the error affects both readings in the same way and to the same extent.

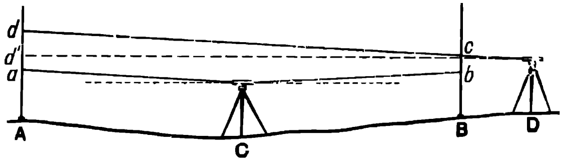


FIG. 109

2. Set the instrument near one of the pegs, and preferably beyond it, as at D, level up, and note the readings *c* and *d* of the staff on the pegs. If the difference between *c* and *d* equals the true difference of level, the adjustment is correct.

Adjustment —1 If not, by applying the true difference of level to the reading *c*, deduce *d'*, the reading of A which would be observed by a horizontal sight from D, and by means of the diaphragm adjusting screws *6* (Figs 90 and 91) direct the line of sight to *d'*, taking care that the bubble is central. In the case of the stage dumpy (Fig 89), use the levelling screws for setting the line of sight on *d'*, and bring the bubble central by the adjusting screws *16*, as before.

Notes.—(1) It is quite possible in this case that the reading *c* will be perceptibly changed in the course of the adjustment owing to the greater distance between B and D. The adjustment should therefore be perfected gradually by repeating the observations from D once or twice

(2) Wheeling of the telescope, with the bubble maintained in the centre of its run, causes an unadjusted line of sight to generate a cone, the axis of which

is the vertical axis of rotation. The true difference of level between two points may therefore always be obtained by placing the instrument equidistant from them. This constitutes one of the most important precautions in levelling practice.

Adjustment of the Wye Level

While the ultimate aim in adjusting the wye level is the fulfilment of the two geometrical relationships given for the dumpy level, the arrangement of the wye instrument alters the procedure, and three adjustments are required. The relationships to be established are :

1 The line of sight should coincide with the axis of the telescope collars, the latter axis being, in a well-made instrument, coincident with the optical axis

2. The line of sight should be parallel to the bubble tube axis

3. The bubble tube axis should be perpendicular to the vertical axis.

1. Adjustment of the Line of Sight.—*Object* —To make the line of sight coincide with the axis of the telescope collars.

Necessity —The fulfilment of this condition is highly important ; otherwise, unless the telescope can be clipped in one position, rotation about its axis would cause the line of sight to generate a cone, and the fundamental requirement of parallelism between the bubble axis and the line of sight would obtain only for a particular position of the telescope in the wyes.

Test and *Adjustment.*—As in adjustment 2 of the wye theodolite (page 93).

2. Adjustment of the Level Tube.—*Object.*—To make the level tube axis parallel to the line of sight The manner in which the telescope is mounted renders it necessary to effect this adjustment in two steps : *A*, to bring the bubble tube axis into the same plane as the line of sight ; and *B*, to place them parallel.

Necessity —As for the dumpy level.

A.—*Test.*—1. Level up carefully

2 Rotate the telescope in the wyes through a small angle. If the bubble remains central, the adjustment is correct.

Adjustment.—1. If not, bring the bubble central by the adjusting screw *8* (Fig 95) controlling the level tube laterally.

2. Repeat the test and, if necessary, the adjustment.

Note.—No reversal being performed in the test, the apparent error is the actual error.

B.—*Test.*—1. Set the telescope parallel to two levelling screws, open the wyes, and level up carefully.

2. Lift the telescope gently from the wyes, and replace it end for end. If the bubble remains central in the reversed position, the adjustment is correct.

Adjustment —1. If not, move the bubble half-way towards the centre by the screw *7* (Fig. 95) at one end of the tube

2. Repeat test and adjustment until correct.

Notes —(1) Since the reversal is made with respect to the wyes, the adjustment really secures parallelism between the level tube axis and the line joining the bottoms of the wyes If, however, the telescope collars are exactly similar, and adjustment 1 is correct, the bubble tube axis will have been placed parallel to the line of sight The error arising from inequality of the collars will probably be negligible for ordinary levelling

(2) The two-peg method used in the same adjustment of the dumpy level may be adopted for step *B* Although less convenient, it has the merit of being unaffected by collar inequality

3. Adjustment of Perpendicularity between Vertical Axis and Level Tube Axis.—As for the stage dumpy level, adjustment 1 (page 116).

Adjustment of the Tilting Level

In the adjustment of the tilting level it is only necessary to secure that the line of sight is horizontal when the bubble is centered In tilting levels of the non-reversible type, the test is performed as for a dumpy level by the method of page 118 (Fig 109) If it is found that the difference of the staff readings observed from D does not represent the true difference of level of A and B, the reading *d′* is computed as before, and the line of sight is directed upon *d′* by means of the tilting screw, the bubble being ignored The adjustment is effected by returning the bubble to the centre of its run by means of the bubble tube adjusting screws

Instruments of the reversible type possess the advantage that the test can be rapidly performed from one position of the instrument, as follows

Test.—1 With the level tube, say, on the left-hand side of the telescope, sight a staff held upon a firm point Centre the bubble accurately, and note the reading

2. Rotate the telescope about its longitudinal axis through 180°, so that the level tube is brought to the other side, and again read the staff. If the readings agree, and the bubble tube is of the type having its upper and lower axes parallel, the line of sight must be horizontal, and the adjustment is correct.

Adjustment.—1. If not, direct the line of sight towards the mean reading by operating the tilting screw.

2 In the case of an instrument having a simple bubble reflector, centre the bubble by means of the adjusting screw connecting the bubble tube to one of its supports. If a Zeiss prismatic reader is fitted, adjust by moving the prism case longitudinally until apparent coincidence of the bubble ends is obtained. In the instrument shown in Fig 100, screw *23* is provided for this purpose, and screws *21* are available if the error is more than can be eliminated by adjustment of the prism case.

3. Repeat the test and adjustment until no error is shown on reversal.

HAND LEVELS AND CLINOMETERS

These levelling instruments are designed with a view to lightness and compactness rather than a high degree of precision, since the absence of a steady support does not allow of the employment of a sensitive bubble tube. They are convenient for rapid work, especially where serious error cannot be accumulated, as in cross sectioning The clinometer is simply a hand level adapted for measuring vertical angles, and, as it can be employed as a hand level, is the more useful instrument

The Hand Level.—The hand level (Fig. 110) consists of a tube, about 4 in. long, circular or square in section, and having a small bubble tube mounted on top. A line of sight parallel to the axis of the bubble tube is defined by a pin hole at the eye end and by a cross bar, or one edge of a rectangular opening, at the other To afford the observer a view of the bubble to enable him to hold the instrument level, the metal body is cut away under the level tube, and a reflecting surface is inserted in the tube at 45° to its axis. This reflector extends half-way across the tube, and distant objects can be viewed through the other half When the centre of the bubble appears opposite the cross-bar sight, or lies on a line ruled on the reflector, the instrument is being held level, and all points intersected by the line of sight are at the same level as the eye Telescopic forms of the instrument are also available

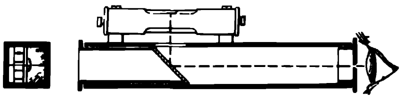
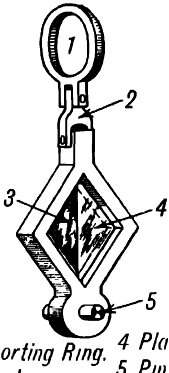


FIG 110 —HAND LEVEL

If provision is made for adjustment of the bubble tube relatively to the line of sight, the test is carried out by the two-peg method as for the dumpy level

The Burel Hand Level.—This form of hand level is based on the fact that a ray of light which is reflected back from a vertical mirror along the path of incidence must be horizontal The instrument (Fig 111) consists of a glass mirror, the frame of which is suspended in gimbals so that the mirror hangs vertically The mirror extends half-way across the frame, and the other half is plain glass, the edge of the mirror forming a vertical or, in some patterns, a horizontal line. To use the instrument, it is suspended from the thumb and held at arm's length in



1 Supporting Ring.
2 Gimbal
3 Mirror.
4 Plain Glass.
5 Pin filed for adjustment of Mirror to Vertical

FIG, 111 —BUREL HAND LEVEL.

such a position that the observer sees the reflected image of his eye at the edge of the mirror. Distant objects appearing through the plain glass opposite this image are at the same level as the observer's eye The instrument proves very rapid in practice, but is unsatisfactory in windy weather.

The Abney Clinometer.—This instrument (Fig. 112), usually known as the Abney level, is deservedly the most popular instrument of its class, and is simply the hand level of Fig. 110 adapted for measuring vertical angles. The bubble tube is pivoted, and carries an index arm perpendicular to its axis Rotation of the milled wheel moves the index over a graduated arc attached to the sighting tube, and, when the bubble is centered, the reading on the arc represents the inclination of the line of sight above or below the horizontal. A small clamp is sometimes fitted to the index arm, and is useful for fixing the index to zero, so that the clinometer can be used as a hand level. The instrument is also made in a telescopic form

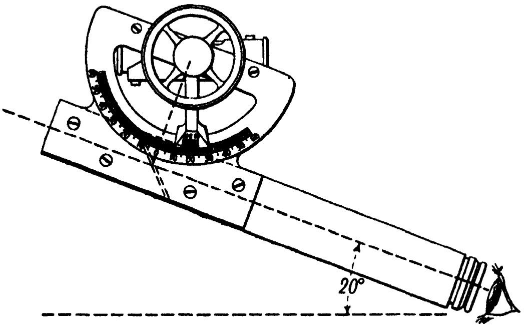


FIG. 112.—ABNEY CLINOMETER.

In using the Abney, the line of sight is directed towards the object, and the milled wheel is turned until the reflected image of the bubble appears in the plane of sight. The required angle is then read on the arc. To test and adjust the instrument, the index is set to zero, and the two-peg method is applied. Alternatively, two points at different elevations are selected, and the vertical angle between them is observed from both. The angle of elevation observed from the lower station should equal the angle of depression taken at the upper. If these differ, their mean is the correct value of the inclination, and the instrument is made to record this by means of the adjusting screws controlling the level tube.

The De Lisle Clinometer.—This instrument is a modified Burel level in which the mirror can be inclined to the vertical by means of a weighted arm. The inclination to the horizontal of the line from the eye to the point at which it appears in the mirror equals the inclination of the mirror to the vertical. To measure a vertical angle, the mirror is tilted until the image of the eye appears opposite the signal, and the angle is read by the position of the arm on a graduated arc.

The Ray Shade Clinometer.—The ray shade of the level has been neatly adapted for rough levelling and clinometric observations at right angles to the telescopic axis. The shade is provided with two narrow slits (Fig. 90) diametrically opposite each other. The centre line of one coincides with the zero of a scale graduated to show the inclination of the line of sight through the slits. When the instrument is levelled up, and the zero of the scale placed opposite an index mark on the telescope barrel, the plane of the slits is horizontal. To measure an inclination from the horizontal, it is only necessary to turn the ray shade until the line of sight intersects the object, when the angle is shown against the index.

The Footrule Clinometer.—This consists simply of a stout 12-inch boxwood rule fitted with a small bubble tube on each part, a graduated arc, and a pair of sights (Fig 113). In sighting, the instrument is held against a rod as firmly as possible with the bubble central in the lower tube, and the rule is opened until the line of sight lies at the required inclination, which is then read on the arc In a common application of the clinometer the sights are not used, and the instrument is placed on a straight-edge laid on the slope to be measured. In this case the rule is opened until the upper part is level.

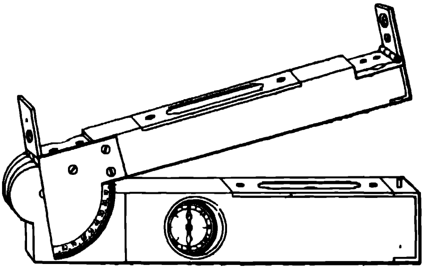


FIG. 113.—FOOTRULE CLINOMETER.

Miscellaneous Levelling Instruments.—The gradiometer is an instrument designed to facilitate the measurement or setting out of long uniform gradients. It consists of a level in which the telescope is so mounted that it may be set at various inclinations to the horizontal, the gradient being registered on a drum. When the index is set to zero, the instrument serves as an ordinary level.

For levelling over a distance of a few feet, the commonest apparatus employed is a straight-edge laid horizontal by means of the familiar mechanic's level, which is simply a bubble tube mounted on a flat base parallel to the axis. Levels can also be transferred for short distances without a spirit level by using a frame having a straight edge square to a plumb line. In an emergency, a horizontal line of sight can be obtained by placing in a pail of water a wooden float with two equal upright pieces, the tops of which can be sighted across. In the water level proper, the horizontal is defined by the water surface in the legs of a U-tube or in a pair of glass tubes the lower ends of which are connected by rubber tubing The latter apparatus is of considerable utility and accuracy.

PRECAUTIONS IN USING INSTRUMENTS

Setting Up.—(*a*) In lifting the instrument from the box, handle it in such a way as to minimise the possibility of strain. It must not be lifted by the telescope, the standards, or axes, but should be supported by the hand placed under the levelling head.

(*b*) Screw the instrument firmly home on the tripod head, so that the connection may not slacken during observations

(*c*) If the instrument has three levelling screws held by a locking plate, see that the plate is gripping the screws and is properly locked, otherwise the instrument will fall off the tripod when lifted.

(*d*) Do not force the levelling screws of a four-screw instrument.

(*e*) After centering a theodolite by means of a shifting head, do not fix the locking nut of the head too tightly Certain forms are easily jammed, and it is well to make trial of the least force necessary to prevent slip and in practice to give only a slightly greater grip than this.

Carrying.—(*a*) Support the instrument against the shoulder as uprightly as possible In the case of the theodolite, place the telescope vertical with its clamp slack, and release the lower clamp.

(*b*) In situations with limited headroom carry the instrument in front, and see that it does not strike branches, etc. In crossing a fence or any awkward place, it is best for one man to go over first and have the instrument handed to him.

(*c*) If the compass has been in use, throw the needle off its pivot before lifting the instrument.

In Wind.—(*a*) Spread the tripod legs well apart, and thrust them firmly into the ground.

(*b*) On a pavement or other smooth surface, if sufficient spread is given to prevent overturning by wind, the instrument may collapse by the feet sliding outwards : they should therefore be inserted in joints or cracks

(*c*) Do not leave the instrument in high wind, even although the above precautions have been taken.

In Rain.—(*a*) Protect the objective as far as possible by extending the sun shade. Use a brush for absorbing moisture from the eye lens. If a brush is not available, a corner of the handkerchief may be used, but the lens should not be rubbed.

(*b*) If work is stopped temporarily, close the sun shade by the shutter, and, if the eyepiece has no cap, make a paper cover and attach it by a rubber band. The telescope of a theodolite should be set vertically, with the eyepiece down. In wet climates a waterproof hood to slip over the instrument is useful.

(*c*) If water enters the telescope tube, it will cause dewing of the lenses, and observations are impossible. This cannot be remedied at once in the field, as the hairs must never be exposed out of doors. The moisture will gradually evaporate if the instrument is left in the sun or in a moderately warm room . in the latter case, drying is hastened by removing the eyepiece.

(*d*) Dry all exposed parts thoroughly before putting away the instrument Use a soft piece of linen in wiping graduated circles.

General.—(*a*) Do not force screws, or apply clamps too tightly

(*b*) A camel-hair brush is best for dusting instruments Levelling and tangent screws are cleaned with a nail brush, and a little oil should be applied, and finally wiped off Oil should not be left on exposed parts as it collects dust Vaseline or watch oil is used to lubricate axes, but this need seldom be renewed Unless the instrument develops stiffness, it should not be taken apart

(*c*) Avoid fingering silver graduated surfaces When tarnished, apply a very little vaseline, and where necessary rub it with the finger, and wipe clean with a soft rag On no account use plate polish

(*d*) For care of telescope, see page 29.

Packing. —(*a*) There is only one position in which an instrument will fit into its box, and the difficulty of packing may be lessened if a note of the correct positions of the various parts is written in the box. The plates of the levelling head must be left approximately parallel, and clamps should be slackened

(*b*) The instrument should pack without forcing, but it may happen that difficulty is experienced owing to shrinkage or warping of the wood of the box The offending part can be discovered by chalking those portions of the instrument held by the bearing pieces inside the lid, and may be cut down a little

(*c*) The cap and ring of the tripod should be kept in the box while the instrument is in use

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CHAPTER II

CHAIN SURVEYING

CHAIN or linear surveying is that method of surveying in which only linear measurements are made in the field Although unsuitable for large areas or difficult country, it is quite well adapted for small surveys on open ground with simple detail, and is largely employed in such circumstances To the beginner it forms a fitting introduction to the study of other methods on account of its simplicity and the general applicability of many of its operations.

Field Party.—The party consists of :

The Surveyor, who determines what measurements are to be taken, directs their carrying out, and records the results in his field book

Two Assistants or Chainmen, who make the measurements.

Except in the case of a survey which may be completed in an hour or two, it is a mistake for the surveyor to work with only one assistant, as he must then take part in the chaining, etc., time is wasted, and it is difficult to keep the notes clean.

Equipment.—*Essential* —One chain or steel band, 66 ft. or 100 ft , with ten arrows ; one linen or metallic tape, 50 ft. or 66 ft. ; six to twelve ranging poles, according to size of survey and character of ground , field book, pencil, and rubber.

Optional.—Cross staff, or optical square, or box sextant ; line ranger ; clinometer ; small compass ; plumb line ; offset rod ; pegs ; flags for attaching to poles , field glass.

THE MEASUREMENT OF A LINE

The ordinary routine of measuring distance, being common to several branches of surveying, will first be described. It will be supposed in the first instance that the ground is level, and presents no obstacles to straightforward measurement.

Ranging the Line.—The terminal points, or stations, between which the distance is required, are marked by ranging poles fixed vertically in the ground. If the distance is short, the measurement may proceed at once, but otherwise it is a convenience in aligning the chain to have one or more intermediate poles set in the line.

The ranging of a line by eye is performed by the surveyor and an assistant. The former remains at one of the stations, while the latter proceeds the required distance along the line and puts himself approximately in alignment He faces the surveyor, and holds a pole vertically and nearly at arm's length, so that his body will not obstruct the surveyor's view The latter directs him to right or left until the pole appears in line with the remote station pole When signalled to mark, the assistant fixes his pole and examines it for verticality He should then wait for a second signal as to whether the surveyor is satisfied with the fixing of the pole

Notes —(1) To obviate errors due to non-verticality of the poles, it is most important that, where possible, ranging should be performed with reference to their lower ends The surveyor should therefore bring his eye sufficiently low by stooping.

(2) For good work he should station himself about a yard behind his pole, and should sight with one eye along the edges of the poles Alternatively, he may bring the mid-point between his eyes into line by finding that position from which, by opening and closing each eye alternately, his pole appears to move equally on either side of the farthest pole The alignment of intermediate poles is then easily judged by sighting them with both eyes.

(3) The assistant should align himself roughly, and should not have to be repeatedly warned that his pole is not plumb It should be held lightly by the forefinger and thumb so that it hangs vertically with the shoe an inch or two off the ground On fixing, it may be tested by a plumb line or by dropping a pebble alongside

(4) In guiding the assistant into line, the surveyor should use a prearranged code of signals, such as—

Rapid sweeps with right (left) hand--Move considerably to right (left)
Right (left) arm extended—Continue to move to right (left)
Slow sweeps with right (left) hand—Move slowly to right (left)
Right (left) hand up and moved to left (right), left (right) hand down and moved to right (left)—Plumb pole in direction indicated
Both hands above head – Correct

Such signals should be made to be seen clearly The arms should be extended clear of the body , signals to move to the left should not be made with the right hand When they have to be read from a considerable distance, signals should be made with a handkerchief

Chaining the Line.—The word "chaining" denotes the measurement of a distance whether it is executed by the chain or the steel band. No explanation is required of the case where the distance is shorter than the length of the chain, but otherwise the chain must be stretched successively in the correct line, and it is important to realise the need for system in this apparently very simple operation if speed is to be acquired and serious error avoided.

Throwing out the Chain.—With both handles in his left hand, the chainman throws out the chain with his right, and, assisted by the second chainman, proceeds to free it from twists and knots. Otherwise, one man may retain the handles, while the other walks forward with the remainder of the chain, paying it out on to the ground. When the chain has been roughly straightened, and examined for bent links and badly opened joints, the work of laying it down successively may proceed If a steel band is used, it must first be unwound and removed from the case.

Lining and Marking —The more experienced of the two chainmen remains at the zero end or rear of the chain, and is known as the follower ; the other, called the leader, takes the forward handle. Before starting, the leader must provide himself with the ten arrows with which to mark the successive positions of his end of the chain, and he and the follower should each have a ranging pole

While the follower holds the rear handle at the terminal point, the leader stretches the chain as nearly in the line as he can judge, and, standing erect, holds his pole for alignment by the follower. It is desired to range a point a little short of the chain length from the follower ; the end of the chain being slack and held in the leader's left hand, the pole may therefore be placed about opposite the handle. When the pole has been lined, the position it occupied is marked by the hole made, or by a scratch, and the pole is removed, but in long grass or heath it should be left in. The follower now holds his handle exactly at the station point, while the leader proceeds to stretch the chain in a straight line over his mark or against the pole. This is accomplished by transmitting a series of gentle undulations along it by shaking the handle up and down, and at the same time bringing it gradually into line Only sufficient tension to keep the chain straight is required. While straightening the chain, the leader should be stooping and holding an arrow hard against the end of the handle and in the same hand, so that he can thrust the arrow vertically into the ground immediately the follower instructs him to mark.

To repeat the procedure for the next chain length, the leader, taking his pole and remaining arrows, walks forward and pulls the chain after him, having first swung it a little out of the line so that the arrow which has been placed will not be disturbed. He should count his paces so that he may know to stop and turn round when the follower, carrying the rear handle, has reached the arrow. The follower ranges the leader by planting his pole behind the arrow, and, having held his handle at the arrow (preferably flush with that side which was against the leader's handle), and got the second length marked by the leader, removes the arrow. The number of arrows in the possession of the follower at any time thus shows the number of completed chain lengths.

Transfer of Arrows —At the end of the tenth chain length the leader has inserted his last arrow. When the chain is next pulled forward, the follower removes the tenth arrow, and marks the place by pushing a nail or pencil into the ground. The two men then meet, and the ten arrows are handed over to the leader, who uses them over again. In a long line a careful record must be kept of the number of such transfers. Occasionally eleven arrows are carried, the additional one being used before the transfer to mark the forward end of the eleventh length.

End of Line.—At the end of a line there will usually be a part chain length to be measured. The leader pulls his end of the chain beyond the station, and then straightens the chain up to the station pole, and reads.

Making up the Chain.—The chain must be coiled up in a particular manner to form a compact bundle which can be tightly held by a strap. Having roughly doubled the chain by pulling it by the middle, the chainman holds it at the 50 tab in the left hand, and with the right grasps it at the joints corresponding to 48 and 52. He passes the four links to the left hand, doubling them as he does so to form a length of one link fourfold. He continues with the remainder, laying each four links obliquely across those in his hand. When finished, the bundle should have an hour-glass shape, all the links bearing on each other at the waist. The strap is passed round the narrowest part and through the handles.

Notes—(1) Much time is saved if the leader can put himself approximately in line. If one or two intermediate poles have previously been ranged in, he can do so accurately, but should always allow the follower to satisfy himself before marking. If no intermediate poles have been set, he should, when first aligned by the follower, sight past the latter's pole and "take a back object," *i e.* note any feature of the landscape which lies approximately in the line produced. He can then place himself nearly in line at every chain length by getting the distant point and the rear pole in range A forward object in line with the other station may also be taken.

(2) Some surveyors do not use poles for the alignment of the chain, the follower sighting the arrow held by the leader against his handle. This is quite satisfactory where the ground is smooth, but in the majority of cases it is unwise to dispense with poles.

(3) The counting of his paces by the leader when pulling the chain forward should not be omitted, as it may save the follower a search for the arrow in long grass. The leader should not be stopped suddenly by the follower pulling the chain, as the jerk contributes to lengthen the chain.

(4) It requires a little practice to be able to stretch the chain quickly. Beginners are apt to shake and tug violently, making it very difficult for the follower to hold at his arrow. The tension should be only sufficient to make the chain lie straight

(5) When chaining on a hard surface, such as that of a road or street, the leader should mark each chain length by a scratch made with an arrow or by a chalk mark, and should lay the arrow on the ground. It is advisable to write the chainage at each mark.

Chaining on Sloping Ground.—If the ground over which a measurement has to be made is not level, means must be taken to obtain the *horizontal* length of the line. This may be done directly by "stepping" the measurement in horizontal lengths, or indirectly by deducing the horizontal equivalent of the distance along the slope.

By Stepping.—*Chaining Downhill.*—In this case the follower's end of the chain is held on the ground, and his routine is as before.

The leader must hold his end, either of the whole chain or of a suitable portion of it, above the ground so that the chain is stretched horizontally in the air (Fig. 114). It is ranged, and the end of the suspended length transferred to the ground by means of a plumb line held by the leader.

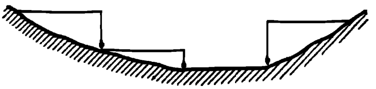


FIG 114 —STEPPING.

Chaining Uphill.—It is more difficult to obtain good results in this case, as the follower's handle is now off the ground, and he must simultaneously plumb the end over the mark and range the leader. It is therefore a great convenience to have sufficient poles in the line that the leader can align himself.

Notes —(1) The length of steps permissible, to yield material accuracy, decreases with increase of the slope of the ground and the weight of the chain or band For convenience of manipulation, the suspended end should not be more than about 6 ft off the ground The error introduced by sag is proportional to $\left(\frac{\text{weight of chain}}{\text{pull}}\right)^2$, so that a light steel band is best for slope work. In place of stepping with the chain, a good method is to align and lay the chain on the ground, and then perform the stepping by a linen or steel tape. This saves the ranging of each step, but, if a badly worn linen tape is used, it may be necessary to correct for the error of the tape

(2) The transfer of the horizontal lengths to the ground may be performed by a pole or by dropping an arrow, ring down, but the plumb line method is the most accurate

(3) It is very difficult for the chainmen to judge when the tape or chain is horizontal The surveyor should therefore stand clear to one side, and direct them Even from such a position, the inexperienced will think the chain horizontal when it really slopes downhill. That it should make a right angle with the suspended plumb line is a good guide

(4) Great care is necessary to keep a correct record of the steps Those in a series should, as far as possible, have the same length, preferably 10, 20, 25, 50, or 100 ft or links Caution must be exercised if the chaining arrows are used to mark the ends of short steps It is better to insert arrows only at the ends of chain lengths and to use cleft twigs or nails at the intermediate points

By Measuring along Slope.—In this method, chaining is performed along the surface of the ground, and the various slopes encountered are measured by clinometer (page 122). If the measurement is composed of a series of lengths l_1, l_2, etc., inclined at θ_1, θ_2, etc , to the horizontal, then

the required horizontal distance $= l_1 \cos \theta_1 + l_2 \cos \theta_2 + \text{etc.}$

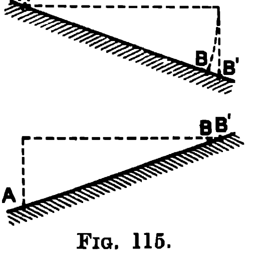


FIG. 115.

If only the total distance is required, the calculation may be made on completion of the chaining, but, when numerous intermediate points have to be located, it is better to make a correction in the field at every chain length. The chain having been stretched in the position AB (Fig. 115), the leading arrow is shifted from B *forward* to B′, where BB′ $=100 (\sec \theta - 1)$ ft. or links, and the next chain length starts from B′. On moderate slopes the chainage of intermediate points may be read directly from the chain with sufficient accuracy for ordinary offsetting. In applying the method, a table should be prepared of the values of BB′ for various slopes.

Relative Merits of Stepping and Measurement on Slope.—Measurement on the ground yields better results than stepping, but is somewhat tedious except on ground characterised by long gentle slopes On short slopes of varying degree the method of stepping is quicker, and is that more generally used in ordinary work. Stepping is useless on very flat slopes, as the sag error may exceed that introduced by assuming the inclined and horizontal lengths equal.

ERRORS IN ORDINARY CHAINING

An examination of the nature and effects of the various sources of error in ordinary chaining is necessary for the due appreciation of the relative importance of the precautions to be observed against them. The difference between cumulative and compensating, and positive and negative errors (page 5) should be kept in mind. Errors and mistakes arise from :

1. Erroneous Length of Chain.
2. Bad Ranging
3. Bad Straightening.
4. Non-horizontality.
5. Sag.
6. Careless Holding and Marking.
7. Variation of Temperature.
8. Variation of Pull.
9. Displacement of Arrows.
10. Miscounting Chain Lengths.
11 Misreading.
12. Erroneous Booking.

1. **Erroneous Length of Chain.**—Cumulative, + or —. This is the most serious source of error in using the wire chain because of its liability to stretch. It should be tested before commencing work, and on important surveys should be compared with a field standard every day or two, while knotting of the rings should be carefully avoided. Steel bands for ordinary work need be tested only occasionally. Measurements obtained with a chain or band which has been found in error can be corrected as follows :

$$\text{Correct distance} = \frac{\text{Erroneous length of chain}}{\text{Nominal correct length of chain}} \times \text{observed distance.}$$

If the chain is too long, the error is negative (the chain not going sufficiently often into the line), while the opposite effect is produced if it is short.

2. **Bad Ranging.**—Cumulative, +. In ordinary work this produces a relatively small error. If a chain length diverges a perpendicular distance d from the correct line, the error in length is practically $d^2/200$ ft. or links. Very refined ranging is unnecessary if only distance is required, but greater caution is called for if offsetting is being performed, as the offsets are thrown in error by the full amount of the divergence

3 **Bad Straightening**—Cumulative, +. The effect produced by the chain lying in an irregular horizontal curve is similar to the last, but is more productive of error as the deviation is not so easily seen.

4. **Non-horizontality**—Cumulative, +. This error is common in stepping, but also arises from disregarding flat slopes. It is the second error above, reproduced on the vertical instead of the horizontal plane, but is much more important in practice.

5 **Sag.**—Cumulative, +. When the chain is stretched in the air either in stepping or in measuring over small undulations or obstructions, it must necessarily sag, and the distance between the ends is less than that read. The error is reduced by suspending short lengths only and pulling firmly

6. **Careless Holding and Marking.**—Compensating, ± The follower may hold his handle to one or the other side of the arrow, and the leader may not thrust his arrow vertically into the ground or exactly at the end of the chain The possibility of inaccurate marking is much increased when plumbing The error of marking developed by inexperienced chainmen is often of a cumulative character, but with ordinary care the distance, as marked, may be greater or less than a chain length, and such errors tend to compensate. If an error of $\pm e$ is made in marking each of N chain lengths, the probable uncompensated error in the length of the line from this cause is $\pm e\sqrt{N}$.

7. **Variation of Temperature**—Cumulative, + or —. The effect of temperature is negligible in ordinary chaining, but must be allowed for in careful work with the steel band If the band is used at a temperature different from that at which it was compared with the standard, the necessary correction can be computed from the known temperature difference and the coefficient of expansion of the material.

8. **Variation of Pull.**—Compensating, ±. Variation in length produced by varying tension is also unimportant in ordinary work. The pull should be maintained at that applied in standardising the chain, but, unless a spring balance is used, it will probably vary on either side, and the errors tend to compensate. The chainmen may, however, persistently apply too great or too small a tension, when the error becomes cumulative.

The following mistakes produce quite irregular effects.

9. **Displacement of Arrows.**—If an arrow has been knocked out of the ground or disturbed by any means, it may be replaced wrongly. Such an accident is easily avoided, but, in view of its possibility, the leader may mark the end of each chain length by a scratch on the ground as well as by the arrow.

10 **Miscounting Chain Lengths** —This serious blunder should be guarded against by seeing that the leader has the full number of arrows on starting, and by both men counting them at each transfer.

11. **Misreading** —The most likely mistake in reading the chain is to confuse the 40 with the 60 tally The position of the 50 tab should be noticed In using a fully graduated band or tape, it is not uncommon for beginners to misread the feet by concentrating their attention on the inches. The surveyor should himself verify readings where possible

12 **Erroneous Booking** —The possibility of booking figures wrongly is obviated by the chainman calling out the measurements loudly and distinctly, the surveyor repeating them as he makes the entry.

Relative Importance of Sources of Error.—It is instructive to compare the conditions under which each source of error, taken by itself, may give rise to a definite amount of error Supposing the 100-ft chain or band to be used, an error of about 1 in 5,000 is produced by each of the following conditions :

1. Length of chain ¼ in wrong
2. Divergence in direction per chain length of 2 ft.
3. Middle of chain 1 ft off the line, the half-lengths being straight.
4. One end of chain 2 ft higher than the other.
5. Sag per chain length of 10½ in.
6 Each marking ± 1¾ in out (taking the line as a mile long).
7. Variation of temperature of 30° Fahr from standard.
8. Variation of pull from standard of 25 lb. and upwards, depending upon cross-sectional area of chain or tape. (The figure is based on the supposition of a light steel tape and a cumulative error.)

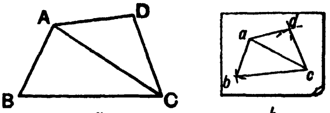
These figures emphasise the futility of elaborate refinement in ranging and marking unless the chain is correct or its error is known. Speed in chaining, as in other surveying operations, is much increased by consistency between the precautions against error and the probability and seriousness of the errors.

Customary Limits of Error.—It is difficult to assess the degree of accuracy attained in linear measurements made in ordinary surveying. Since the routine does not ensure the elimination of constant errors, a knowledge of the discrepancies between successive measurements of the same distance affords no guide to their real accuracy. The total error is composed of cumulative errors, both positive and negative, as well as compensating errors, but in expressing the value of ordinary chaining the latter are neglected, and error is regarded as proportional to distance.

The ratio of error depends so greatly upon the care taken in standardising the chain that general figures are of little value. For measurements by wire chain on average ground, the most commonly quoted ratio is 1 in 1,000. This is difficult to attain in rough country, but on flat ground 1 in 2,000 should be reached without special precautions. With a steel band the errors should not exceed about half those.

CHAIN SURVEYING—PRINCIPLES

Surveys with Straight Boundaries.—The simplest possible survey is that of a triangular plot with straight boundaries. If the horizontal lengths and sequence of the three sides are recorded, sufficient information is available to enable a plan of the area to be drawn by the method of Fig. 1*a*.



a *b*

FIG. 116.

If, however, the area has more than three straight boundaries, it is no longer sufficient to measure the lengths of the sides only, as an infinite number of figures could be drawn satisfying the data. The field measurements must be so arranged that the area can be plotted by laying down triangles Thus, if either diagonal of the four-sided field ABCD (Fig 116 *a*) is chained, as well as the sides, the plotting can be performed, preferably by first laying down the diagonal in a convenient position on the paper and erecting the triangles upon it (Fig 116 *b*).

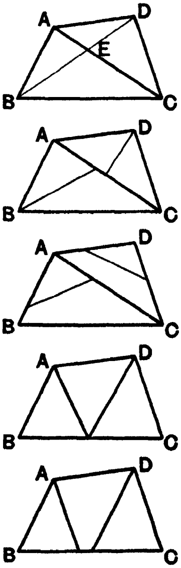


FIG. 117.

Check Lines.—The surveyor should be able to guarantee that, within permissible errors of measurement and plotting, the plan produced does actually represent the area surveyed. Consideration of Fig 116 will show that a mistake made in the measurement or plotting of any of the lines will cause a distortion of the figure which may pass unnoticed. Such mistakes could be eliminated by performing the whole work twice, but may be discovered with less trouble by making cross measurements, called check or proof lines, which, although they may not be essential to the plotting of the chain lines, serve to indicate the correctness of the work by their falling into place on the drawing. Fig 117 shows various ways in which the survey of Fig 116 could be checked : the method to be used should depend upon local circumstances.

Notes —(1) In the first method, the intersection E of the diagonals should be marked by a pole, and its chainage along each diagonal noted Triangles ABC and CDA can then be checked independently, and a mistake localised

(2) None of the methods shown affords an absolute check, as it is possible for two errors to combine and permit a proof line to fit in They are, however, to be regarded as sufficiently good, as their object is to ensure that gross mistakes will not pass undiscovered, and these should be of sufficiently rare occurrence as to preclude the possibility of balance

(3) If, on plotting, it is found that a mistake has been made in the field, careful examination of the notes may reveal how and where it has occurred It is usually necessary, however, to rechain the doubtful portion, hence the importance of confining the effects of possible mistakes by distributing check lines to all parts of a survey When surveying far from headquarters, the accuracy of the chaining may be verified by plotting the lines to a small scale before leaving the ground

Offsets.—In the more general case in which the lines to be reproduced on paper are not all straight, the above methods are applied to the measurement of a framework of survey lines which can be plotted and checked, and from which the various details are located The most rapid and commonly used system of location is that of Fig. 1*b*, the perpendiculars being called *offsets*, but points may also be tied in from the chain lines by the method of Fig 1*a*

Fig 118 represents a field with four irregular sides ABCD is a framework, the lines of which lie alongside the features to be surveyed From these lines as many points can be located as are necessary to define the irregularities of the boundaries In plotting, the triangles are laid down as before, and from their sides short perpendiculars are erected in the proper positions, and the lengths of the offsets are scaled The boundaries, etc , are then drawn through the points obtained.

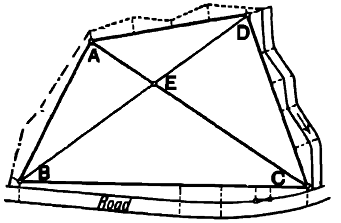


FIG. 118

Points to which Offsets are Taken.—To survey a straight line from an adjacent chain line, it is sufficient to determine correctly the positions of both ends by offsets. These points being plotted, the straight line joining them represents the feature surveyed, the measurement and plotting of additional offsets imparting no further information.

In surveying an irregular line, it is treated as divisible into a series of lengths, each sensibly straight, and a sufficient number of offsets is taken to locate those lengths Offsets must therefore be measured to every point at which the line has a marked change of direction. The degree to which minor irregularities may be neglected depends upon the character of the line being surveyed and also upon the scale to be used in plotting. Fewer offsets are required for surveying indefinite lines, such as the margins of woods or marshes, or those subject to variation, such as shore lines, than in the case of, say, a well-marked boundary equally irregular. The scale of plotting should be kept in view, as on it depends the refinement of detail which can be reproduced on the plan.

In the survey of lines of regular curvature, *e g.* railway lines, it is sufficient to take offsets at regular intervals.

CHAIN SURVEYING—FIELD WORK

Reconnaissance.—On arriving on the field, the surveyor should first of all walk over and thoroughly examine the ground with a view to determining how he may best arrange the work The importance of this step is sometimes overlooked by the beginner, but the utility of a thorough reconnaissance cannot be overemphasised, the time spent being amply repaid in the greater ease with which the survey can be executed The positions of stations are selected and marked, the poles being used to test intervisibility. During the reconnaissance the surveyor should prepare a sketch of the ground, showing the arrangement of lines and the numbering or lettering of the stations

Selection of Stations.—In examining the ground for a good arrangement of survey lines, the surveyor should endeavour to meet the following requirements

1. Survey lines should be as few as practicable, and such that the framework can be plotted
2. If possible, a long line should be run roughly through the middle of the area to form a " backbone " on which to hang the triangles
3. Triangles should be well-conditioned.
4. Each portion of the survey should be provided with check lines
5. As few lines as possible should have to be run without offsets.
6. Offsets should be short, particularly for locating important features.
7. Obstacles to ranging and chaining should be avoided as far as possible.
8. Lines should lie over the more level ground.
9. In lines lying along a road, the possibility of interruption of the chaining by passing traffic is to be avoided by running the line at one side of the road.

Well-conditioned Triangles.—The third of the above requirements refers to the shape of the triangles, which should be such that the distortion produced by errors in measurement and plotting will be a minimum. It can be shown that a point located by the intersection of two arcs is least displaced by errors in the radii when the arcs intersect at 90°. If the three sides of a triangle are equally subject to error, the three angles should approach 90° as nearly as possible, *i e* the triangle should be equilateral, and triangles approximating to this form are termed "well-conditioned" An endeavour should always be made to avoid using triangles having one or two angles less than 30° · if they must be used, such badly-conditioned triangles require additional care in chaining and plotting

Marking Stations.—Survey stations should be marked to enable them to be readily discovered during the progress of the survey, and preferably in a manner which will render them available after the lapse of some time, in case it may be necessary to repeat a portion of the work or to extend the survey. In soft ground wooden pegs about 18 in long by 1½ in square are suitable They should be driven firmly with only a small projection above the ground. In roads and streets it is necessary to substitute nails or spikes driven flush The difficulty of recovering the position of a nail is avoided by noting the tie measurements to it from two, or preferably three, well-defined and permanent points near it In important surveys it is highly expedient to locate the principal stations in this manner as a precaution against displacement of the peg In many surveys pegs are dispensed with, and the stations are marked by inserting a twig into the hole made by a ranging pole. The marking is made conspicuous if the twig is cleft to receive a piece of paper Plasterers' laths are sometimes used, as they are sufficiently straight to be used for ranging as well as marking. On turf a good marking is obtained by cutting out a sod in the form of an arrow or an acute isosceles triangle with the apex at the station.

Running Survey Lines.—The routine of running a survey line comprises the chaining of the line and the location from it of the adjacent detail Having stretched the chain and inserted the forward arrow, the leader leaves the chain lying on the ground, and returns to assist the follower with the offsetting. He takes the ring of the tape, and holds it at the various points to right and left of the chain as directed by the surveyor, while the follower remains at the chain with the tape-box The latter holds the tape at right angles to the chain, estimating the perpendicularity by eye in the general case, and calls out to the surveyor the lengths of the offsets and the chainages from which they are projected. The distances along the survey line at which fences, streams, etc , are intersected by it must also be observed and noted.

Notes.—(1) The surveyor should be on his guard to detect gross mistakes in reading He should remain beside the follower to check the readings and see that the tape is held perpendicular to the chain line and horizontal on slopes.

(2) Offsets should be taken in order of their chainages, and, before allowing the chain to be moved forward, the surveyor should make sure that no offsets have been overlooked.

(3) A common tendency is to take too many offsets. No attempt should be made to record irregularities too small to be shown on the scale to which it is proposed to plot.

(4) On flat ground offset measurements should be made on the surface of the ground ; on slopes the chainmen should use poles for plumbing.

(5) The accuracy required in the perpendicularity of offsets increases with the length of the offset, the scale of plotting, the importance of the feature offsetted, and the angle between the chain line and the line being located. The cross staff, optical square, or box sextant may be used in setting out long offsets, but in general the perpendicularity is estimated The correct position of the tape can be determined by swinging it through a small angle about the ring as centre The smallest tape reading against the chain is the required perpendicular distance.

(6) The accuracy required in measuring offsets depends on the first three factors above In particular, by keeping in view the scale to which the plotting is to be performed unnecessary refinement will be avoided Assuming about 01 inch as the smallest distance on the paper which can be distinguished in plotting, the following figures indicate the refinement of measurement required.

If the scale is 1 in = 100 ft or more, measurements should be recorded to the nearest foot or link

If the scale is 1 in = 50 or 66 ft , measurements should be recorded to the nearest ½ ft or link

If the scale is 1 in = 20 to 40 ft , measurements should be recorded to the nearest ¼ ft or ½ link

If the scale is 1 in = 8 or 10 ft., measurements should be recorded to the nearest inch

It is, however, better to be over- than under-accurate in view of the possibility that the survey, or a part of it, may be required on a larger scale than was originally intended.

(7) The allowable length of offset depends upon the accuracy desired, the method of setting out the perpendicular, and the scale of plotting Survey lines should be so arranged that offsets to important objects are as short as possible, but for the survey of indefinite features long offsets, up to, say, 100 ft , are permissible without instrumental setting out Progress is considerably delayed, however, when offsets exceed the length of the tape, and it is false economy to reduce the number of survey lines at the expense of convenient offsetting

(8) Special care is necessary in offsetting to buildings, and the measurements should be made self-checking by the use of ties and by noting the chainages at which the directions of the walls cut the survey line. Buildings should be taped round , those of irregular outline may have to be enclosed by subsidiary chain lines from which to offset

(9) As a large part of the time occupied on a chain survey is spent in offsetting, the necessity for systematic routine is evident The chainmen should not be allowed to exchange their duties, and the one who observes the measurements should be instructed to call them out in a uniform manner, thus—" Fence, 9½ off 285."

Note-keeping.—The field book, about 9 in. by 5 in., and opening lengthwise, may be of unruled paper, but more commonly each page has a single red line, or two red lines about ⅜ in. apart, ruled down the middle The single line represents the survey line, and against it are entered the total length of the line and the distances along the chain at which offsets are taken, intermediate stations are established, and fences, streams, etc., are crossed. The space on either side is available for sketches of the various features located from the survey line and for notes of the offset distances. In the double line system, the space between the lines is reserved for distances along the chain, which are thus kept entirely separate from other dimensions. In using a plain page book, the surveyor draws down the page either one or two pencil lines as preferred. This system has the great advantage that if more detail has to be surveyed on one side of a line than on the other, overcrowding of the notes is avoided by ruling the lines near one side of the page.

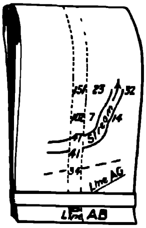


FIG 119
FIELD BOOK

On commencing the chaining of a line, a fresh page should be started, and the designation of the line is noted at the foot. Booking proceeds from the bottom of the page upwards (Fig 119), as it then progresses naturally with the chaining, and the right and left sides in the book correspond with those in the field when looking in the direction of chaining As the various features within offsetting distance are reached, the surveyor sketches them and enters the chainage and length of each offset as shown in Figs 120 and 121 For uniformity and ease in plotting, offsets should be entered as distances from the chain line only.

Notes —(1) The complete record includes (*a*) a general sketch of the lay-out of the lines, (*b*) the details of the lines, (*c*) the date of the survey, (*d*) a page index of the lines; (*e*) the names of the members of the party Some surveyors ink in their notes after plotting This is desirable on pages which have suffered from rain, but otherwise is a matter of choice

(2) The two essentials in booking are accuracy and clearness The surveyor, although he may intend to plot the work himself, should always aim at producing notes which could be plotted without difficulty by a draughtsman quite unfamiliar with the ground, as the field book may be referred to, and extracts made, at a future date This desideratum is promoted by (*a*) neat figuring and legible writing, (*b*) lucid sketches; (*c*) clearness in representing the points to which offsets have been taken; (*d*) the insertion of explanatory notes where misunderstanding might occur, (*e*) leaving nothing to the memory; (*f*) keeping the book clean.

(3) The making of satisfactory sketches proves troublesome to the inexperienced. The tendency is to allow insufficient room for sketches of intricate detail, so that dimensions cannot be entered clearly No attempt should be made to sketch strictly to scale. The size of complicated parts should be exaggerated, the curvature of flat curves should be increased, and angles which are nearly 90° or 180° should have their divergence from those values emphasised On the other hand, long straight lines should be shown shortened A comparison of the sketches in Figs 120 and 121 with the corresponding features on the scale diagram, Fig 123, will exhibit the allowable distortion. It should be observed that, in the double line system, a line crossed by the chain is apparently broken at the intersection, because the survey *line* is represented in the book by a *space*, which has no physical existence on the ground.

(4) The pencil should be of good quality, to avoid smearing, and rather harder than is usually preferred for writing, to prevent the notes being washed out by rain. In wind, it is a convenience to have a strong rubber band to hold the leaves of the field book (Fig. 119).

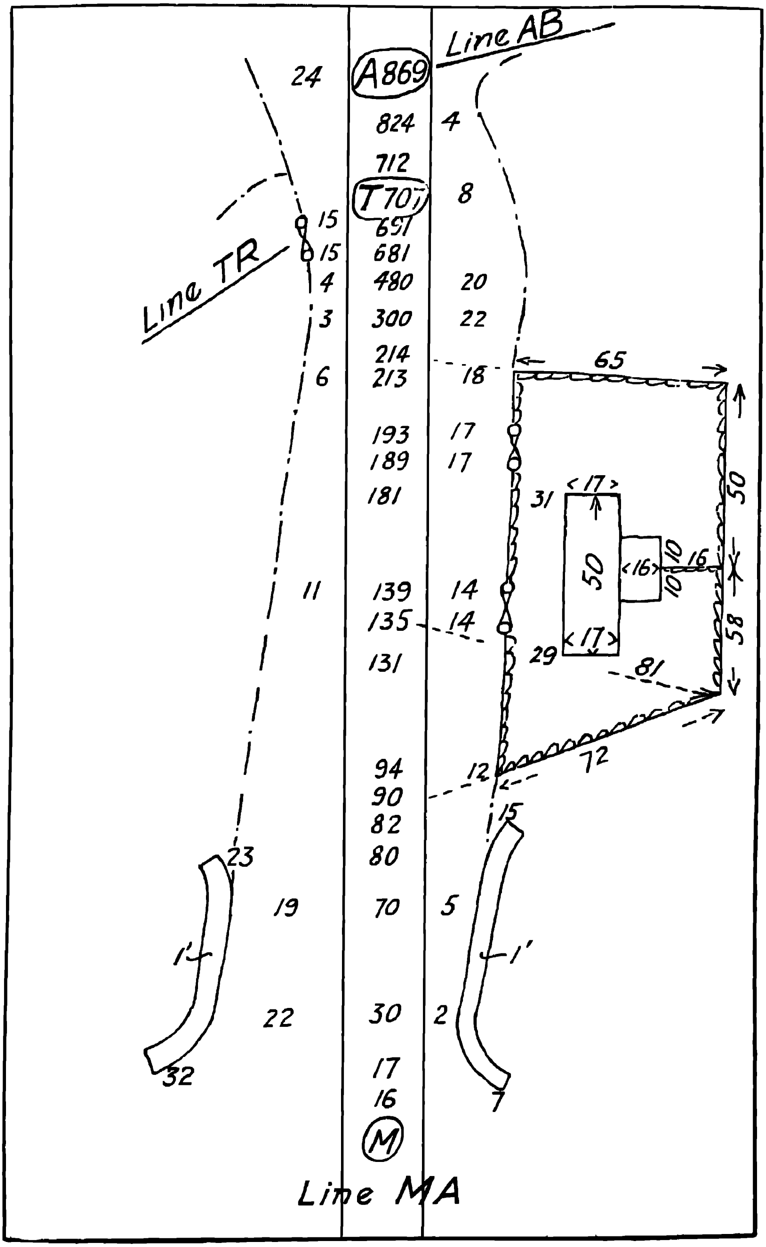


FIG. 120.—DOUBLE LINE BOOKING.

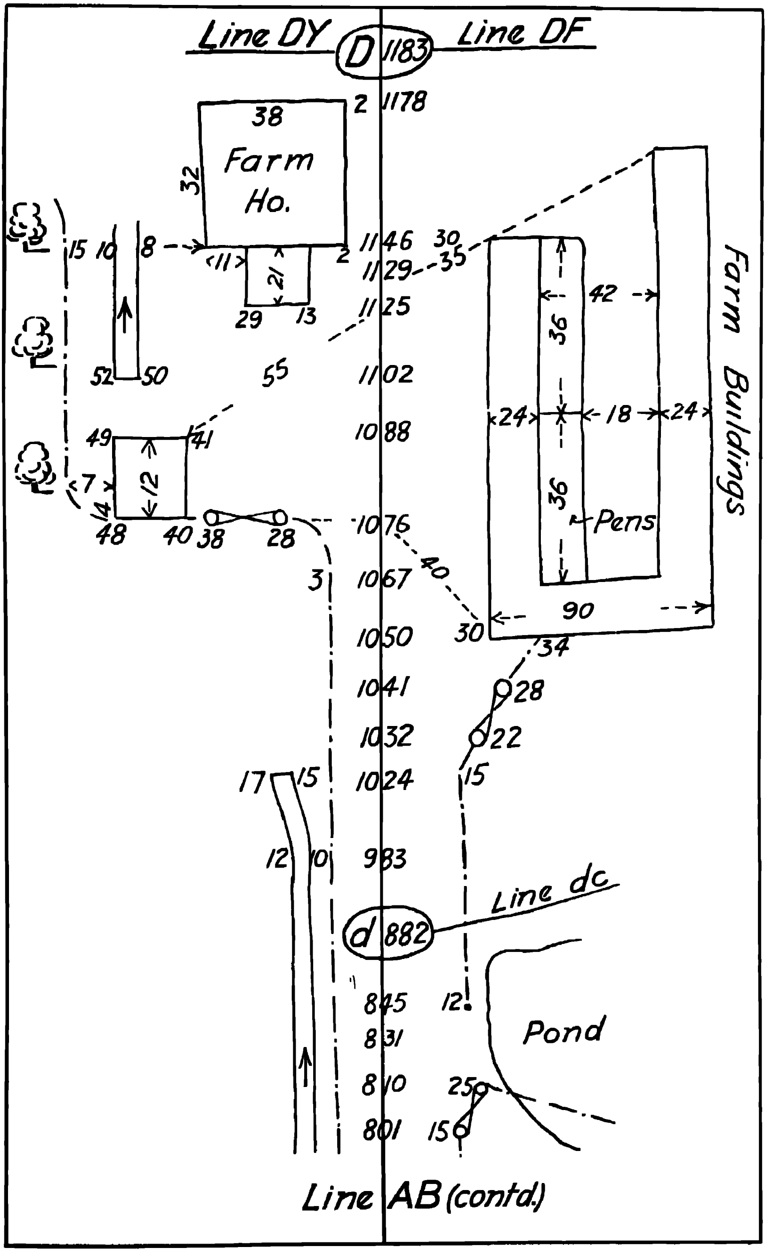


Fig. 121.—Single Line Booking.

Examples of Chain Surveys.—Fig 122 shows a small survey of an irregularly shaped parcel of ground. The noticeable features are: (*a*) the provision of a long backbone line AB; (*b*) the running of lines CH and EK merely as proof lines, to check triangles AEF and DJB respectively; (*c*) the manner of plotting and checking triangle FGH by locating G on CH produced and checking its position by FG; (*d*) the survey of the straight hedge to the right of CH simply from its intersections with lines AB and EF.

A farm survey is illustrated in Fig. 123. AB has been adopted as a base line in preference to a diagonal extending right across the survey, as the outline would not permit of well-shaped triangles by the latter arrangement. Having fixed AB, there is little choice as

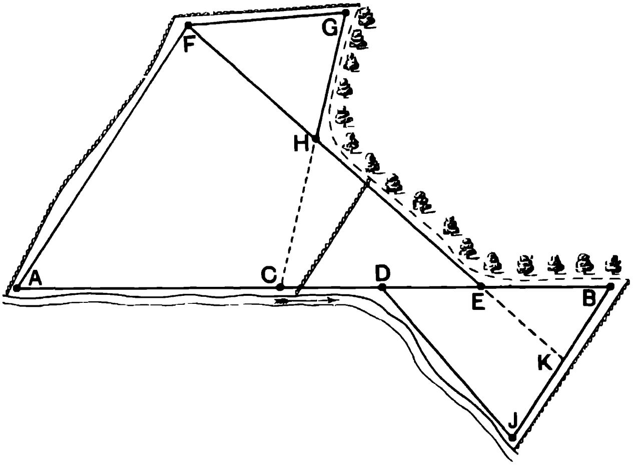


Fig 122.—Small Chain Survey

regards the positions of D, F, M, G, H, and J The line CK is inserted for offsetting to the pond and to the piece of rough ground: its extension KL is merely a proof line to check triangle AMF. No proof line is required in triangle DFB, as it forms part of triangle AFB, which is checked by DF and CK The narrow area ABZX is best surveyed and checked as shown. The lines are self-checking, while the fact that Y is in range with FD affords a further check, so that the chaining of a line between C and Y is unnecessary. The arrangement of lines for the survey of area XVRM is preferred to running diagonals XR and VM, as being more convenient for the location of the outline of the wood. Triangle PQN, with the proof line OQ, is introduced to avoid long offsets to the stream.

Survey of Woods, etc.—Chain surveying methods may prove tedious when applied to the survey of a dense wood, a pond, or other area across which lines cannot be run. In such cases a system of lines must be laid out to surround the area. Wherever possible,

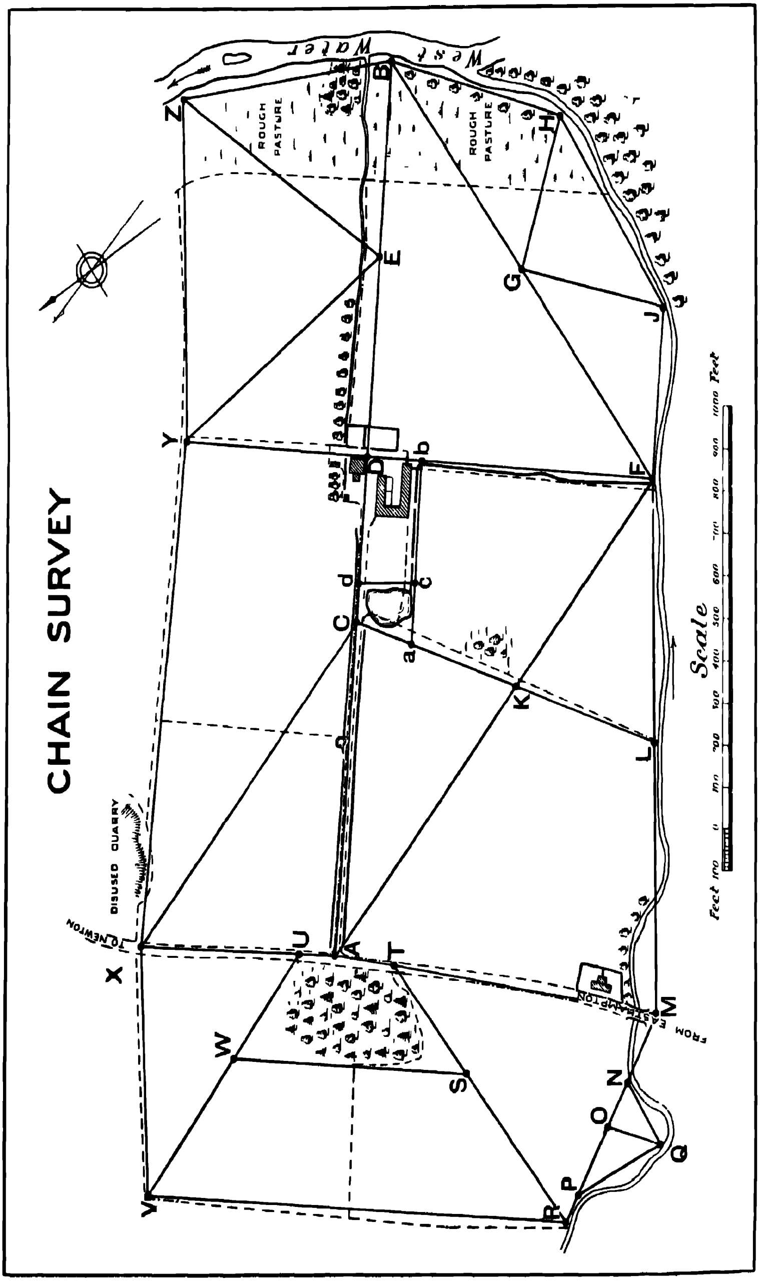

CHAIN SURVEY
West Water
ROUGH PASTURE
ROUGH PASTURE
DISUSED QUARRY
TO NEWTON
FROM EASTHAMPTON
A
B
C
D
E
F
G
H
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
a
b
c
d
Scale
Feet 100 0 100 200 300 400 500 600 700 800 900 1000 Feet

FIG 123.

the main lines should form a single triangle, checked by proof lines at one or more corners, and from the main lines subsidiary triangles may be projected inwards where required to shorten the offsets. Otherwise, as in Fig 124, the sides of the enclosing polygon must be fixed relatively to each other by forming triangles at the corners. This example could be plotted by first drawing AB and constructing the triangles AEC and DFB. On producing AE and BF to G and H respectively, the measured length GH should fit in between G and H A better check is, however, available by chaining triangle GKL On plotting this triangle on base GK, and producing GL to H, the station H so obtained should coincide with its position as given by the line BH It is desirable for the rapid location of a mistake that each of the triangles should be checked by a proof line.

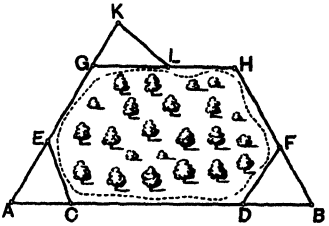


Fig 124 —Chain Survey of Wood.

Survey of Narrow Belts.—Fig 125 illustrates how the above principle is applied to the chain survey of a long narrow strip of country, the width of which can be controlled by offsets from a central chain line The introduction of triangles between the survey lines in this, as in the previous example, is simply the linear survey method of enabling the angles between the lines to be plotted Such surveys are more accurately and rapidly executed by measuring the directions of the lines by theodolite

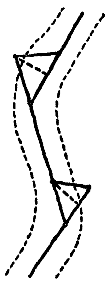


Fig 125

MISCELLANEOUS FIELD PROBLEMS

It is frequently impossible to arrange a survey so that all the lines can be run in the straightforward manner previously described. While the difficulties which may be encountered are most rapidly and accurately surmounted by the aid of an angular instrument, they may also be solved by a routine involving only the essential equipment used in chain surveying, and these methods will be considered here. Many of them involve the setting out of perpendiculars and parallels, and such operations will be treated first.

To Erect a Perpendicular to a Chain Line from a Point on It.—(*a*) Select points B and C on the line on either side of, and equidistant from, the given point A (Fig 126). Pin down the ring of the tape at B, and, having the end held at C, hold the mid-point D so that both halves are stretched tight. D is then on the required perpendicular.

Alternatively, sweep arcs of equal radii with the tape from B and C, scratching the arc on the ground with an arrow, and mark the intersection D.

By similarly locating D′ on the other side of A, a longer base is available for production.

(*b*) With A as centre, sweep an arc of 30 ft radius, and from B, 40 ft. along the chain from A, sweep an arc of 50 ft. radius C, the cut of these arcs, is on the perpendicular AC (Fig. 127), since $50^2 = 30^2 + 40^2$. The ring of a 100 ft linen tape may be pinned at A and the 80 mark held at B, while the 30 mark is placed so that the two parts are taut. A steel tape cannot be bent sharply at C, but, by holding the 100 mark at B, C is obtained as the position of the 30 and 50 marks held together with a 20-ft loop between (Fig. 128). Any multiples of 3, 4, and 5 can, of course, be similarly used, and the sides of the triangle are proportionately reduced when using a shorter tape. Integral values for the sides of right-angled triangles are proportional to $(2n+1)$, $2n(n+1)$, and $(2n^2+2n+1)$, where n is integral or fractional, but, of the various series of integers available, 3, 4, 5, and 20, 21, 29, corresponding to $n=1$ and $n=\frac{3}{4}$ respectively, yield the best intersections. With the latter, the whole of a 50 or 100 ft. tape is stretched (Fig 129).

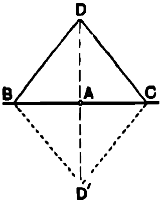


FIG. 126.

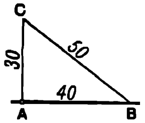


FIG. 127.

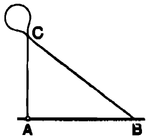


FIG 128

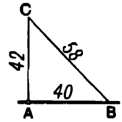


FIG 129.

To Drop a Perpendicular to a Chain Line from a Point Outside It.—i. *When the Point is Accessible.*—(*a*) With the given point A as centre, swing an arc to cut the chain line at B and C (Fig. 130). D, the mid-point of BC, is the foot of the required perpendicular.

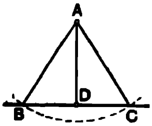


FIG. 130.

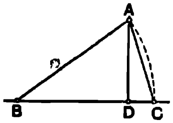


FIG. 131.

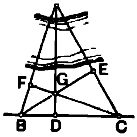


FIG. 132.

(*b*) Measure the distance from A to any point B on the line (Fig. 131). Set off BC on the chain equal to BA. Measure AC.

Then $CD = \dfrac{AC^2}{2BC}$.

ii. *When the Point is Inaccessible*—(*a*) By repeated trial of erecting perpendiculars from BC, find that which passes through A

(*b*) Select points B and C on the line (Fig. 132). Set out BE and CF perpendicular to AC and AB respectively. Mark their intersection G. AG produced to D is the required perpendicular

To Run a Parallel to a Chain Line through a Given Point.—i. *When the Point is Accessible.*—(*a*) From the given point A drop a perpendicular AB to the given line BC (Fig. 133). From C, as far along BC as convenient, erect a perpendicular CD equal to AB. AD is the required parallel Alternatively, measure the sides of a triangle A*ef*, with apex at A, and reproduce the triangle at D*gh*.

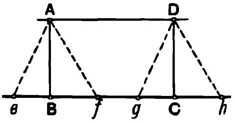


FIG 133

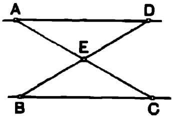


FIG 134

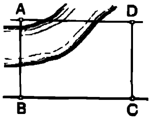


FIG 135

(*b*) From A measure AC to any point C on the line (Fig. 134) Mark E, the mid-point From any point B on the chain measure BE, and produce BE its own length to D

ii. *When the Point is Inaccessible*—Proceed as in (*a*) above, obtaining the inaccessible perpendicular distance AB (Fig 135) by the methods of page 148.

Obstacles.—The more important obstacles may be classed as :

1. Those which obstruct ranging but not chaining.
2. Those which obstruct chaining but not ranging.
3 Those which obstruct both

1. Obstacles which Obstruct Ranging but not Chaining.—This type of obstacle, in which the ends of a line are not intervisible, is difficult to avoid except in flat country, as it commonly occurs in the form of an intervening hill Two cases may occur .

(1) Both ends may be visible from intermediate points on the line.

(2) Both ends may not be visible from any intermediate point.

Case (1) Having plumbed the terminal poles A and B (Fig. 136), the surveyor C and his assistant D, each with a ranging pole, proceed up the hill, and take up positions such as C_1, D_1, in plan, as nearly in the line as they can judge, and such that C can see both B and D, while D can see A and C. C directs D to D_2, in line with B, and then D guides C into position C_2, in line with A. C now

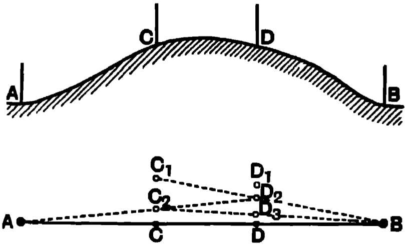


FIG. 136.

brings D to D_3, and so on alternately. When C and D are simultaneously satisfied that the other is in line with the remote pole, they are both in the line AB, and fix their poles. This method is also serviceable in ranging a line across a hollow.

Case (2). The same principle is applied in this case by gradually bringing a series of intermediate poles into range with each other and with the stations. This involves either additional assistance or considerable delay in walking from pole to pole, but is often the most suitable method.

When only the length of the line is required, or if offsets have to be taken from only a part of it, the method of the *Random Line* is best. Proceeding from station A, a line such as AB′ (Fig. 137) is ranged out at random, but as nearly towards B as can be judged. It is chained out to B′, where B′B is perpendicular to AB′, and B′B is measured. Then $AB = \sqrt{(AB')^2 + (B'B)^2}$. If C′ is a point of known chainage on AB′, a point C can be located on AB by the perpendicular offset $C'C = \frac{AC'}{AB'} \times B'B$ In this manner a sufficient number of points can be obtained on AB to complete its ranging, and, if necessary, AB is then chained for offsetting If, however, the detail to be offsetted is confined to a part CD of the line, it will suffice to chain between those points, since their chainages can be deduced from those of C′ and D′ If the detail can be located satisfactorily from AB′, time is saved by offsetting from that line, the plotting being performed by first obtaining B on the paper from the computed length AB.

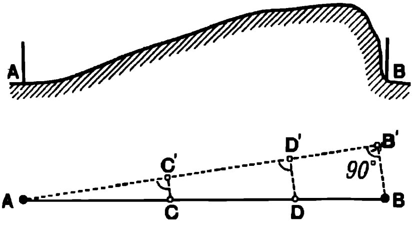


FIG 137

2. Obstacles which Obstruct Chaining but not Ranging.—The typical obstacle of this class is a sheet of water the width of which in the direction of measurement exceeds the length of the chain or tape The problem consists in finding the distance between convenient points on the survey line on either side of the obstruction. Two cases may be distinguished :

(1) In which the obstacle can be chained round.

(2) In which the extent of the obstacle prevents this.

Case (1) Figs. 138 to 143 illustrate six methods of finding the distance between poles A and B on either side of a pond.

(*a*) Set out equal perpendiculars AC and BD (Fig 138) ; then CD = AB.

(*b*) Erect perpendicular AC (Fig. 139), and measure AC and CB , then $AB = \sqrt{BC^2 - AC^2}$.

(*c*) Find, by optical square or cross staff, a point C which subtends 90° with A and B (Fig. 140). Measure AC and CB; then $AB = \sqrt{AC^2 + CB^2}$.

(*d*) Set out a straight line CAD (Fig. 141) Measure AC, AD, BC, and BD; then $AB = \sqrt{\frac{BC^2 \times AD + BD^2 \times AC}{CD} - AC \times AD}$.

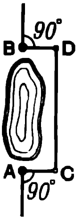


FIG 138.

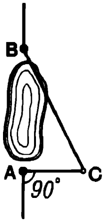


FIG 139

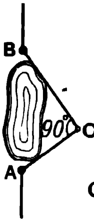


FIG 140

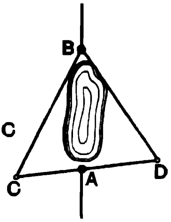


FIG 141.

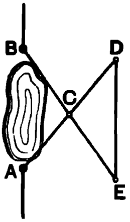


FIG 142

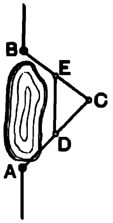


FIG 143.

(*e*) Mark a point C (Fig 142) Range D in line with AC so that CD = AC Range E with BC, making CE = BC, then ED = AB

(*f*) Select a point C (Fig 143), and measure AC and BC Mark D and E such that $CD = \frac{CA}{n}$ and $CE = \frac{CB}{n}$; then AB = *n*DE.

In each case the obstacle is surveyed from the auxiliary lines Method (*d*) is suitable for a wide obstacle, since offsets can be taken on each side, but long offsets are avoided in the other methods by repeating the work on the other side

Case (2). A river is typical of this class of obstacle.

(*a*) From A and another point C on the line (Fig 144) erect perpendiculars, or any parallel lines, AD and CE, such that E, D, and B are in line. Measure AC, AD, and CE; then $AB = \frac{AC \times AD}{CE - AD}$.

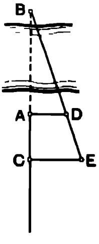


FIG 144.

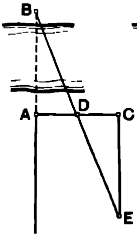


FIG. 145.

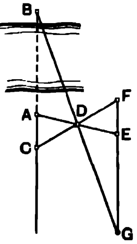


FIG. 146.

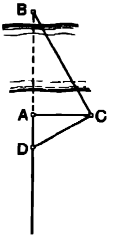


FIG. 147.

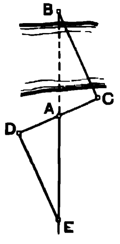


FIG. 148.

(*b*) Set out a perpendicular AC (Fig 145), and mark its mid-point D. From C erect CE perpendicular to AC, and mark E in range with BD; then CE = AB. More generally, if D lies anywhere on AC, $AB = \frac{AD \times CE}{CD}$.

(*c*) Select a point D (Fig 146). Measure AD and CD, and continue them their own lengths to E and F. Find G in range with BD and EF ; then EG = AB.

(*d*) Erect a perpendicular AC (Fig. 147), and, using an optical square or similar instrument at C, find D on the chain line so that BCD is a right angle Measure AC and AD ; then $AB = \frac{AC^2}{AD}$.

(*e*) By instrument locate C which subtends 90° with AB (Fig 148). Range D with AC, so that AD = AC Find E at which the line is cut by the perpendicular DE , then AE = AB.

3. Obstacles which Obstruct Ranging and Chaining.—In this case the obstruction cannot be seen through, but the methods of Figs 144 to 148 are applicable if, owing to the contour of the ground, the line can be ranged beyond the obstacle In the general case, however, the problem consists both in prolonging the line beyond the obstacle and finding the distance across it

(*a*) From A and B on the line erect equal perpendiculars AC and BD (Fig 149). Range E and F with CD, and set out EG and FH perpendicular to DF and equal to AC G and H are in the line, and CE = AG

(*b*) Select a point C (Fig. 150). Measure AC and BC, and mark D and E so that $CD = \frac{AC}{n}$ and $CE = \frac{BC}{n}$. Range F and G with DE. Measure CF and CG, and produce them to H and K, making CH = *n*CF and CK = *n*CG H and K are in the line, and AH = *n*DF.

(*c*) Set out a triangle BCD (Fig 151). Continue BC and BD to E and F, making BE = *n*BC and BF = *n*BD Mark G on EF so that EG = *n*CA. Similarly locate K. G and K are on the line, and AG = (*n*—1)BA.

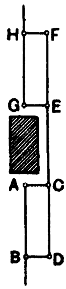


FIG. 149.

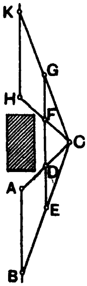


FIG 150.

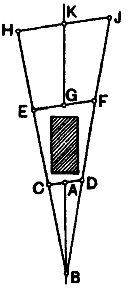


FIG. 151.

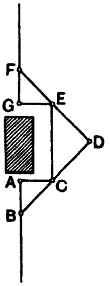


FIG. 152.

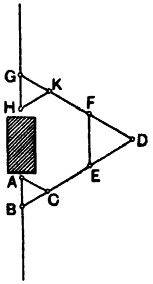


FIG. 153.

(*d*) Erect a perpendicular AC, and mark B so that AB = AC (Fig. 152). Produce BC to D. Set out DF perpendicular to DB, making DF = DB and DE = DC. From F and E swing arcs of radii = AC, obtaining intersection G. G and F are in the line, and CE = AG.

(*e*) By swinging the tape, construct an equilateral triangle ABC

(Fig. 153). Produce BC to D. Form an equilateral triangle DEF, and produce DF to G, making DG = BD. Obtain H on the line by an equilateral triangle GKH. AH = BD − BA − HG.

None of these methods ensures much accuracy in the prolongation of the chain line, and this type of obstacle should therefore be studiously avoided unless an angular instrument is available.

PLOTTING

Instruments.—The draughtsman will be provided with a set of the usual drawing instruments, which should include drawing pens, compasses with pen and pencil points and lengthening bar, dividers, spring bows, pricker, protractor, scales, set-squares, French curves, and colour brushes. In place of a T-square, which is of minor utility in plotting surveys, a 24-in. parallel ruler should be used. For drawing long straight lines a steel straight-edge is required, 6 ft being a serviceable length. Beam compasses are necessary for describing arcs of large radius, and are very useful in plotting various kinds of surveys. For drawing railway lines, a set of railway curves is essential. These are flat strips of pearwood, celluloid, or vulcanite, each cut to a distinct curvature. A useful set consists of 100 curves of radii from 1½ in. to 240 in.

Scales.—Drawing scales should be of well-seasoned boxwood or ivory and of superior quality. The most useful length is about 12 in , and a flat section with two bevelled edges accommodating two different fully divided scales proves more accurate and convenient than oval and triangular forms, having respectively four and six different scales. A useful pattern has one edge divided to chains and links with the same scale converted to feet on the other, so that dimensions laid down in links can be readily scaled in feet.

The more commonly used scales are 10, 20, 30, 40, 50, 60, 80, 100, 200, etc., ft. or links to 1 in. It is to be noticed that one set of graduations is employed for plotting to several scales decimally connected, *e.g.* the 20 scale, having 20 divisions to 1 in., can be used as a scale of 2, 20, 200, or 2000 ft. or links, according as the smallest division is treated as ·1, 1, 10, or 100 ft. or links.

Since Ordnance Survey maps are extensively used, alterations, new works, etc., have frequently to be plotted to the Ordnance scales. The larger scales are :

60 in. to one mile or 1/1,056,
50·688 ,, ,, 1/1,250,
25·344 ,, ,, 1/2,500,
6 ,, ,, 1/10,560,
1 ,, ,, 1/63,360.

The 1/1,056 and 1/1,250 scales apply to town maps only, and the first is now available only for London. Town maps were formerly published on a scale of 1/500, but this practice has been discontinued.

Materials.—*Paper.*—Only the best hand-made paper should be used for survey plans other than those intended merely for temporary use. Plans which will be subjected to much handling are rendered more permanent by having the paper mounted on holland or cotton. Mounting is also necessary for very large plans and long rolls, and should always be done before plotting, otherwise the plan will suffer distortion.

Notes—(1) Paper expands under the influence of a damp atmosphere, and contracts when dried The variations are less marked in seasoned than in fresh paper, and are also reduced in mounted papers Tight rolling of a drawing tends to stretch it, so that it is desirable that drawings should be kept flat

(2) Mounted sheets and rolls are sold in various sizes and qualities, but mounting may at times have to be performed by the draughtsman A sheet of the fabric, a few inches longer and wider than the paper, is tacked on to a table at intervals of 6 in or less round the margin, and is then thoroughly damped A quantity of flour paste is freshly prepared, and this is well rubbed into the back of the paper with a stiff brush or by the fingers Superfluous paste is brushed off to leave a uniform film over the whole sheet The paper is stretched on the cloth, and, by means of a squeegee roller or the edge of a large clean set-square, worked from the middle of the sheet towards the margin, the paste is pressed into the interstices of the fabric, and all surplus paste is squeezed out at the edges until the paper is perfectly flat The whole is left to dry before the tacks are removed, and the sheet is finally trimmed, or a narrow margin of cloth is left for pasting on the front of the sheet to protect the edges

If a large mounted sheet is to be prepared by joining individual sheets, unsightly overlaps are avoided if the sides to be joined are each given a feather edge This is effected by cutting each sheet half-way through along a straight line near the margin On tearing off the strip by pulling on the side of the paper away from the knife cut, the paper is brought to an edge, and these edges are to be overlapped in mounting

Pencils.—It is uneconomical to use other than superior pencils. 4H is the most suitable hardness for plotting, but 2H or 3H may be employed for sketching, lettering, etc A chisel point may be used for drawing straight lines, but for all other work a conical point is required Points should be kept very sharp by the use of sandpaper, pencil points on compasses requiring similar attention.

Ink—The ink should be good quality Indian or Chinese ink, preferably prepared fresh each day from the stick. Most bottled waterproof inks, however, prove quite satisfactory, and prepared inks of various colours are often required.

Preliminary Considerations.—*Scale.*—The scale is usually definitely, or at least approximately, fixed from considerations of the extent and purpose of the survey before the field work is undertaken. If a choice remains, the final selection may be governed by the size of sheet required to contain the survey.

Position of Survey on Sheet—Having drawn a border line giving a margin of 1 in. to 1½ in round the sheet, the position of the survey, title, scale, north point, etc., within the rectangle must be arranged so that the drawing will present a balanced appearance. Whenever possible, the north side of the survey should lie towards the top of the sheet The title may be placed either centrally at the top or in the lower right-hand corner. The scale is usually drawn at the bottom.

In the absence of an existing plan of the area, a suitable position of the survey on the sheet is best determined by first plotting the outline roughly on tracing paper to the scale of the drawing. This plot is placed over the drawing, and, when in a suitable position, the direction of the principal survey line and the position of one of its stations are pricked through. If the preliminary plot is prepared on a smaller scale, a suitable position must be estimated within a border rectangle similarly reduced

The Pencil Plot.—*Plotting Survey Lines*—The longest line is first drawn, and its total length, as well as the positions of intermediate stations, is carefully scaled Stations are marked with a pricker or a fine conical pencil point, and the marks are rendered conspicuous by having a small circle sketched round them. The triangles attached to the base line are then erected by describing short intersecting arcs with the lengths of the sides as radii, beam compasses being used if necessary. When these triangles are checked by fitting in the proof lines, the remaining stations are similarly laid down The whole framework must be plotted and checked before the filling in of detail is begun As the accuracy of the finished drawing is largely dependent upon the precision with which the framework is plotted, care should be exercised in scaling dimensions, and pencil lines should be very fine.

Plotting Detail—There are two ways of plotting offsets. In the first, the chainages from which offsets were taken are marked out along the survey line, and the lengths are scaled off at right angles. In plotting long offsets and those which were set out by instrument, a pencil line should be drawn perpendicular to the survey line by set-square, but otherwise it is sufficient to estimate the perpendicularity by eye.

In the second method, a short scale, called an offset scale, is used as shown in Fig 154. The ordinary scale is laid parallel to the survey line and so that (*a*) the zero of the offset scale coincides with the line, (*b*) the chainage of the working edge of the offset scale can be read on the long scale The long scale being held by weights, the offset scale is slid along to the various chainages, and the offset lengths can be pricked off rapidly.

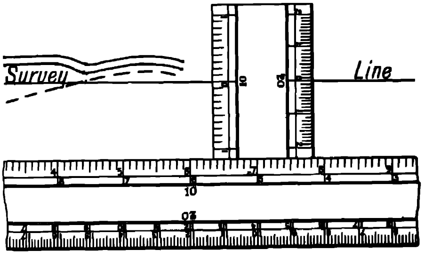


Fig. 154.—Use of Offset Scale.

Where much detail has to be shown, offset points should be joined up as they are plotted, and in doing so it should be remembered that changes of direction occur only at the offsets. The more common mistakes in plotting detail are : (*a*) plotting offsets from wrong points ; (*b*) plotting offsets on wrong side of survey line ; (*c*) omitting offsets ; (*d*) joining up wrong points ; (*e*) scaling chainages from wrong end of line.

Inking In.—When inking in the drawing, the pencil lines must be followed exactly. Lines should be rather finer than are suitable for structural or mechanical drawing on account of the smaller scales, but very fine lines should be avoided as they often present a ragged appearance and become indistinct when the plan gets dirty. Shade lining may be adopted in representing buildings. A writing pen should never be used in place of the drawing pen, as the lines produced have a varying thickness.

Black ink being used for existing features, new works are distinguished by being shown in red. On large scale plans railway lines are often drawn in blue. Telegraph poles may be joined by a thin blue or green line to indicate the line of wires. Survey lines are not inked in, but it is sometimes a convenience to have the positions of stations recorded, say, by a small red circle.

Conventional Symbols.—Symbols, more or less standardised by custom, are employed to suggest various features. The form taken by some of these depends upon the scale : thus, on a small scale it is usual to represent fences, hedges, and walls by a solid line, but on a large scale these are differentiated. Fig. 155 illustrates the more common symbols used in ordinary work. The convention of showing trees in elevation instead of in plan is generally favoured, as it is easier to make the result look effective : the trunks are sketched parallel to the side borders of the drawing. In large scale plans of railways, the running edge, not the centre line, of each rail is represented. A list of symbols relating to signals and other details of railway track has been drawn up by the British Engineering Standards Association, and forms the subject of their Specification No. 376, Part 1 (1930).

A considerable number of abbreviations and signs is adopted on the Ordnance maps, and one must be familiar with them in order to read the "25-in." and 6-in. maps correctly. They are explained in a booklet, "A Description of the Ordnance Survey Large Scale Maps," published by the Survey.

Colouring.—The application of large washes of colour to survey plans is unnecessary, and is open to the objection that it distorts the paper. Roads should be shown up in pale burnt sienna, water is indicated by an edging of Prussian blue, preferably shaded off, and buildings should be given a wash of grey, formed of very much diluted black ink. Boundaries have commonly to be made conspicuous by edgings of colour. Tints should be prepared from

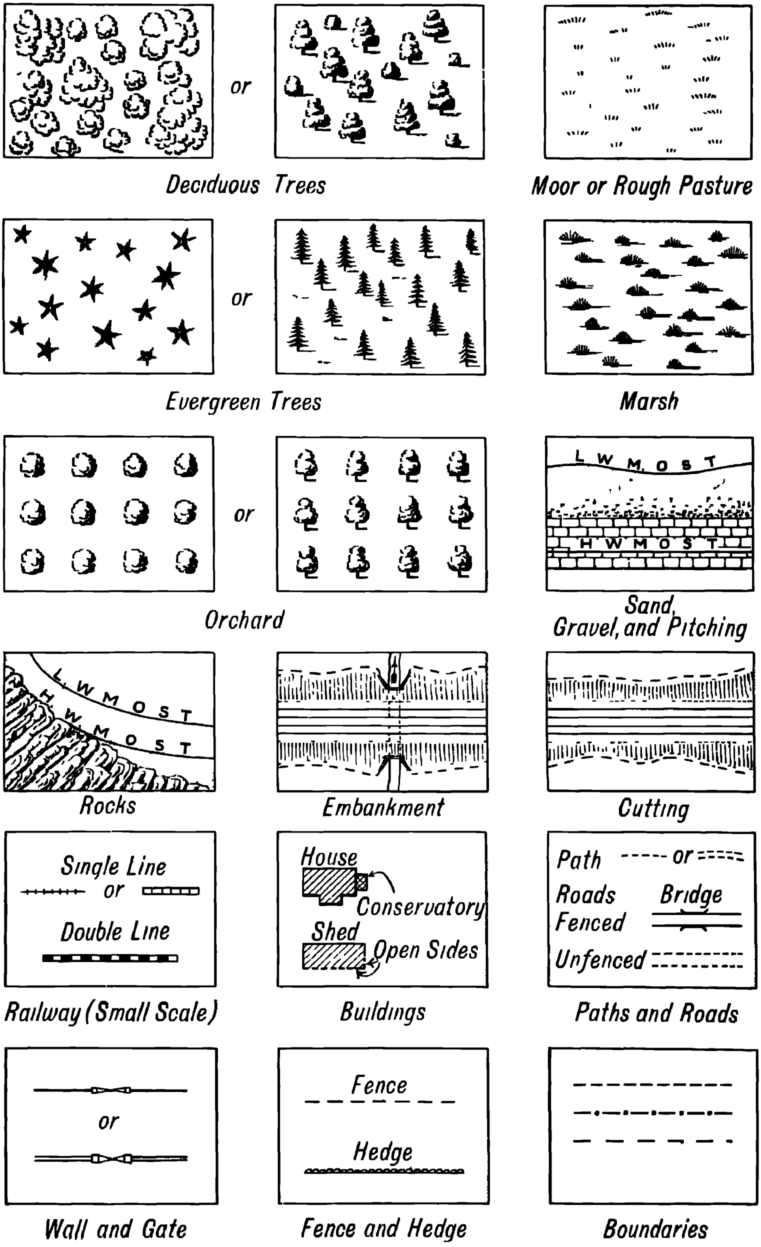


FIG. 155.—CONVENTIONAL SYMBOLS.

the best cake colours only, and should be applied with a large sable brush.

Lettering, etc.—Where appearance is important, neat hand printing is preferable to stencilling. To ensure proper spacing, letters should first be sketched in with pencil between top and bottom guide lines. Only severely simple styles of lettering are required, and the beginner should practise these either from a lettering copy-book or from well-finished drawings. Block characters, in the form of upright or sloping capitals and sloping smalls, are most commonly used, and, on account of the uniform thickness of the lines forming them, are much easier to draw than Roman characters. The title should be formed of upright capitals, while the use of upright and sloping capitals and italics of various sizes imparts sufficient variety to the appearance of the notes throughout the drawing The size of letters should accord with the size of the sheet and with the scale, and should indicate roughly the relative importance of the descriptions As far as possible, lettering should be parallel to the top and bottom borders ; otherwise, descriptions should be capable of being read from the lower right-hand corner of the sheet

The scale must always be drawn, and its value should be written above it. A plain north point should be placed in a convenient position, its direction being obtained from a field observation of the magnetic bearing of any of the lines.

Reducing and Enlarging Drawings.—To reproduce a drawing on a different scale from the original, the most accurate method is to replot the work from the field notes It is especially desirable to adopt this method for enlargements, in preference to utilising the original drawing, in order to avoid multiplying the errors of plotting of the latter

For reductions, or enlargements only slightly greater than the original, a simpler method is to rule a network of squares on the original drawing or on a sheet of tracing paper covering it. On the new sheet a similar series of squares is drawn, the linear dimensions of the latter being greater or smaller than those of the first, according to the enlargement or reduction required. By reference to the sides of the squares, the detail may be transferred from the one sheet to the other by scaling, by means of proportional compasses, or simply by estimation.

The reduction or enlargement of drawings may also be performed entirely by mechanical means by the use of either the pantograph or the eidograph. These instruments are, however, not extensively employed.

CHAPTER III

THEODOLITE AND COMPASS TRAVERSING

A TRAVERSE survey is one in which the framework consists of a series of connected lines, the lengths and directions of which are measured The system of fix involved is therefore that of Fig. 1*c*. When the lines form a circuit which ends at the starting-point, the survey is termed a closed traverse : otherwise it is unclosed. The latter type is applied in the survey of long strips of country, but for wide areas the survey usually takes the form of a network of closed traverses, with or without unclosed figures in parts.

Scope.—There is a considerable range in the character of traverse surveys according to the instruments used and the degree of accuracy necessary. In countries unsuited for triangulation, extensive theodolite traverses are required for the establishment of control points from which subsidiary surveys for the mapping of detail may proceed. In such primary traverse work great refinement is called for in both angular and linear measurements In small surveys, particularly if they form closed figures, or if they are run between points whose relative positions have been otherwise determined, the precision aimed at may be considerably less, and at the bottom of the scale we may have, as in certain exploratory surveys, distances estimated from the rate of march, and directions taken by compass towards sound signals instead of to visible points

In the present chapter are considered the methods ordinarily used in general cadastral and large scale engineering surveys, the applications of traversing in small scale mapping being dealt with in Vol II, Chap. VI.

Comparison with Chain Surveying.—Angular surveying is of much wider applicability than linear surveying, and is susceptible of greater accuracy, while in most cases the field work can be more rapidly executed. The routine of chaining and offsetting has to be performed just as in chain surveying, but the arrangement of the survey lines is quite different. The running of lines from which no offsets can be taken, as is frequently necessary for the plotting and checking of a chain survey, is obviated, and the lines can be arranged to follow the detail. The employment of angular instruments is essential in the many cases where it is impossible, on account of the shape of the survey, the character of the ground as to obstacles, etc., to lay out a satisfactory triangular system of chain lines.

BEARINGS

Direction.—The directions of survey lines may be defined (*a*) relatively to each other, (*b*) relatively to some reference direction, or meridian. In the first case they are expressed in terms of the angles between consecutive lines, and in the second by bearings

The use of a meridian of reference from which the directions of survey lines may be measured has so many advantages, particularly regarding the facilities afforded for checking and plotting, that this system is adopted in preference to the other. Only in small closed surveys involving few instrument stations should it be regarded as permissible to dispense with the establishment of a meridian.

The reference direction employed may be one of the following :

(*a*) True meridian.
(*b*) Magnetic meridian.
(*c*) Any arbitrary direction.

True Meridian.—The true or geographical meridian passing through a point is the line in which the earth's surface is intersected by a plane through the north and south poles and the given point. It therefore lies truly north and south The determination of its direction through a station involves astronomical observation, and is described in Vol. II, Chap. II.

The meridians converge from the equator to the poles, and consequently the true meridians through the various stations of a survey are not parallel to each other. All the survey lines, however, are to be referred to one meridian, *viz.* that through the initial station, or station at which the meridian has been established. The bearings of lines situated east or west of the initial station therefore differ from their azimuths, or directions from their respective local meridians. In consequence, a line common to two adjoining surveys is usually designated by different bearings in the two surveys. For ordinary small surveys the discrepancy is slight, and, when necessary, the correction for convergence (Vol. II, Chap. IV) can be applied.

The direction of true meridian at a station is invariable, and a record of true bearings therefore assumes a permanence not otherwise possible. This is a matter of considerable importance for large surveys in unmapped or imperfectly mapped country. In engineering location surveys the adoption of true meridian may save much time in retracement of the lines during final location and construction, more particularly if the ground is rough or densely wooded. For small surveys, on the other hand, true bearings need be used only if they can be measured from a meridian already established.

Magnetic Meridian.—The magnetic meridian of a place is the direction indicated there by a freely floating and properly balanced magnetic needle, uninfluenced by local attractive forces. Magnetic meridian does not coincide with true meridian except in certain localities, and the horizontal angle between the two directions is termed the *Magnetic Declination*, or declination of the needle. The amount and direction of the declination is different at different parts of the earth's surface : in some places the needle points west, and in others, east of true north. Its value at any place may be determined by making observations for true meridian, or may be interpolated approximately by reference to published isogonic charts.

Isogonic lines, or isogons, are imaginary lines passing through points at which the magnetic declinations are equal at a given time. Those through places at which the declination is zero are termed agonic lines. Across Great Britain and Ireland the value of the magnetic declination in 1931 ranges from about 11° W. at Dover to about 17° W in the west of Ireland, and the isogons have a bearing of roughly 16° to the east of true meridian.

Variation of Declination.—The declination at any place is not constant, but is subject to fluctuations, which may be divided into : (1) regular or periodic variations ; (2) irregular variations.

(1) This class of variation may itself be analysed into several components of different periods and amplitudes, but only two of them, secular and diurnal variation, are sufficiently pronounced to merit attention by the surveyor.

(*a*) *Secular Variation* is a slow continuous swing having a period of several centuries. Thus, at London, previous to about the year 1657, the declination was easterly and decreased annually. In 1657 the needle pointed towards true north Thereafter the declination gradually increased westwards until 1819, when a maximum westerly declination of about 24½° was attained. Since then the declination west has been decreasing until at present, 1931, its value at Greenwich is 12° 0′ W , with an annual decrease of 11′. While in this country the magnetic meridian has, within the recorded cycle, moved from one side of true meridian to the other, similar records in other places show that a complete swing may be performed on one side of the meridian and that the range and period of the oscillation vary in different localities. The change produced annually by secular variation amounts in different places to from 0 to about ± 12 min., but does not remain constant at any place. The annual change is greatest near the middle point of the complete cycle, and is least as the extreme limits are reached.

(*b*) *Diurnal Variation* is an oscillation of the needle from its mean position during the day. The amount of this variation ranges from a fraction of a minute to over 12 min. at different places, being greater in high latitudes than near the equator, and more in summer than in winter at the same place. In the northern hemisphere, the needle is east of its mean position during the night, and attains its maximum easterly position at about 8 a.m. It then moves westwards, and is farthest west from its average position at about 1 p.m. The mean position of the magnetic meridian occurs in this country at about 10 a.m., and again between 6 and 7 p.m. The direction of the daily swing is reversed in the southern hemisphere.

(2) *Irregular Variations* are caused by magnetic storms. A variation within a quarter of an hour of as much as 5° has been recorded, but this is very exceptional, and variations exceeding 1° are rare.

Arbitrary Meridian.—For small surveys, especially in mapped country, any convenient direction may be assumed as a meridian. This artificial meridian is usually the direction from a survey station either to some well-defined and permanent point or to an adjoining station. It is desirable that its magnetic bearing should be known. An arbitrary meridian has the merit of being invariable, and its direction can be recovered when required if the station or stations defining it are permanently marked or fixed by ties from permanent objects. If it is subsequently found necessary, the bearings can be converted to true bearings by the establishment of a true meridian.

Designation of Bearings.—Bearings are specified on either of two systems of notation: (*a*) the whole circle system; (*b*) the quadrantal system. In the whole circle or azimuth method, bearings originate from north, which is marked 0° or 360°, and are measured clockwise from the meridian, through E., or 90°, S., 180°, and W., 270°. In the quadrantal system, they are numbered in four quadrants, increasing from 0° to 90° from N. to E., S. to E., S. to W., and N. to W. Thus, if O (Fig. 156) is a survey station, and ON the meridian through it, the bearings of the lines from O are:

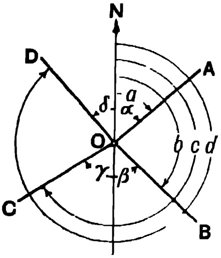


FIG. 156

Line.	Whole circle bearing.	Quadrantal bearing.
OA	*a*	N. α E.
OB	*b*	S. β E.
OC	*c*	S. γ W.
OD	*d*	N. δ W.

Note.—Quadrantal bearings are never reckoned from the E. and W. line, so that the letter which precedes the figure must be either N. or S.

Comparative Merits of Whole Circle and Quadrantal Reckoning.—In the whole circle method a bearing is completely specified by an angle, and the convention of reckoning clockwise from N. is so simple that the noting of the cardinal points as required in the quadrantal method must strike one as unnecessary trouble. The former system lends itself to the measurement of bearings on a continuously graduated circle as fitted to the theodolite and some compasses The circles of many compasses are, however, divided in quadrants, and in using such instruments quadrantal reckoning is naturally adopted. The fact that quadrantal bearings never exceed 90° is an advantage in extracting the values of their trigonometrical functions from ordinary tables, but the alternate clockwise and anti-clockwise direction of increase of angle in the different quadrants is sometimes inconvenient Thus, if the true bearings of Fig 156 have to be converted to magnetic bearings, given that the declination of the needle is 16° W , each of the whole circle reckonings has to be increased by 16° (subtracting 360° if the result exceeds 360°), but in the quadrantal method the correction is positive in the 1st and 3rd, and negative in the 2nd and 4th quadrants Care must be exercised that the appropriate cardinal points are applied to the resulting figures

Back Bearings.—The bearing of a line designated by the stations between which it lies is to be taken as that from the first station mentioned. Thus, in referring to the bearing of OA (Fig 156), the sign of the direction is from O towards A The bearing from A to O is termed the back, or reverse, bearing of OA, which latter may be distinguished as the forward bearing By referring to a parallel meridian through A, it will be evident that the back and the forward bearings of the line differ by 180°.

In whole circle reckoning, the back bearing of a line is obtained from the forward bearing by applying 180° To apply 180°, add when the given bearing is less than 180°, and subtract when the given bearing exceeds 180°. In the quadrantal system, it is only necessary to change the cardinal points by substituting N for S. and E. for W , and *vice versa* Thus, if the bearing of a line AB is observed as 300°, or N. 60° W., the back bearing of AB, or the forward bearing of BA, is 120°, or S. 60° E

Reduced Bearings.—In finding the values of the trigonometrical functions of a whole circle bearing exceeding 90°, one must refer in the tables to the corresponding angle, less than 90°, which possesses the same numerical values of the functions. This angle is called the reduced bearing, and is that between the line and the part of the meridian, whether the N. or S end, lying adjacent to it. It is therefore the angle used in the quadrantal reckoning of bearings.

To Reduce Whole Circle Bearings.—The following rule is applied :

If the bearing lies between 0° and 90°, no reduction is required.
„ „ 90° and 180°, subtract it from 180°.
„ „ 180° and 270°, subtract 180° from it.
„ „ 270° and 360°, subtract it from 360°.

It follows that the reduced bearing of a line lying due N. or S. is 0°, while that of a line due E. or W. is 90°.

FIELD WORK OF THEODOLITE TRAVERSING

Field Party.—The minimum party consists of the surveyor and two chainmen, and this is sufficient for small surveys, if no clearing is necessary In large theodolite surveys, particularly when rapid progress is important, the party may consist of a chief, an instrument man, a note-keeper, two chainmen, and a few labourers, the number depending upon the nature of the ground.

The chief of party directs the survey, and, in particular, reconnoitres the forward ground, fixes the position of stations, and sees to their being pegged and flagged The instrument man is responsible for the angular observations and notes, and the note-keeper directs and records all linear measurements The labourers clear away underbrush for the line of sight where necessary, fetch poles, etc., and one of them is given the duty of driving pegs and erecting signals.

Equipment.—For the linear measurements, the equipment is as for chain surveying (page 127) In theodolite traversing, the steel band is to be preferred to the wire chain, because the linear work usually contributes more than the angular observations towards the total error.

The theodolite should be a transit of 4-in. to 6-in size, with either vernier or microscope reading Except in city work, a prismatic compass may be carried for use on subsidiary traverses. By including traverse or trigonometrical tables in the miscellaneous equipment, the accuracy of closed circuits can be tested before leaving the field.

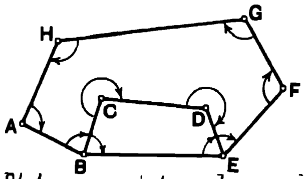
Selection of Stations.—As in linear surveying, considerations of easy chaining and short offsets should be given due weight An unobstructed view between adjacent stations is necessary for the instrumental observations, and, where possible, stations should be arranged to permit of check sights (page 173). The ground at stations should be firm to afford an unyielding support for the instrument, and should be moderately level rather than on a steep slope. Unless in exceptional circumstances, stations should not be established in situations, such as on roads, streets, or railways, where the observations will be delayed, and the instrument possibly endangered, by passing traffic.

Stations may be marked by pegs as described on page 138, but more permanent marks are often required for some or all of the stations of a traverse. These may take the form of a bolt or spike set in a block of concrete or a large stone. Reference marks to aid in the recovery of the station point should be established in its vicinity, their distance and bearing from the station being measured. The notes should include a detailed description of the site of each station with particulars of the reference marks.

Order of Field Work.—If the party is small, it is preferable in minor surveys to complete the observation of bearings before the chaining is begun, or *vice versa*. Extensive surveys may be made in sections in the same manner. A full party, however, is divided into two groups, the first selecting stations, clearing, and observing bearings, while the other follows with the chaining.

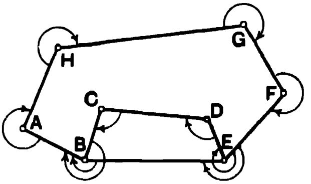
Theodolite Observations without Reference Meridian.—The horizontal angles measured at the several stations may be either (*a*) included angles, (*b*) deflection angles. An included angle is either of the two angles formed at a station by two survey lines meeting there A deflection angle is that which a survey line makes with the preceding line produced beyond the station occupied, and its magnitude is the difference between the included angle and 180°.

The minimum routine to be adopted for the observation of included angles is given on page 79, and it is often sufficient in small surveys when the linear measurements are made by chain. It is, of course, preferable, especially in hilly country, to observe each angle " face right " and " face left " (page 85) and to read both verniers, not only to minimise the effects of instrumental error, but also to provide a check against mistakes in reading. In measuring deflection angles, having bisected the back station by using the lower clamp and tangent screw, the telescope is transited, and is then pointed to the forward station by means of the upper clamp and tangent screw. The measurement is either right- or left-hand from the production of the back line Included angles can be measured clockwise or counter-clockwise from either the back or the forward station, but it is well to adhere to the regular routine of measuring from the previously occupied station and in a clockwise direction, since the graduations of the theodolite circle increase in this direction This does not necessitate that the telescope should always be turned clockwise, although it is better to wheel it in a constant direction. Figs 157 and 158 show that, in a closed polygon,



Stations occupied in order A B C--

FIG. 157.



Stations occupied in order A H G--

FIG. 158.

angles measured clockwise from the back station are either interior or exterior according to the direction of progress round the survey. Interior angles are obtained by proceeding counter-clockwise round the figure, but these will be exterior to subsidiary circuits as at C and

D. Whether included or deflection angles are observed, the notes must always include a sketch showing clearly the angles measured.

Included angles are to be preferred to deflection angles. The latter are sometimes used in railway surveys, in which a series of traverse lines may make small deflection angles with each other, but they are open to the objections that right- and left-hand angles may be confused in booking and that the transiting of the telescope introduces possible errors of non-adjustment if observations are not made on both faces.

Observation of Bearings.—The routine at the initial station of the survey differs from that at the remaining stations in that the observations are there directed to referring the first survey line to the meridian. At the second station the bearing of the second line is measured only indirectly from the meridian by making use of the known bearing of the first line. The observations at the subsequent stations are similar to those at the second To avoid repetition in the following descriptions of the observations, it is supposed that a single measurement only of each bearing is made, but the observations may be duplicated by being taken on both faces of the instrument.

Procedure at Initial Station.—Having set the theodolite over the station, and performed the other temporary adjustments, the surveyor must proceed to orient the instrument with respect to the meridian selected. The theodolite is said to be oriented when the horizontal circle is clamped in such a position that the vernier reads zero when the line of sight is directed along the meridian. On pointing the telescope in any other direction, the same vernier will record the whole circle bearing of the line from the station to the object observed, provided the lower plate remains undisturbed. The manner of orienting the instrument depends upon the meridian adopted.

Orienting on True Meridian.—In establishing true meridian at the initial station, what is determined, as the result of calculation based upon the astronomical observations, is the true bearing of a line from the station to a reference mark or signal set up for the purpose. As illustrated in Fig. 159, the point M has been used in this manner, and the true bearing of AM is computed The necessity for making astronomical observations is sometimes obviated when working in a country provided with a system of trigonometrical stations established by the State. If one of these is conveniently near the survey, it may be used as the initial station A, and the true bearing from A to the next trigonometrical station M is extracted from the published tables of positions.

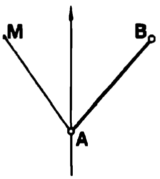


FIG. 159.

To orient the theodolite by use of the bearing AM, one of the verniers is first set to read that bearing, and the vernier plate is clamped to the circle. With the lower clamp slack, the telescope is then turned towards M. When M appears near the intersection of the hairs, the lower clamp is tightened, and the line of sight is brought exactly on the signal by the lower tangent screw. This completes the orientation, since the vernier still reads the true bearing of the line along which the line of sight is directed. Under these conditions, if the upper clamp is released, and the circle is kept fixed, the telescope will point towards true north when the vernier is set to zero or 360°. To observe the bearing of the survey line AB, the telescope, carrying the vernier plate with it over the fixed circle, is directed to B by the upper clamp and tangent screw. The bearing is then read off on the vernier previously used.

Orienting on Magnetic Meridian.—In orienting from magnetic meridian, reference is made to the compass on the theodolite. The compass box, if of circular form, is mounted on the vernier plate, as in Fig. 53, while the trough pattern is either connected to the vernier plate by being screwed to a standard or is attached to the lower or graduated plate. In the case where the compass is connected to the vernier plate, the line joining the N. and S. graduations in the circular form, or that joining the zeros of the scales in the trough form, bears a fixed relationship to the line of sight of the telescope, and is intended to be parallel to it. When the compass is attached to the lower plate, the line of zeros is parallel to the line of sight only when the horizontal verniers read 0° and 180° respectively

To orient, with either arrangement, the vernier is first set and clamped to zero, the lower clamp being slack The needle is then lowered upon its pivot, and the instrument is turned about the outer axis until the N. and S graduations are brought opposite the ends of the floating needle The lower clamp is then tightened, and the lower tangent screw is used to bring the zero graduation to exact coincidence with the point of the needle The instrument is now oriented, since the line of sight is directed towards magnetic north while the vernier reads zero. To observe the bearing AB, it is only necessary to set the line of sight on B by means of the upper clamp and tangent screw, and note the reading of the vernier previously set to zero. The result of the observation may be checked by a glance at the reading of the north end of the needle in the circular box form.

Notes.—(1) It is to be noticed that when the needle rests on its pivot, the instrument can be rotated about it without disturbing its direction.

(2) The needle cannot be set properly if it is looked at from one side. The eye should be in the vertical plane of the needle as nearly as possible.

(3) When the plate has been oriented, the telescope may be pointing south, in which case it must be transited before observing bearings.

(4) While the needle of a trough compass is more sensitive than the shorter needle of a circular box, neither is capable of defining the meridian with the

refinement with which angular measurements can be made by the theodolite. This is not highly important, since in theodolite surveys, as distinct from compass surveys, the bearings of all lines after the first are measured from the bearing of the first line, and the needle is consulted at subsequent stations only as a check Inaccurate orientation at the first station therefore merely turns the whole survey through a small angle, but does not distort it.

(5) If the value of the magnetic declination is known, the needle may be used for approximately orienting to true meridian by setting off the declination It is, however, better, as regards facility for applying the compass check, to adhere to magnetic bearings in the field, subsequently converting them to true bearings if required

Orienting on Arbitrary Meridian.—Having set and clamped the vernier to zero, it is only necessary to sight the object or station defining the direction from A of the adopted meridian When the signal is bisected by the use of the lower clamp and tangent screw, the circle is oriented The upper clamp is then released, and bearing AB is observed as before

Procedure at Second and Subsequent Stations.—The routine at these stations is independent of the meridian adopted, but several methods of observation are available, as follows

(1) By carrying forward the bearing
   (*a*) Direct method involving transiting of the telescope.
   (*b*) Direct method without transiting
   (*c*) Back bearing method
(2) By measurement of angles
   (*a*) Included angles
   (*b*) Deflection angles.

Carrying Forward the Bearing.—While the measurements in these methods are virtually those of the angles between the lines meeting at the various stations, the instrument is so manipulated that the required whole circle bearings are read directly on the vernier In following the methods, it is necessary to distinguish between the two horizontal verniers They are designated here as 1 and 2, the former being supposed used at the initial station. If they are not given distinctive marks on the instrument, they can easily be distinguished by their positions with respect to the vertical circle or a plate level.

In each of the three methods, after observing the bearing of AB from A, the lower clamp is released, and the instrument is carried to B with vernier 1 kept clamped at the reading obtained. A signal is left at A.

Direct Method with Transiting.—(*a*) Set up and level the instrument at B. See that vernier 1 still records the bearing AB. If the plate has slipped during transfer of the instrument, correct the reading by the upper tangent screw

(*b*) By using the lower clamp and tangent screw, sight back on A.

(*c*) Transit the telescope. The line of sight has now the same direction as it had at A, and vernier 1 still records the bearing AB. The instrument is therefore oriented.

(*d*) Bearing BC can now be observed by releasing the upper clamp and sighting C, the upper tangent screw being used in bisecting C. Vernier 1 now records the bearing BC.

(*e*) This reading being maintained on the vernier, the instrument is transferred to C, and the routine is repeated. On transiting at C, the instrument is returned to the same face as was used at A.

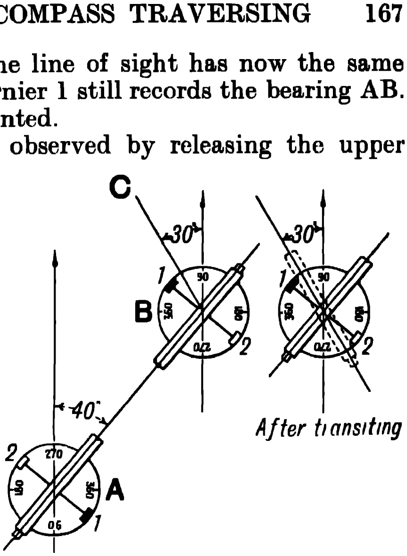


FIG. 160

Example.—Fig. 160 illustrates the case where the bearings of AB and BC are respectively 40° and 330°. It will be seen that, on taking the backsight BA, the verniers occupy the same positions relative to the telescope as at A, but, since the telescope is at 180° to its previous direction, the orientation of the circle is 180° different from what it was at A. This is neutralised on transiting the telescope. To bring the telescope into the position shown dotted, in order to sight C, it must be turned counter-clockwise through 70°, or clockwise through 290°, either movement resulting in vernier 1 (shown in black) being brought opposite the reading 330° as required.

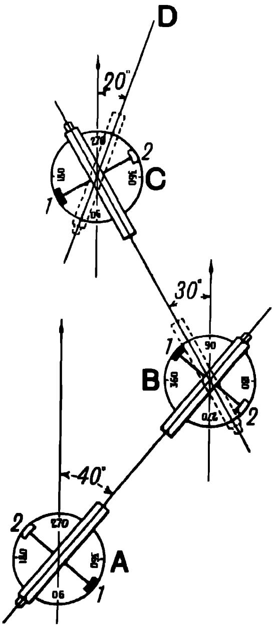


FIG. 161

Direct Method without Transiting.—The manipulation of the instrument at B is similar to that in the previous method, except that the telescope, instead of being transited after the backsight is taken, is turned directly on to C. The difference of 180° in the orientation of the circle at B from its orientation at A therefore remains uncompensated, and the reading of bearing BC on vernier 1 is 180° out. A correction of 180° has therefore to be applied to the reading or readings taken at B, adding the correction if the observed value is less than 180°, and subtracting if the reading exceeds 180°. At C the orientation, being 360° out, is correct, and the results need no adjustment The application of 180° is therefore necessary only at the 2nd, 4th, 6th, etc , stations occupied.

Example —Fig. 161 shows this system applied to the previous case To sight C from B, the telescope must now be turned through 110° clockwise, or 250° counter-clockwise, and vernier 1 then reads 150°, which falls to be increased by 180° Following the process to station C, let the bearing CD be 20°. After backsighting on B, the rotation necessary to bring the telescope into the position shown dotted, in order to sight D, is 130° counter-clockwise, or 230° clockwise, and the vernier then reads 20°

Notes —(1) Some surveyors adopt the routine of reading opposite verniers alternately to eliminate the 180° correction, but it is simpler to read one vernier throughout and note the corrected values

(2) Notwithstanding that a number of successive instrument stations may lie in the same straight line, the correction must be applied at alternate stations

Back Bearing Method.—(*a*) Set up and level the instrument at B as before

(*b*) *Before* sighting back on A, set vernier 1 to read the back bearing of AB, and fix the upper clamp

(*c*) By using the lower clamp and tangent screw, sight back on A The instrument is now oriented, since vernier 1 records the bearing BA, along which the line of sight lies

(*d*) Release the upper clamp, and, without transiting, direct the telescope towards C Clamp, and adjust by the upper tangent screw Vernier 1 records the bearing BC

(*e*) Apply the same method at all the subsequent stations.

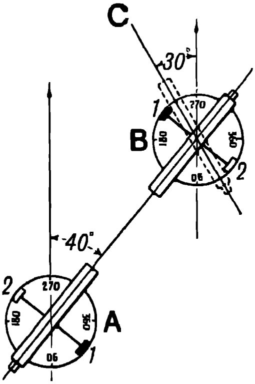


FIG 162

Example —Fig 162 shows that, on backsighting with vernier 1 set to the back bearing, the circle has exactly the same orientation at B as at A To turn the telescope into the dotted position for sighting C, the same rotation is required as in the last case, but vernier 1 gives the required bearing directly.

Precautions in Carrying Bearings Forward. — (1) It is necessary to guard against using the wrong clamp and tangent screw. The routine at all stations consists of the two steps : (*a*) orientation by backsighting ; (*b*) measurement of a bearing or bearings. In backsighting, only the lower clamp and tangent screw must be used to bring the intersection of the hairs on to the signal, and these must not be touched again until the observations at the station are completed.

(2) If the telescope is set upright in carrying the instrument, it

should be restored to its previous position before backsighting. Confusion may arise by inadvertently transiting

(3) When several bearings are measured from one station, the round of bearings should be completed by observing the backsight a second time in order to detect possible movement of the circle The observation to the station to be next occupied may then be repeated, so that the vernier may be left at the required reading.

Relative Merits of Methods of Carrying Bearings Forward.—In point of speed there is little difference between the three methods, as the time occupied in the first two in checking the vernier reading before backsighting is not much less than that required for setting the back bearing The first method is probably the most mechanical, but, if only single observations are made, the transiting of the telescope introduces possible errors of non-adjustment, and in respect of accuracy the others are preferable On the whole, the second method is the most satisfactory The necessity for applying 180° at every second station is not likely to lead to error, as an omission to apply the correction is easily traced

Bearings from Angles.—Whether included or deflection angles are measured, the procedure at the second and subsequent stations is precisely the same as when working without a reference meridian (page 163) The bearing of the first line, however, is measured, and those of the remaining lines are computed from it by applying the angles This is best performed with the aid of a sketch roughly in proportion If a meridian is drawn through each station, the equality of the alternate angles, a (Fig 163), due to the parallelism of successive meridians suggests the manner of using the data

The calculation may be formulated as follows

Let AB = the known bearing,
BC = the required bearing,
c = the *clockwise* angle ABC,
d = the deflection angle.

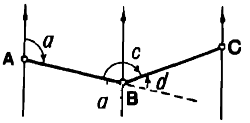


FIG. 163.

(1) Bearing BC = Bearing BA + c,
(subtracting 360° if the result exceeds 360°)

(2) Bearing BC = Bearing AB $\pm d$,
(using the + sign if d is clockwise from AB produced, and the — sign if d is counter-clockwise, as in Fig. 163 ; and adding 360° if the result is negative, and subtracting it if the result exceeds 360°).

Quadrantal bearings should be converted to whole circle reckoning before applying the formulæ.

In the reverse process the formulæ become ·

(3) c = Bearing BC — Bearing BA,
(adding 360° if the result is negative).

(4) d = Bearing BC — Bearing AB,
(if d is positive, it is clockwise from AB produced).

Survey of Detail.—The bulk of the detail is located by offsets, but the radiation, or angle and distance, method is available for detail near instrument stations. In this, bearings to the various points are observed, and the lengths of the radial lines from the instrument are measured. Unimportant detail and distant and inaccessible points may be fixed by the intersection of bearings from two instrument stations

In city surveying, where there is a large amount of detail, the best procedure is to survey only the frontages of the buildings during the running of the traverse lines The miscellaneous detail can be subsequently located from the buildings and from subsidiary traverse lines projected where possible towards the back of the main buildings This system is particularly advantageous if the main survey has to be executed during the night, when traffic is suspended.

Note-keeping.—The field notes should always include a sketch of the framework of survey lines, roughly to scale, so that the relative directions of the lines may be shown approximately correctly The stations should be lettered or numbered on the sketch, and the observed values of the angles or bearings may be noted in their proper places In the case of bearings from which co-ordinates are to be calculated, it is preferable to enter the results in tabular form as they are observed The booking of offsets is performed exactly as in chain surveying

ERRORS

Sources of Error in Theodolite Traversing.—Errors of linear measurement are dealt with on page 132 Those to which the angular observations are liable may be treated as

(1) Instrumental errors ,

(2) Errors and mistakes in setting up and manipulating the instrument ;

(3) Observational errors ;

(4) Errors due to natural causes

1. Instrumental Errors.—The effects of residual errors of adjustment, as well as of non-adjustable errors, can be satisfactorily reduced only by the adoption of a system of multiple observations, either on the repetition or direction principle (Vol II, Chap. III). In employing any of the methods which have been described above for carrying forward the bearing, it is advisable to measure each bearing twice, face right and face left, and to read the opposite verniers at each observation. It is unusual to take more than one face right and one face left observation in ordinary traversing ; but if the work is required to be of a precise character, and suitable precautions are taken in the linear measurements, a greater number of angular observations are taken for averaging. In such a case.

the included angles between the lines should be measured, the bearings being deduced from the average values. In thus endeavouring to obtain results of superior accuracy with a small theodolite, the necessity for rigidity and stability in the instrument and tripod must be recognised.

2. Errors of Manipulation.—(*a*) *Defective Centering* —If the centre of the instrument is not vertically over the station point, the angle or bearing is not measured from the point to which it is presumed the foresight has previously been taken. The angular error produced depends upon the error of centering, the lengths of sights, the magnitude of the angle being measured, and the position of the instrument relatively to the station and the points sighted, no error being introduced if the four points are concyclic Under the worst circumstances, an error of centering of about $\frac{1}{6}$th in may produce an error of 1 min in the angle measured if the sights are only 100 ft. long, but centering would have to be at least 9 in out to produce a similar error with sights a mile long Considering the ease with which centering may be performed, the error is not likely to be appreciable in ordinary work, except in very short courses (*cf.* page 259)

(*b*) *Defective Levelling* —Careless levelling, particularly of the plate level perpendicular to the telescope, may be productive of more serious error than the last, especially if the points sighted are at considerably different elevations In the absence of a striding level, the trunnion axis of the telescope is made horizontal by reference to this plate level Although the plate levels are in adjustment, and the levelling of the instrument has been carefully performed, unequal settlement may cause the bubbles to move out of centre, and, if the instrument is not relevelled, the dislevelment of the horizontal axis introduces error whenever the inclination of the telescope is changed

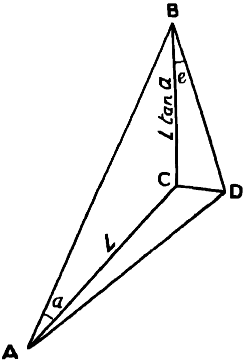


FIG. 164.

Let *e* be a small angular error of dislevelment of the horizontal axis, or the inclination to the vertical of the plane in which the line of sight travels when the telescope is elevated or depressed If a sight is taken from A (Fig 164) to an object B at an inclination of *a* to the horizontal, the direction AB or AC is erroneously recorded by the horizontal circle as AD, since a horizontal line of sight has to be placed along AD in order that B may be bisected on elevating the telescope. The error of direction is therefore

$$\tan^{-1}\frac{\mathrm{CD}}{\mathrm{AC}} = \tan^{-1}\frac{L\tan a\tan e}{L} = e\tan a, \text{ since } e \text{ is small.}$$

The error in the value of an angle or bearing is the algebraic difference of the errors developed in the backsight and foresight, so that the greater the change of inclination of the telescope for the two observations the greater is the error. In the most precise angle measurements corrections are computed from the striding level readings (Vol. II, Chap. III).

(c) *Slip*—Observations are vitiated if the orientation of the instrument is disturbed by the instrument turning · (a) on the tripod head by not being screwed firmly home, so that it slackens back , (b) on the lower parallel plate of a four-screw head by the levelling screws not being turned until they grip , (c) by neglecting to clamp a shifting head , (d) by insufficient tightening of the lower clamp

(d) *Using Wrong Tangent Screw*—If the upper tangent screw has been used in backsighting, the mistake will be discovered by a glance at the vernier before sighting forward, but the mistake of turning the lower one in bisecting the forward signal is not evidenced until a check sight reveals a discrepancy

3. Observational Errors.—These consist of errors and mistakes in sighting and reading.

(a) *Inaccurate Bisection of Signal*—This may occur through defective vision or carelessness, particularly as regards the elimination of parallax The bisection of a pole should always be made at the point of intersection of the hairs, as the vertical hair may not be truly vertical

(b) *Non-verticality of Signal*—Poles should always be sighted as far down as possible When the foot cannot be seen, there is every likelihood of error being introduced through non-verticality, unless the pole has been tested by plumb line The error is of very common occurrence, and since the error of direction of the line of sight, *viz* $\tan^{-1}\dfrac{\text{error of verticality}}{\text{distance}}$, is inversely proportional to the distance, particular care is necessary with short sights. A deviation of 1 in in the position of the point sighted produces at a distance of 100 ft an error of bearing of nearly 3 min.

Whether the error is caused by non-verticality or by the signal not having been erected at the point intended to be used as an instrument station, its effect may be eliminated by centering the instrument at the forward station with respect to the point to which the foresight was taken.

(c) *Errors of Reading.*—The precautions to be observed in reading verniers and micrometers have been dealt with on pages 40 and 68.

4. Errors due to Natural Causes.—*Wind.*—It is impossible to perform accurate work in a high wind because of the vibration of the instrument If careful centering is required, the plumb bob and line must be sheltered.

High Temperature—On hot sunny days the heating of the ground during the midday hours causes warm air currents to ascend, and the irregular refraction produced gives rise to an apparent trembling of the signal near the ground. The effect is avoided if the line of sight is at all points above the disturbance, which may be taken as appreciable only within a height of 3 ft. from the ground.

The effect of the sun shining on one side of the instrument is to throw it out of exact adjustment, but in ordinary traversing uncompensated errors from this source are negligible

Haze.—A hazy atmosphere increases the probability of inaccurate bisection of the signal, and may necessitate a suspension of the work.

Field Checks in Closed Traverse.—Let AB...KA (Fig 165) be a closed circuit. If there is no intervisibility except between adjacent stations, then, on proceeding from the initial station A and carrying the bearing forward to B, C, etc , all the bearings will have been determined when the observations at K are completed The instrument should, however, be again set up at A, and the bearing AB read from a backsight on K. Any difference between the now observed value of bearing AB and that first determined represents the angular error accumulated in the circuit. More usually the bearings of both AK and AB will be determined at the first occupation of A, and the error is discovered at K by comparison of the value of bearing KA with that previously obtained for AK.

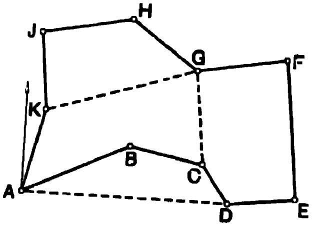


FIG 165.

The magnitude of the angular closing error will reveal whether any gross mistake has occurred If the error is appreciably greater than that to be expected from accidental errors of angle measurement, the observations must be repeated. It is therefore desirable to be able to detect a mistake as soon as committed, or at least to localise it, so that it may be discovered and eliminated without undue additional labour. The following methods of checking are directed to this end.

Double Observations.—The method of obtaining each bearing from observations on both faces of the instrument is valuable because it reduces the effects not only of instrumental error but also of accidental errors of observation. On completing the backsight at each station, the same face is used in sighting the forward station or stations. The telescope is then reversed, the circle is oriented by a backsight on the opposite face, and the foresights are repeated. The method should always be used on an extended traverse. It is not regularly applied in small closed surveys, but the precaution 3 (page 169) should always be adopted.

Cross Bearings.—In place of checking the measurements at each station, possible mistakes may be localised if the survey is divided up by observing cross bearings between such non-adjacent stations as are intervisible. Thus, in Fig 165, the signal at D being visible from A, a measurement of bearing AD is included in the observations at A. On reaching D, after having occupied stations B and C, a sight is taken on A, and, if the bearings DA and AD differ by 180°, the circuit observations up to D are checked. Cross bearings CG and GK similarly serve to verify the angular work from C to G and from G to K respectively.

Check bearings are also useful for the location of a mistake in the linear measurement When the magnitude of the closing error shows that a mistake in chaining has occurred, each of the compartments into which the survey is divided by the cross bearings can be tested for closure either by means of co-ordinates or by plotting It will then be seen which portion of the circuit is affected by the mistake.

Compass Check.—If the theodolite is fitted with a circular compass box, a reading of the compass affords a rough but useful check on each observation If the bearings are being referred to magnetic meridian, the compass readings should be the same as the observed bearings , otherwise the difference should be the constant angle between the magnetic and reference meridians. The compass is insufficient to check the bearings nearer than to about half a degree, but it serves to show up serious mistakes, and the check should not be neglected if opportunities for taking cross bearings are few. The trough form of compass is less direct in this respect, but serves to verify the orientation of the instrument in terms of magnetic meridian, except at those stations where the orientation of the circle is 180° from that at the initial station.

Included Angles.—If measurement is made of the angles between the lines instead of their bearings, the angular closing error is found at the last station occupied by comparing the sum of the interior included angles with $(2n - 4) \times 90°$, where n is the number of stations in the circuit.

Deflection Angles—If deflection angles are measured round a closed figure, the difference between the sum of the right-hand angles and that of the left-hand angles should equal 360°.

Linear Measurement.—A check on the accuracy of the linear work can be secured in the field only by adopting a system of double measurement The two measurements should preferably be of similar precision and in opposite directions. If time does not permit of this, check measurements by tacheometry, or even by pacing, will serve to detect gross mistakes. If no field check is applied, the degree of consistency in the linear work will not be known until the closing error is computed.

Field Checks in Unclosed Traverse.—In the case of unclosed traverse, opportunities may present themselves of observing check bearings as in closed traverse. Thus, in Fig. 166, illustrating part of an enclosed traverse, if E is visible from A, bearing AE may be observed from A, and, on reaching E, the orientation of the instrument and the angular work between A and E will be checked by a backsight on A The forward and back bearings between E and H serve to check the carrying forward of the bearing from E to H. Mistakes in the linear work may also be discovered by means of these check bearings on computing the co-ordinates or on plotting.

An alternative method consists in observing at intervals the bearing to a prominent object to one side of the traverse. Thus, in Fig 166, the bearing of a point P is observed from stations such as A, E, and H The co-ordinates of P are obtainable from the observed parts of figure ABCDEP. A computed value of bearing HP is derived from the co-ordinates of H and P, and agreement between this value and that observed serves to test the traverse from A to H If the co-ordinates are not computed, the check may be applied on the drawing by trying if the rays from A, E, and H intersect in one point

Practically, it will rarely be possible to maintain either of the above systems of checking throughout the complete survey. Double observation of each bearing and distance is therefore specially desirable in unclosed traverse to avoid the carrying forward of a gross mistake.

A thorough check is afforded in the case where the traverse is run between the stations of a refined triangulation Constant errors as well as inconsistencies in the linear measurements then contribute towards the closing error, which may be regarded as a real index of the quality of the work.

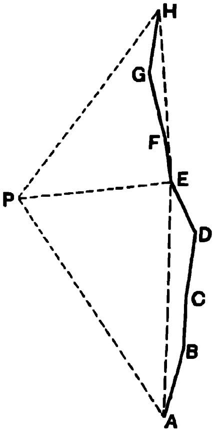


FIG. 166.

In the case of very long unclosed traverses which do not proceed from and terminate on triangulation stations, recourse is had to astronomical checks Errors of bearing are kept within narrow limits by frequent observations for azimuth, but the checking of the linear work involves observations for latitude or, in the case of surveys running nearly east and west, for longitude difference. The precision with which these quantities can be determined by the usual field methods is insufficient to afford a real check on the distance between the places of observation, unless the latter are at sufficiently wide intervals to permit of the propagation of a considerable error in the traverse (Vol. II, Chap. VI).

Allowable Error of Traverse.—It is difficult to estimate the probable errors in position of the stations of a traverse survey, since the linear and angular measurements, particularly the former, are affected by constant as well as accidental errors.

If the constant errors of the angular work of theodolite traverse are treated as negligible, then, since each bearing is derived from the preceding one, the probable error of bearing of any line equals the square root of the sum of the squares of the probable errors of the angles measured in carrying the bearing from the initial station to that line. With a 5-in vernier theodolite, the probable error of each angle may be taken as lying between the limits, 10″ and 40″, its value depending upon the quality of the instrument, refinement of vernier reading, precision of centering, routine of observation, nature of signals, length of sight, state of atmosphere, and the personal factor In reality, the method of observation used in small traverses is insufficient to ensure the elimination of cumulative errors, such as those due to imperfect adjustment of the instrument, and errors of bearing are likely to be greater than those given by the law of probable error In the case of compass traversing, since an error of bearing is not carried forward, the probable error of each bearing should be the same, so long as the conditions for sighting are constant.

If the traverse were affected only by errors of bearing, each station would be displaced relatively to the preceding one in a direction lateral to that of the line joining them and by an amount given by the product of the error of bearing by the length of the line. Errors of linear measurement cause a displacement along the line by an amount proportional to the square root of the distance for accidental errors and proportional to the distance in the case of constant errors. The values of both these errors are difficult of estimation The actual error of closure of a closed traverse is an imperfect index, since it is unaffected by purely constant errors, but only by variation in the cumulative errors and by accidental errors. To enable a satisfactory estimate to be formed of the probable amount of accidental error developed, the linear measurement would have to be performed with a degree of consistency seldom or never attempted in traversing.

In setting limits of allowable error, we must therefore be guided entirely by the results obtained in practice. For a closed traverse executed with a 5-in. vernier theodolite and chain, the ratio of the error of closure (page 184) to the perimeter of the traverse should not exceed 1/2,000 in moderately easy country, but will probably have a greater value if the ground is rough or the lines very short. If the steel band is used, the ratio of error should be only half as great, and may be less than 1/10,000 under favourable conditions, as in city surveying. The actual displacement of any point in an unclosed traverse, or of an intermediate point on a closed circuit, would be expected to have a greater ratio to the length of traverse up to it on account of the influence of constant errors of linear measurement.

COMPASS TRAVERSING

In theodolite surveying, direct reference is made to the meridian at the initial station only, and from the observed bearing of the first line those of the others are obtained mechanically or by calculation by the methods already described In compass surveying, on the other hand, every bearing may be observed directly from magnetic meridian, established at each station by floating the needle Alternatively, those forms of compass which are fitted with a vernier, connected with the line of sight and moving over a fixed graduated circle, can be used in the manner of a theodolite to measure angles and to carry forward the bearing, the needle only being consulted as a check at stations after the first These two methods of compass traversing are distinguished as free needle and fixed needle surveying.

Fixed Needle Surveying.—In deciding upon which of the two systems to adopt, consideration may be given to the following.

Advantages of Fixed Needle Work —(1) The precision attainable, although inferior to that of theodolite surveying, is greater than in free needle working

(2) Readings are not influenced by local attraction.

Disadvantages —(1) Most forms of vernier compass are heavier and less portable than those adapted for free needle observations only

(2) The time taken to observe is greater than in free needle work.

The fixed needle system is used extensively in mine surveying, but the general surveyor has recourse to the compass only in cases where lightness of equipment and speed are of greater account than refinement of observation For this reason, the free needle method is that more generally followed As the methods of observation used in theodolite surveying (other than that involving transiting of the telescope) are equally applicable to fixed needle surveying, the latter need not be further considered

Relative Merits of Theodolite and Free Needle Surveying.—*Advantages of the Compass.*—(1) Some forms, notably the prismatic compass, are very light and portable

(2) Observations are more rapidly completed than with the theodolite.

(3) Each bearing is determined independently of the others, and errors do not accumulate, but tend to compensate.

(4) The bearing of a line can be observed from any point on it.

(5) Intermediate points can be established quickly on a long line of known bearing which cannot be ranged completely from one end

(6) The bearings of all the courses of a traverse can be observed

by occupying every second station only, but this procedure is not recommended because of the possibility of local attraction

Disadvantages —(1) Under the best conditions, readings cannot be relied upon to closer than about 10 min The uncertainty arises both in the estimation of the position of the needle and in the existence of diurnal variation (page 159)

(2) Precautions must be taken against the effects of local attraction

Application of Compass Surveying.—The portability of the compass makes it specially suitable for reconnaissance and exploratory surveys, where rapidity of observation is important Short compass traverses may, however, be introduced with advantage between stations of a theodolite survey for the rapid survey of detail which need not be recorded with a high degree of precision The time saved by using the compass in place of the theodolite is greatest when survey lines have to be short on account of obstructions or irregularities of detail The method is therefore suitable for tracing streams, particularly in wooded gorges, irregular shore lines, clearings in woods, etc

Local Attraction.—The magnetic needle does not record the direction of magnetic meridian when under the influence of attracting bodies in its vicinity Local attraction, as it is termed, may arise from mineral deposits in the ground, particularly magnetic iron ore, but is also caused by the proximity of steel structures, rails, electric cables conveying current, fences, lamp-posts, iron pipes, etc Care should be exercised to see that the chain, clearing axes, and similar articles are at a safe distance from the needle, and that the observer has no sources of attraction, such as keys, knife, etc., about his person. The possibility of local attraction is much greater in cities than in the country, but in no case can compass bearings be regarded as reliable unless the routine of observation is designed to detect its presence and eliminate its effects.

Detection of Local Attraction.—To test for local attraction, it is only necessary to observe the bearing of each line from both its end stations If the forward and back bearings differ by 180°, it may be taken that no local attraction exists at either station, provided the compass is free from instrumental error and that no mistake is made On finding that the back bearing does not agree with the forward bearing within the limits of permissible error of reading, the latter should be checked if it is thought improbable that the disagreement proceeds from local attraction. The discrepancy may have been caused by avoidable attraction from articles on the person, chains, etc. If not, local attraction exists at one or both stations, and the two bearings should be noted for subsequent correction.

Elimination of Effects of Local Attraction.—The deflection of the needle at a station under the influence of local attraction causes the bearings read there to originate from a direction other than magnetic meridian The error is the same for each bearing read at the station, and from the erroneous bearings the true included angles at the affected stations can be deduced in the usual manner. Starting from a bearing unaffected by local attraction, and applying these included angles, the correct bearings are obtained.

A more rapid method of correction is illustrated in the following example

Line	Observed Bearing.	Correction.	Corrected Bearing
AE	319°	−2½°	316½°
AB	72¾°	−2½°	70¼°
BA	252°	−1¼°	250¼°
BC	349°	−1¾°	347¼°
CB	167¼°	0	167¼°
CD	298½°	0	298½°
DC	118½°	0	118½°
DE	229°	0	229°
ED	48°	+1°	49°
EA	135½°	+1°	136½°

The readings have been estimated to the nearest ¼°, and the bearing of each line has been observed twice, the first letter in the designation of the line showing the station of observation On examining the values of the observed bearings, it will be seen that only in the case of the line between C and D are the forward and back bearings consistent It may therefore be taken that stations C and D are *both* free from local attraction Consequently all bearings observed at C and D are correct, and their values are transferred to the third column. Now, since the correct bearing of DE is 229°, that of ED must be 49°, but, as it was read as 48°, station E is influenced by local attraction, and a correction of +1° must be applied to all readings taken at E. The corrected bearing of EA is therefore 136½°, and consequently that of AE must be 316½°, giving a correction for station A of −2½°. On applying this correction, and obtaining the adjusted value 70¼° for bearing AB and 250¼° for BA, the correction at station B is found to be −1¾°, so that bearing BC should be 347¼°, which agrees with the reading CB taken at the unaffected station C.

Notes —(1) If the bearings are expressed quadrantally, care must be exercised to apply the corrections in the proper direction, since the numerical values of bearings increase clockwise in the 1st and 3rd quadrants and counter-clockwise in the 2nd and 4th Positive corrections are applied clockwise, and negative corrections counter-clockwise Expressed in quadrantal reckoning, the above table becomes —

AE	N 41° W	−2½°	N. 43½° W.
AB	N. 72¾° E.	−2½°	N. 70¼° E.
BA	S. 72° W.	−1¾°	S. 70¼° W., etc.

(2) If no two bearings agree within the limits of permissible error, the corrections should be made from the mean value of the bearing of that line in which there is least discrepancy between the foresight and backsight readings.

(3) In exceptional cases a large source of attraction may exercise a constant effect at all points on a line, and permit of agreement between the forward and back bearings. If those bearings are regarded as correct, and are used as a basis for the correction of others, then all stations really free from local attraction will be found to require a correction of constant amount If this occurs, it is better to assume that such constant correction should be zero, otherwise the bearings are corrected to a meridian other than magnetic meridian.

(4) The effects of local attraction can be eliminated only if the bearings of adjacent lines are observed under the same conditions Bearings for correction should therefore be observed at stations, and not at intermediate points on the lines.

Field Work of Compass Surveying.—The chief difference between compass and theodolite field work lies in the method of observing the bearings and the necessity, in the former case, for reading and booking both the forward and back bearing of each line.

The accuracy possible in taking compass bearings does not warrant great refinement in linear measurement Ordinary chaining is sufficiently accurate, while the less exact methods of pacing, etc. (Vol. II, Chap. VI), are sometimes employed The allowable length of offsets may be increased, and in small scale work detail may be located by pacing offsets, by estimating distances, or by intersecting compass bearings from two points on the survey line

Sources of Error.—The effects of local attraction being assumed eliminated, errors may be classed as instrumental and observational. Accurate work is impossible with a sluggish needle, and this defect is most likely to arise from dulling of the pivot, which must therefore be protected from unnecessary wear Even when in adjustment (page 97), it is found that most compasses possess an individual error, generally due to non-coincidence between the magnetic and geometrical axes of the needle, so that different instruments give different values for the bearing of a line If the same compass is used throughout, this defect does not influence the accuracy of a survey, but would affect the result of an observation for the value of the magnetic declination.

Observational errors are those of sighting and reading. The former are not peculiar to compass surveying, and are of less importance than the latter. In estimating the reading, parallax should be avoided by bringing the eye into the vertical plane of the needle. In taking important readings it is well to tap the compass box when the needle has come to rest, to overcome pivot friction. A system of double reading may also be adopted by displacing the needle after the first reading and taking a second. The possibility of deflection of the needle by sources of attraction about the person, or by electrification of the glass cover, should not be overlooked.

Limits of Precision.—If the position of the needle can be estimated to the nearest 10 min , the error of a reading should not exceed 5 min On account of diurnal variation, however, readings can seldom be relied upon to less than 10 min. As this corresponds to a lateral deviation of 1 in 344, free needle surveying is evidently inapplicable to many surveys. The figure also suggests the inconsistency of combining refined taping with compass bearings.

CO-ORDINATES

Latitude and Departure.—The *Latitude* of a survey line is its projected length upon the reference meridian, and the *Departure* of the line is its projected length upon a line at right angles to the meridian These two quantities are therefore rectangular co-ordinates of the survey line and of the end station of the line referred to its origin.

Use of Co-ordinates.—A knowledge of the values of the co-ordinates of a survey is of great value on account of the variety of uses to which it can be put Co-ordinates are employed :

(1) In closed circuits, to determine, before plotting, the actual linear closing error arising from errors of chaining or angular observation, or both

(2) To distribute the effects of this error, if within the permissible error of closure, by applying corrections to the various lines in accordance with the probable manner in which the total error has been propagated

(3) To verify, from the results of angular observations to a side object, the accuracy of the linear and angular work of an unclosed traverse

(4) To plot the stations of closed or unclosed traverses.

(5) To calculate the area within a closed circuit, independently of the plot.

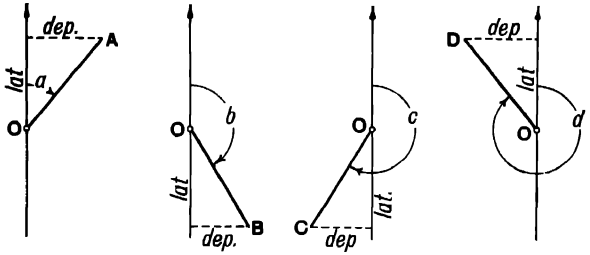
(6) To obtain the length of inaccessible lines, etc.

Calculation of Latitudes and Departures.—Fig. 167 shows different cases, a line being assumed in each quadrant. Evidently for case (*a*), if L = the length of the survey line,

$$\textit{Latitude} = L \times \cos\ (\textit{whole circle bearing}),$$
$$\textit{Departure} = L \times \sin\ (\textit{whole circle bearing}).$$

In the remaining quadrants the whole circle bearings exceed 90°, but the same formulæ apply with the usual trigonometrical convention regarding signs. To prevent confusion in extracting the values of those functions for bearings over 90°, the corresponding angles less than 90° should first be tabulated. An inspection of the figure will show that these angles are the reduced bearings of the lines (page 161). The above formulæ therefore become

$$\textit{Latitude} = L \times \cos\ (\textit{reduced bearing}),$$
$$\textit{Departure} = L \times \sin\ (\textit{reduced bearing}).$$



1st. Quadrant Bearing 0° to 90°
2nd Quadrant 90° to 180°
3rd Quadrant 180° to 270°
4th. Quadrant 270° to 360°

FIG 167.

Due attention must be paid to signs, the trigonometrical convention being adopted as follows .

Quadrant	Latitude	Departure
1	+	+
2	—	+
3	—	—
4	+	—

It is to be noted that the latitude is zero for bearings of 90° (E) and 270° (W), while the departure is zero for bearings of 0° (N) and 180° (S.)

In place of affixing signs, some surveyors designate + latitudes as northings, — latitudes as southings, + departures as eastings, and — departures as westings. Corresponding to the above table, we have on this system .

Quadrant	Latitude	Departure
1	Northing	Easting
2	Southing	Easting
3	Southing	Westing
4	Northing	Westing

If quadrantal bearings have been observed, no reduction of bearings is, of course, required, and the cardinal directions serving to specify a bearing at once suggest the names or signs of the co-ordinates.

The computed values of latitudes and departures are to be set out in a table showing the lengths and bearings of the survey lines Logarithmic calculation is usually preferred to the use of natural trigonometrical functions, and the values of the logarithms should be entered in the table as an aid to the rapid checking of the calculations.

The labour of multiplying is also avoided by the use of traverse tables. These give the latitudes and departures of lines usually for lengths of 1, 2, 3, etc , up to 10 units, the tabulated values being simply the natural cosines and sines of angles from 0° to 90°, multiplied by those lengths. The table used should show the values for every minute of arc expressed to four or five places of decimals, the former being sufficient for all ordinary purposes. To find the latitude and departure of a given line, the tabular values for the given reduced bearing are extracted for each digit of the given length, and, on properly placing the decimal points, the required co-ordinates are given by summing the parts

Example —The following figures are extracted from a traverse table, for bearing 36° 41′.

1		2		3		4		5	
Lat	Dep	Lat	Dep	Lat	Dep.	Lat.	Dep	Lat	Dep
0 8019	0 5974	1 6039	1 1948	2 4058	1 7922	3 2078	2 3896	4 0097	2 9870

If it is required to determine to the first place of decimals the co-ordinates of a line having this reduced bearing and of length 534·1 ft , the additions are .

	Lat	Dep
500	400 97	298 70
30	24 06	17 92
4	3·21	2 39
1	08	06
534 1	428 3	319 1

Tabulation of Co-ordinates.—The table may be arranged as follows :

Line	Length	Bearing	Reduced Bearing	Log {Cos / Length	Log {Sine / Length	Latitude	Departure
	Ft						
AB	412	132° 17′	47° 43′	9 8279 2 6149 2 4428	9 8691 2 6149 2 4840	−277 2	+304 8
BC	275	64° 28′	64° 28′	9 6345 2 4393 2 0738	9 9554 2 4393 2 3947	+118 5	+248·1
Etc							

Alternatively, the co-ordinates are entered without signs thus :

Latitude		Departure.	
N	S.	E	W.
—	277 2	304 8	—
118 5	—	248·1	—

The columns in which the logarithmic calculations are shown have no equivalent when traverse tables or natural functions are used When bearings are deduced from included or deflection angles, the observed angles are entered in a column between the length and bearing columns above Additional columns are to be reserved for quantities required in plotting the survey lines (page 192) and, if necessary, for area calculations (pages 287 and 289).

SURVEY ADJUSTMENT

Closing Error.—The algebraic sum of the latitudes and that of the departures of all the lines forming a closed circuit should each be zero, since a return to the starting-point in the field produces equality between N and S latitudes and between E. and W. departures If the tabulated co-ordinates do not add up to zero, the respective algebraic sums represent the latitude and departure of the line required to close the polygon, or

$$\text{Closing error} = \sqrt{(\text{Latitude error})^2 + (\text{Departure error})^2},$$

and the relative error of closure is this quantity divided by the perimeter of the circuit

The closing error arises from errors both of chaining and angular measurement It should always be determined in order to detect gross mistakes in the field work and as a check on the calculation of co-ordinates, but the inability of the closing error to reveal constant errors of chaining must be kept in view Permissible values of errors of closure are given on page 176

When the magnitude of the closing error indicates that a mistake in chaining has taken place, it is frequently possible to ascertain in which line it has occurred by comparing the closing errors in latitude and departure with the co-ordinates of the several lines The ratio between the closing errors of the co-ordinates will be the same as that between the co-ordinates of the affected line. If the mistake cannot be located and its occurrence explained, reference to the cross bearings will enable the surveyor to ascertain the compartment of the survey which requires to be rechained.

Balancing.—By balancing or adjusting a survey is meant the elimination of the closing error by distributing corrections to the various lines in accordance with the probable manner in which the total error has been propagated. In many cases balancing is unnecessary, but it is required (*a*) when the scale of plotting permits the closing error being shown, (*b*) when the co-ordinates are to be used for purposes other than plotting In formulating rules for the distribution of the error, account must be taken of the relative contributions to the closing error made by the angular and linear measurements. As these are unknown, a reasonable assumption must be made which will lead to an easily applied method of correction.

Methods.—The oldest and best known rule is that of Bowditch, which states :

As the sum of all the distances is to each particular distance, so is the whole error of latitude (or departure) to the required correction in latitude (or departure). Or,

Correction in latitude (or departure) of a line

$$= \frac{\text{Length of that line}}{\text{Perimeter}} \times \text{Total error in latitude (or departure).}$$

This rule is based on the assumption that the error in the position of the stations is due equally to chaining and angular measurement. The validity of this assumption depends upon the field methods of linear and angular measurement and the instruments used It is quite justifiable in compass surveys, particularly with free needle, but for theodolite surveys it overestimates the probability of angular error Bowditch's rule should therefore be used exclusively for the adjustment of compass traverses, and on this account it is sometimes called the compass rule

In theodolite surveys, the angular closing error is known, and it should first be distributed before computing the co-ordinates. Assuming that the probability of error at one instrument station is the same as that at another, equal corrections should be applied to the observed values of included or deflection angles If, however, bearings have been carried forward in the field, since the first bearing involves the measurement of one angle, the second that of two angles, and so on, the corrections should increase in arithmetical progression

Having computed the co-ordinates and found the errors of latitude and departure, the adjustment is commonly made according to the following rule ·

As the arithmetical sum of all the latitudes (or departures) is to each particular latitude (or departure), so is the whole error of latitude (or departure) to the required correction in latitude (or departure). Or,

Correction in latitude (or departure) of a line

$$= \frac{\text{Lat. (or dep) of that line}}{\text{Arithmetical sum of all lats. (or deps.)}} \times \text{Total error in lat. (or dep.).}$$

Weighting.—Since it is unlikely that all the measurements have been performed under the same conditions, allowance may be made for the relative difficulties by distributing the corrections in accordance with the estimated probability of error in the various lines. This operation is termed weighting, and consists in assigning to each line a factor, or weight, representing its relative liability to error as judged from the conditions under which the length and bearing were measured In estimating the relative difficulty of the linear measurements, consideration is given to roughness of ground and the existence of obstacles, while, as regards the angular work, difficulty of sighting and difference of level between the points observed are kept in view. It is usual to assign unit weight to the line considered least subject to error, the weights of the others being estimated by comparison.

To use Bowditch's rule with weighted lines, each distance is multiplied by its weight, giving the weighted length of the line, then Correction in latitude (or departure) of any line

$$= \frac{\text{Weighted length of that line}}{\text{Sum of weighted lengths of all lines}} \times \text{Total error in lat. (or dep.)}.$$

In theodolite surveys, each latitude and departure is multiplied by the weight of the corresponding line, and

Correction in latitude (or departure) of any line

$$= \frac{\text{Weighted lat. (or dep.) of that line}}{\text{Arithmetical sum of weighted lats. (or deps.)}} \times \text{Total error in lat. (or dep.)}.$$

Examples.—The following cases will serve to illustrate the application of the rules.

(1) *Adjustment of Prismatic Compass Traverse by Bowditch's Rule.*

Line	Length in Ft	Bearing	Reduced Bearing	Latitude	Departure	Corrected Latitude	Corrected Departure
AB	345	173°	7°	−342·4	+ 42·0	−341·7	+ 42·4
BC	122	110°	70°	− 41·7	+114·6	− 41·5	+114·7
CD	412	125°	55°	−236·3	+337·5	−235·5	+338·0
DE	286	200°	20°	−268·8	− 97·8	−268·3	− 97·5
EF	380	310°	50°	+244·3	−291·1	+245·0	−290·6
FG	387	209°	29°	−338·5	−187·6	−337·8	−187·1
GH	370	282°	78°	+ 76·9	−361·9	+ 77·6	−361·4
HJ	440	30°	30°	+381·0	+220·0	+381·9	+220·5
JA	564	23°	23°	+519·2	+220·3	+520·3	+221·0
	3,306			− 6·3	− 4·0	0	0

The actual closing error $= \sqrt{6{\cdot}3^2 + 4{\cdot}0^2} = 7{\cdot}5$ ft., representing a ratio of error of $\frac{7{\cdot}5}{3{,}306}$, or about $\frac{1}{440}$.

Applying the rule,

$$\text{Latitude correction for AB} = \frac{345}{3{,}306} \times 6{\cdot}3 = \text{say, } 0{\cdot}7,$$

$$\text{Departure correction for AB} = \frac{345}{3{,}306} \times 4{\cdot}0 = \text{say, } 0{\cdot}4,$$

and so on for the other lines. The corrections must be applied to the co-ordinates in such a manner that the corrected values will balance. In this case both errors are negative, due to a deficiency in the positive co-ordinates and an excess in the negative co-ordinates. The former therefore fall to be increased, and the latter diminished, by the corrections. The adjusted co-ordinates are either tabulated as shown, or the corrections may be made in red ink in the latitude and departure columns.

(2) *Adjustment of Theodolite Traverse.*

Line.	Length in Feet	Bearing	Correction.	Corrected Bearing	Reduced Bearing	Latitude	Departure	Corrected Latitude.	Corrected Departure
AF		149° 0′ 30″							
AB	580	62° 34′	0	62° 34′	62° 34′	+267·2	+514·8	+267·1	+514·4
BC	672	101° 21′	0	101° 21′	78° 39′	−132·3	+658·8	−132·3	+658·3
CD	431	143° 8′ 30″	30″	143° 9′	36° 51′	−344·9	+258·5	−345·0	+258·3
DE	724	206° 53′	30″	206° 53′ 30″	26° 53′ 30″	−645·7	−327·5	−645·9	−327·7
EF	516	251° 18′	1′ 0″	251° 19′	71° 19′	−165·3	−488·8	−165·4	−489·1
FA	1,192	328° 59′ 30″	1′ 0″	329° 0′ 30″	30° 59′ 30″	+1,021·8	−613·8	+1,021·5	−614·2
						+0·8	+2·0	0	0

The bearing AF, taken at starting, does not agree with that of FA, which contains the angular error propagated throughout the traverse, and the error of 1′ is distributed in arithmetical progression in 30″ stages. The arithmetical sums of the latitudes and departures obtained from the new bearings are respectively 2,577·2 and 2,862·2, so that

$$\text{Latitude correction for AB} = \frac{267{\cdot}2}{2{,}577{\cdot}2} \times 0{\cdot}8 = \text{say, } 0{\cdot}1,$$

$$\text{Departure correction for AB} = \frac{514{\cdot}8}{2{,}862{\cdot}2} \times 2{\cdot}0 = \text{say, } 0{\cdot}4,$$

and so on.

(3) *Adjustment of Example* (2) *by Weighting.*

If weights, as shown, are assigned in accordance with the relative difficulties in the linear measurement of the several lines, the adjustment is performed as follows :

Line	Weight	Latitude	Departure	Weighted Latitude	Weighted Departure	Corrected Latitude	Corrected Departure.
AB	3	+267·2	+514·8	801·6	1,544·4	+267·1	+514·2
BC	1	−132·3	+658·8	132·3	658·8	−132·3	+658·5
CD	1	−344·9	+258·5	344·9	258·5	−345·0	+258·4
DE	2	−645·7	−327·5	1,291·4	655·0	−645·9	−327·8
EF	1	−165·3	−488·8	165·3	488·8	−165·3	−489·0
FA	2	+1,021·8	−613·8	2,043·6	1,227·6	+1,021·4	−614·3
		+0·8	+2·0	4,779·1	4,833·1	0	0

Applying the rule, we have

$$\text{Latitude correction for AB} = \frac{801{\cdot}6}{4{,}779{\cdot}1} \times 0{\cdot}8 = \text{say, } 0{\cdot}1,$$

$$\text{Departure correction for AB} = \frac{1{,}544{\cdot}4}{4{,}833{\cdot}1} \times 2{\cdot}0 = \text{say, } 0{\cdot}6,$$

and so on,

Remarks on Balancing.—Since the relative precision of the angular and linear measurements of ordinary traversing is unknown, the reliability of the above methods cannot be assessed. Bowditch's rule is generally accepted as the best for compass work, and is sometimes used for theodolite surveys Its application necessarily alters the directions of all the lines, and the changes produced may be unwarrantably great for theodolite bearings The method given for balancing a theodolite traverse is better, but is theoretically unsound because the adjustment of the latitudes and departures alters the corrected bearings by different amounts, bearings of 0°, 90°, 180°, and 270° remaining unchanged. As a consequence, the amounts of the alterations in bearing depend upon the choice of meridian Thus, if a traverse forms a rectangle, adjustment does not change the angles if the meridian is parallel to two of the lines, but does so if any other meridian is used. If, in the latter case, it is not considered justifiable to alter the bearings, they can be referred for purposes of adjustment to a meridian parallel to two of the sides The general case of balancing any survey without interfering with the observed bearings does not admit of a simple solution

In balancing ordinary small surveys in practice, little consideration need be given to the relative merits of different rules It is usually sufficient to distribute the error by rough mental proportion In adjusting theodolite surveys, it is well to notice where corrections may be introduced with most effect towards the balance of co-ordinates, since corrections so placed have least influence on the bearings.

Balancing of Interlocking Nets.—In the case of a number of traverses connected to each other at several points, any attempt at rigorous adjustment would be unwarrantably troublesome and unnecessary for ordinary purposes It is sufficient that the more important circuits should be adjusted before those in which errors would have less serious consequences. If the traverses cannot be so differentiated, the larger should be balanced before the smaller. Lines common to two or more traverses, when once corrected, must not be subjected to further adjustment.

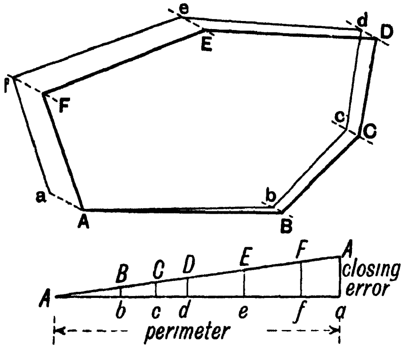


FIG. 168.

Graphical Adjustment.—The adjustment of surveys plotted by angle and distance methods is usually performed graphically, since co-ordinates are not available for correction In Fig 168, the polygon Abcdefa as plotted is found to have a closing error aA, which represents the combined error of field work and plotting. This error is distributed by shifting each plotted station in a direction parallel to that of aA and by an amount proportional to its distance along the traverse from the initial station A. ABCDEFA is the adjusted traverse. In place of calculating the corrections, they may be obtained by graphical proportion as shown.

PLOTTING

Methods of Plotting Survey Lines.—The origin of a survey line being located on the paper, the line can be plotted either by setting off its direction and length, or by fixing the position of its terminal point by co-ordinates. The first system, termed the angle and distance method, involves the laying down of an angle which, according as a reference meridian has or has not been adopted for the field observations, will be either (*a*) the bearing of the line, (*b*) the angle, included or deflection, from the preceding survey line In the co-ordinate system, the direction and length of the line are used in the calculation of co-ordinates, and are not directly plotted

Angle and Distance Methods.—The more commonly used angle and distance methods of plotting an angle or bearing are

(*a*) By protractor
(*b*) By the tangent of the angle.
(*c*) By the chord of the angle

By Protractor.—The protractor furnishes the most rapid method of laying down angles, but, while the degree of accuracy afforded is sufficient for many purposes, the best results cannot be expected, particularly in plotting lines much longer than the radius of the protractor With the ordinary simple pattern, of about 6 in diameter, angles cannot be protracted nearer than to about 10 or 15 min , which accords with the accuracy of free needle readings, but not of theodolite work Protractors of 9 to 12 in diameter with vernier reading to 1 min are more suitable for plotting theodolite surveys, but in using such protractors it is doubtful if full advantage can be taken of the vernier owing to the probability of error in centering A very useful and inexpensive form of protractor has the graduations printed on cardboard, and may be obtained in diameters of from 12 to 18 in.

By Tangents—In this method, the angle θ to be plotted is set out by geometrical construction with the aid of a table of natural tangents. From the station point a base length, scaling a round number of units, is set off along the given side of the angle, and from the end of the base a perpendicular is erected of length $=$ base $\times \tan\theta$. The line joining the station to the point thus obtained includes θ with the given side. The length of base should be such that the hypoteneuse obtained exceeds the length of the survey line to be plotted along it. If the angle exceeds one or more right angles, these are laid off by set-square, and the method is applied to the remainder. In constructing angles of nearly 90° it is better to plot the complement of the angle.

With care, the tangent method is capable of higher accuracy than direct protracting.

By Chords —In this geometrical method, an arc of any convenient radius r, preferably 10 or 100 units, is first swept out with the station point as centre A second arc, centered at the point in which the first cuts the given side, is then described with radius equal to the chord subtended by the angle to be plotted The line joining the station to the intersection of the arcs defines with the given side the required angle The chord length $= 2r \times \sin\frac{\theta}{2}$. Various mathematical tables give the lengths of chords corresponding to unit radius If the angle exceeds a right angle, the construction should be applied only for the part less than 90°, as thereafter the intersections become unsatisfactory The accuracy attainable is not materially different from that of the tangent method

Plotting Angles and Bearings.—The manner of applying the above methods to the plotting of a survey line from the preceding one will be evident Results of angular measurements, noted in tabular form, should always be accompanied by a sketch, to which the draughtsman must make constant reference to avoid setting off angles in the wrong direction

In plotting bearings, two methods may be distinguished

(1) A meridian is drawn in a convenient central position on the sheet, and at an assumed point on it the bearings of all the lines to be plotted are set out from it These directions are then transferred to their proper positions, as required (Fig 169).

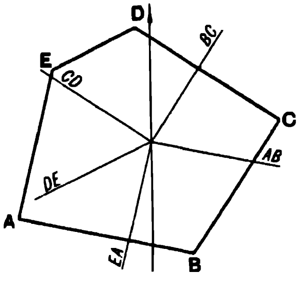


FIG. 169.

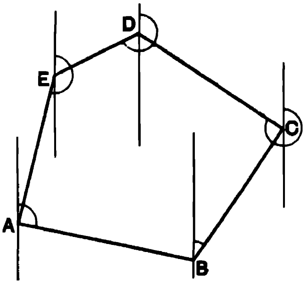


FIG 170.

(2) Each bearing is plotted from a meridian through the origin of the line and parallel to the reference meridian (Fig 170).

The first method is the more rapid. By using a circular protractor, all the bearings can be plotted from one setting of the protractor. In the tangent method the same bases can be used for various bearings, while in applying the chord method the several chord lengths can be marked off on one circle. Great care is necessary in transferring the directions · a heavy parallel ruler gives the best results In the second method there is the same need for accuracy to secure parallelism between the successive meridians.

Plotting by Co-ordinates.—The co-ordinate method is recognised as the most accurate and useful method of plotting traverse lines from bearings. Two systems are used

(1) Each station is plotted from the preceding one by setting out the latitude and departure of the line between them in directions respectively parallel and perpendicular to the reference meridian.

(2) Each station is plotted independently by ascertaining its position relatively to two assumed co-ordinate axes respectively parallel and perpendicular to the reference meridian.

In plotting closed surveys by either method, the closing error, if appreciable, should first be eliminated by balancing, and the adjusted values of the latitudes and departures used

First Method.—In this method, direct use is made of the latitudes and departures as shown in Fig 171. Having assumed a point to represent the first station A and a line *ae* showing the direction of the reference meridian, B is plotted by setting off in the proper directions the latitude A*a* and departure *a*B of line AB. C is similarly located from B, and so on

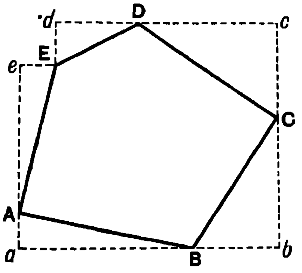


Fig. 171

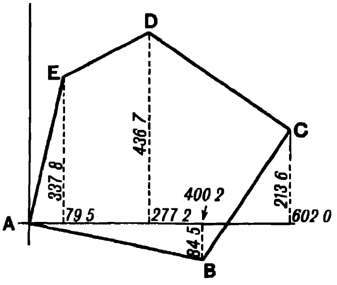


Fig. 172

Second Method.—The co-ordinate axes, to which the positions of all stations are to be referred, are taken through one of the stations, called the origin, which is usually either the initial or the most westerly station. The station co-ordinates are termed total latitudes and total departures, and these must be computed and entered in tabular form before plotting.

Total Latitudes and Departures.—The total latitude (or departure) of any station is the algebraic sum of the latitudes (or departures) of all the lines from the origin to that station.

These are included in the survey table as shown in the following example, in which the latitudes and departures are already balanced.

Line	Length	Bearing	Reduced Bearing	Latitude	Departure	Stn	Total Latitude	Total Departure
	Ft.							
AB	409	101° 55′	78° 5′	− 84.5	+400.2	A	0	0
BC	360	34° 6′	34° 6′	+298.1	+201.8	B	− 84.5	+400.2
CD	394	304° 29′	55° 31′	+223.1	−324.8	C	+213.6	+602.0
DE	221	243° 26′	63° 26′	− 98.9	−197.7	D	+436.7	+277.2
EA	347	193° 14′	13° 14′	−337.8	− 79.5	E	+337.8	+ 79.5

Each total co-ordinate being obtained from that preceding by application of a latitude or departure, the arithmetical work is completely checked if the total co-ordinates of the last station are respectively equal in magnitude, but opposite in sign, to the latitude and departure of EA. When the co-ordinates originate at the most westerly station, as in the example, the total departures are all positive.

The plotting now consists in drawing the co-ordinate axes accurately at right angles and setting off along one of them the total co-ordinates in that direction, as in Fig. 172. From these points, lines parallel to the other axis are erected, and the second co-ordinates are scaled off, the points so obtained representing the various stations.

Precautions in Co-ordinate Plotting.—To obtain good results, pencil lines should be fine, and co-ordinate lengths should be very carefully scaled. Inaccuracy in the directions of co-ordinate lines is the most likely source of error, particularly in the second method, which requires the plotting of longer lines than the first. Accuracy depends largely upon the care taken to secure perpendicularity between the reference axes. Long perpendiculars should be set out by the use of beam compasses rather than by set-square. When a survey covers a large area of paper, so that the station co-ordinates are long lines, it is better to construct a rectangle to enclose the survey. Before using this rectangle, its accuracy should be verified by checking the equality of its diagonals. Stations are then plotted from the nearest sides of the rectangle by subtracting, if necessary, the tabular co-ordinates from the lengths of the corresponding sides of the rectangle. Distances to be scaled can be still further reduced by subdividing the bounding rectangle into smaller rectangles, from the sides of which the plotting can be performed.

Relative Merits of the Two Co-ordinate Systems.—The second method is much more commonly used than the first, but each has advantages. Errors of scaling are revealed in either system by measuring the length of each course as soon as plotted, but this test does not afford an absolute check on the bearings. In the first method, small errors of plotting are carried forward, and in a closed survey the closing error due to inexact plotting is discovered, since the co-ordinates used have been balanced. In the most refined plotting this is an advantage, the cause of the discrepancy being searched for and eliminated Since small errors are not accumulated in the second method, it is to be preferred for plotting unclosed traverse

Advantages of Co-ordinate Plotting.—(1) The table of total latitudes and departures exhibits the extent of the survey, and the position of the point representing the origin of co-ordinates may be selected so that the survey will fall centrally on the sheet.

(2) By calculation of the co-ordinates the closing error can be ascertained, and adjusted if necessary, before plotting commences. In angle and distance methods, plotting must be completed before the closing error is discovered, and then it is not known to what extent the error is due to inaccurate field work or plotting

(3) The plotting of the stations of a large survey can proceed simultaneously on different sheets or on different parts of the same sheet

OMITTED MEASUREMENTS

If, for any reason, it is impossible to measure all the lengths and bearings of a closed traverse, the values of the missing quantities (lengths or bearings) can be calculated, provided they do not exceed two in number Since the observed and omitted quantities are parts of a closed polygon, the algebraic sum of all the latitudes and that of the departures are each zero We have therefore the two independent equations .

$$L_1 \cos \theta_1 + L_2 \cos \theta_2 + \text{etc} = 0,$$
$$L_1 \sin \theta_1 + L_2 \sin \theta_2 + \text{etc.} = 0,$$

where L_1, L_2, etc., are the lengths, and θ_1, θ_2, etc., the bearings of the lines, and from these equations two unknowns can be obtained

If only one measurement is missing, a check on the accuracy of the traverse is still available, but in the case of two omissions all errors propagated throughout the survey are thrown into the calculated values of the omitted data, and no check, other than by repeating the field work, is possible. The omission of measurements merely to save field work is not therefore to be regarded as generally allowable

The most important application of the method occurs when a traverse is run between two points on either side of an obstacle for the special purpose of determining the length or direction of the straight line joining them. A similar problem arises in computing check measurements. Six cases may occur, according as the quantities required are

(1) Length of one line.
(2) Bearing of one line
(3) Length and bearing of one line
(4) Lengths of two lines
(5) Bearings of two lines
(6) Length of one line and bearing of another

Calculation of Unknowns.—To find the missing quantities, the latitudes and departures of the lines completely measured are first calculated These will not both sum up algebraically to zero, since, owing to the omissions, the polygon is not closed. The latitude and departure required to be added to these sums to make both zero represent the co-ordinates of the gap in the polygon

Case (1) The latitude, l, and departure, d, of the omitted length are equal in magnitude and opposite in sign to the algebraic sums of the co-ordinates of the measured lines, and

$$\text{Omitted length} = \sqrt{l^2+d^2}.$$

If the field measurements have been made without error, the calculated value of the omitted length should be such that multiplying it by the cosine and sine of its bearing will give l and d respectively A check on the field work is therefore provided On including the calculated line in the table, and computing its co-ordinates from its observed bearing, the survey may be balanced in the usual manner, but such adjustment is unsatisfactory, since the calculated length necessarily contains the errors made in the other lines.

Case (2) The bearing of a line whose co-ordinates are known can be obtained from the relation,

$$\text{Tan reduced bearing} = \frac{\text{departure}}{\text{latitude}}$$

The signs of the given latitude and departure show in which quadrant the bearing lies, so that the whole circle bearing is at once deduced

In this case, therefore, $\tan \theta = \frac{d}{l}$, and a check on the field work is afforded by comparing the measured length with $\sqrt{l^2+d^2}$.

Case (3) The length and bearing of the line completely omitted are obtained as in cases (1) and (2), and as shown in the example on page 196.

Case (4) If L_1 and L_2 are the unknown lengths, and θ_1 and θ_2 their bearings, then

$$L_1 \cos \theta_1 + L_2 \cos \theta_2 = l,$$
$$L_1 \sin \theta_1 + L_2 \sin \theta_2 = d,$$

l and d being, as before, equal in magnitude, but opposite in sign, to the respective algebraic sums of the known latitudes and departures. L_1 and L_2 are obtained by solving the simultaneous equations. The solution is independent of whether the unknown sides of the polygon adjoin each other or not. The two equations reduce to one when the unknown lengths are parallel, and this case is indeterminate, except in the unlikely event of the area of the polygon being known, the unknown lengths not being adjacent

Case (5) The same simultaneous equations can be solved for θ_1 and θ_2, provided their quadrants are known

Case (6) On solving the equations, two values for the unknown bearing and length are obtained If either is known approximately, it will, in general, be seen which of the solutions is that wanted.

Alternative Method for Cases 4, 5, and 6.—The following solution of the problem is sometimes preferable In Fig. 173, the incompletely measured lines GH and HA adjoin each other From the co-ordinates of the survey lines from A round to G, the length and bearing of the line GA are calculated by the method of case (3). The triangle GHA can then be solved for the two unknowns. In case (4) the internal angles of the triangle must be deduced from the known bearings , in case (5) all the sides are known ; while in case (6) two sides and the angle opposite one of them are known. If the bearing of GH and the length of HA are measured, the two solutions are shown by the two possible positions, H and H′, of the unfixed station

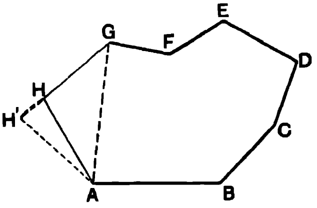


Fig 173

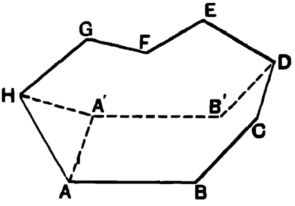


Fig 174.

When the two incompletely measured sides do not adjoin each other, as CD and HA in Fig 174, they may be brought into the same triangle by imagining the intervening lines shifted parallel to themselves in a direction parallel to one of the unknowns. In the example illustrated, AB and BC are moved to A′B′ and B′D, so that AA′ is equal and parallel to CD. On joining HA′, the triangle HA′A is formed, the sides HA and AA′ of which are the two lines affected by the omissions HA′ is completely known since it is the closing side of a traverse A′B′DEFGH, of which full particulars are available. The triangle can therefore be solved for the missing data as before It will be observed that no triangle would be available if CD and HA were parallel.

Example.—The application of the most important practical case is illustrated in the following example

The length and bearing of a line AE cannot be measured directly, and the traverse noted in the first three columns of the following table is run from A to E Calculate the required quantities

Line	Length in feet	Bearing	Reduced Bearing	Latitude	Departure
AB	172	14° 31′	14° 31′	+166.5	+ 43.1
BC	236	319° 42′	40° 18′	+180.0	−152.6
CD	474	347° 15′	12° 45′	+462.3	−104.6
DE	150	5° 16′	5° 16′	+149.4	+ 13.8
				+958.2	−200.3

Having computed and tabulated the co-ordinates, their algebraic sums are obtained, then

Closing length EA $= \sqrt{(958.2)^2 + (200.3)^2} = 978.9$ ft,

and tan reduced bearing of EA $= \dfrac{200.3}{958.2} = .2090 = \tan 11^\circ 48'$

To close the polygon, a negative latitude and positive departure are required, so that the closing line EA lies in the second quadrant

Whole circle bearing of EA $= 180^\circ - 11^\circ 48' = 168^\circ 12'$,

but that of AE is required, and this has the value,

$$168^\circ 12' + 180^\circ = 348^\circ 12'$$

Note—The above method of obtaining the required bearing is adopted to emphasise the fact that the formulæ for cases (2) and (3) give the *forward* bearing EA required to close the polygon The co-ordinates for the back bearing AE are the algebraic sums of the tabulated co-ordinates, without change of sign

MISCELLANEOUS PROBLEMS IN THEODOLITE SURVEYING

By means of the theodolite, obstacles to ranging or chaining may be more rapidly and accurately surmounted than by the use of chain surveying equipment only. The examples given below also include some problems frequently encountered in the course of setting out works While all possible cases cannot be included, the solutions given will suggest the geometrical methods to be applied in particular circumstances

Obstacles.—*Obstacles which Obstruct Ranging*—(*a*) When an intermediate point can be selected from which the ends are visible, the line may first be ranged by eye (page 147) On setting the theodolite in this line, it may be adjusted to the exact line by trial and error, until, on sighting one end station and transiting, the line of sight is found to cut the other station.

(*b*) From A (Fig 175) project a random line AC′B′ by the theodolite in the estimated direction towards B, leaving a point C′

visible from A. Measure AB′ and the offset B′B. Compute $a = \tan^{-1}\frac{BB'}{AB'}$, and, with the theodolite at A, set off a from AC′, and establish points on AB.

(*c*) Select a point C (Fig. 176) from which A and B are visible. Measure AC, CB, and c. Solve for a and b, and set off one or both of these angles at A and B respectively.

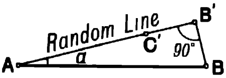


FIG. 175

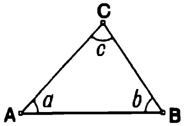


FIG. 176

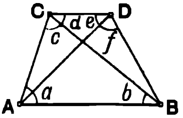


FIG. 177

(*d*) If AB is very long, select two intervisible points C and D (Fig. 177) from both of which A and B are visible. Measure angles c, d, e, and f. Solve triangle ACD for AC and AD, taking CD as unity, and from triangle BCD obtain BC and BD. Now solve triangle ACB for a and b, and check by solving triangle ADB. By orienting on C, set off a and b from A and B respectively, and establish points on AB. This method will not be very satisfactory if CD is short, but is useful for clearing AB.

(*e*) If intervisible points C and D cannot be obtained, run a traverse from A terminating on B, and compute the bearing of AB. Set off this bearing from A.

Obstacles which Obstruct Chaining.—(*a*) The methods of page 148, involving the use of right angles, are available, the theodolite being employed to erect perpendiculars.

(*b*) When both points, A and B (Fig. 178), are accessible, set out an equilateral triangle by laying off a and b each $= 60°$, obtaining the intersection C. Measure AC, and check by BC. Alternatively, set out any triangle ABC, and measure a sufficient number of the parts to enable the triangle to be solved for AB.

(*c*) When B is inaccessible, set out AC perpendicular to AB (Fig. 179). Measure AC and c, and solve for AB. Alternatively,

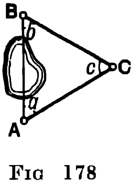


FIG. 178

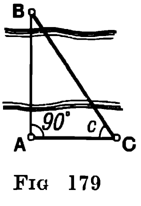


FIG. 179

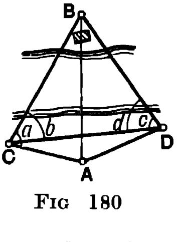


FIG. 180

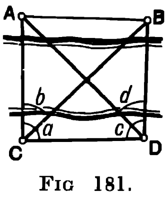


FIG. 181.

measure out AC in any convenient direction, observe the angles at A and C, and solve for AB.

(*d*) When B is inaccessible and invisible from A, select two intervisible points C and D (Fig. 180) from which A and B are visible. Measure CD, AC, and AD and angles *a*, *b*, *c*, and *d*. Solve triangle BCD for BC and BD, then solve triangle ABC for AB, and check from triangle ABD Time is saved in the measurements if C, A, and D are collinear

(*e*) When A and B are both inaccessible, select two points, C and D (Fig 181) from which both A and B are visible Measure CD and angles *a*, *b*, *c*, and *d*. Solve triangle ACD for AC and AD, and BCD for BC and BD Obtain AB by solving triangle ABC, and check from triangle ABD.

Perpendiculars and Parallels.– *To Set Out a Perpendicular to a Given Line from a Given Point* —(*a*) When both the point P and the line AB are accessible, select a point A on AB, and measure *a* (Fig 182) At P sight A, and set off $b=(90^\circ-a)$, then PC is the perpendicular Alternatively. measure *a* and AP, and compute AC

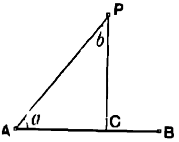


FIG 182

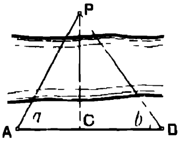


FIG 183

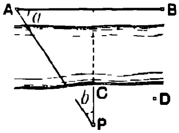


FIG 184

(*b*) When P is inaccessible, select two points A and B on AB (Fig. 183) Measure AB, *a*, and *b* Then $AC=\dfrac{AB\tan b}{\tan a+\tan b}$

(*c*) When P is accessible, but AB is not, deduce *a* (Fig 184) by observations from P and a point D, as in Fig 181 From P sight A, and set off $b=(90^\circ-a)$, then PC is the direction of the perpendicular

(*d*) When both P and AB are inaccessible, set out any accessible line parallel to AB (see below), and proceed as in (*b*)

To Set Out a Parallel to a Given Line from a Given Point —(*a*) When both the point P and the line AB are accessible, select a point A on AB, and measure *a* (Fig. 185). From P sight A, and set off $b=(180^\circ-a)$ PC is the required parallel

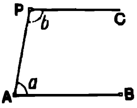


FIG. 185.

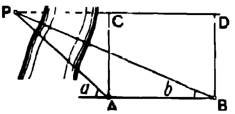


FIG 186

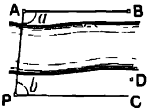


FIG 187

(*b*) When P is inaccessible, select points A and B on AB (Fig 186), and measure AB, *a*, and *b*. The perpendiculars AC and BD to the required parallel $=\dfrac{AB}{\cot b-\cot a}$.

(*c*) When P is accessible, but AB is not (Fig 187), deduce *a* by observations from P and a point D, as in Fig 181 From P sight A, and set off $b = (180° - a)$; then PC is the required parallel.

EXAMPLES

1 A, B, C, D are four survey stations At station B the angle ABC is measured clockwise, and found to be 196° 53′ On setting up at C, the angle BCD, also measured clockwise, is observed to be 142° 17′ If AB is adopted as the meridian, what are the bearings of BC and CD ?

2 The total latitudes of two stations on a survey are 8,257 and 7,542 ft , and their total departures are 1,321 and −146 ft respectively Calculate the distance between them and the bearing from the first to the second.

3 Calculate the closing error of the following traverse

Line.	Length in feet.	Bearing
AB	520	92°
BC	634	174°
CD	580	220°
DE	1,232	279°
EA	1,348	48°

4 The following bearings and lengths represent the observed values for the first three lines of a traverse survey The bearings were referred to magnetic meridian, which was 16° 35′ west of true north at the place Convert the observed bearings to true bearings Tabulate the co-ordinates of the lines derived from the true bearings and the total co-ordinates of the stations referred to *A* as origin

Line	Length in feet	Magnetic Bearing
AB	738	33° 21′
BC	560	349° 46′
CD	893	15° 10′

5 The bearings of a theodolite traverse have been referred to magnetic meridan at the initial station, and the co-ordinates of a certain line are found to be 796 ft S and 368 ft. W Calculate the co-ordinates of this line with reference to true meridian at the initial station if the direction of magnetic north there is 13° 10′ west of true north.

6 The following are the particulars of a small traverse survey in which the bearings have been referred to magnetic meridian, the value of the magnetic declination being 15° 30′ W Convert the observed bearings to true bearings, calculate the co-ordinates of the lines, and find the error of closure.

Line.	Length in feet	Magnetic Bearing.
AB	470	N 6° 2′ W.
BC	637	S 82° 10′ E.
CD	432	S 2° 38′ W.
DA	565	N. 84° 40′ W.

7. The following traverse is carried round an obstruction in a line EF .

Line	Length in feet	Bearing
EG	200	S. 38° 21′ E
GH	580	N 75° 46′ E.
HF	82	N. 11° 8′ W.

Compute the angles GEF and HFE and the distance EF,

8. A determination of the distance between two mutually visible points *A* and *E* is required, but cannot be made with sufficient accuracy by direct measurement. Obtain the distance from the following notes of a traverse run from *A* to *E*:—

Line	Bearing	Distance in feet
AE	360° 0′	
AB	346° 18′	2,386.4
BC	73° 57′	583.2
CD	296° 33′	401.8
DE	18° 21′	1,156.4
EA	180° 0′	

9. A "measured mile" of 6,080 ft. in the direction of magnetic N. and S. is to be marked off on shore by erecting guide poles at either extremity to form parallel lines. From a point A on the line of the first two signals a traverse is run to fix a point D on the second line, the notes being: AB, 970 ft., 357°, BC, 4,632 ft., 331°, CD, 5°. The bearings are measured from an arbitrary meridian, the magnetic bearing of which is N. 23° E. Find the necessary length of CD. (R.T.C., 1920.)

10. The length and bearing of a line *AB* cannot be observed directly, and the following data are obtained by observations from two stations, *C* and *D*.

Line	Length in feet	Bearing
CA	236	258° 35′
CD	1,142	10° 20′
DB	455	274° 15′

Calculate the length and bearing of *AB*.

11. Particulars of part of a traverse survey are:

Line	Length in feet	Bearing
AB	486	342° 24′
BC	1,724	14° 35′
CD	1,053	137° 20′

Calculate the distance between a point E on AB, 326 ft. from A, and a point F on CD, 400 ft. from C.

12. An underground traverse has been run between two survey stations situated at the bottom of two mine shafts A and B.

The co-ordinates of A are 7,260′ N. and 19,340′ E. and of B 9,760′ N. and 21,840′ E. A surface traverse has been run from A to B and the co-ordinates have been calculated to the same axes. The co-ordinates of A are 7,260′ N. and 19,340′ E. as before, but the co-ordinates of B now are 9,660′ N. and 21,912′ E.

Assuming the surface traverse to be correct, what is the error in the bearing of B from A as determined by the underground traverse? (Inst. C.E., 1918.)

13. A four-sided traverse *ABCD* has the following lengths and bearings:

Side.	Length in feet	Bearing
AB	500	roughly East
BC	245	178°
CD	not obtained	270°
DA	216	1°

Find the exact bearing of the side *AB*. (Univ. of Lond., 1913.)

14 For the following traverse compute the length CD, so that A, D, and E may be in one straight line

Line	Length in feet	Bearing
AB	340	85°
BC	506	32°
CD	—	350°
DE	622	18°

15 A straight tunnel is to be run between two points *A* and *B*, whose co-ordinates are given in the annexed table .

Point	Co-ordinates N	Co-ordinates E
A	0	0
B	3,014	256
C	1,764	1,398

It is desired to sink a shaft at *D*, the middle point of *AB*, but it is impossible to measure along *AB* directly, so *D* is to be fixed from *C*, a third known point Calculate

(*a*) The co-ordinates of *D*

(*b*) The length and bearing of *CD*

(*c*) The angle *ACD*, given that the bearing of *AC* is 38° 24′ E of N (Univ. of Lond , 1916)

16 The following traverse is run from A to E, between which there occur certain obstacles

Line	Distance in feet	Bearing
AB	426	38° 20′
BC	518	347° 55′
CD	606	298° 12′
DE	430	29° 46′

It is required to peg the point midway between A and E Calculate the length and bearing of a line from station C to the required point

17 It is required to set out a line AC, of length 2,068 ft , at right angles to a given line AB This is done by traversing from A towards C, the observations being as follows

Line	Length in feet	Bearing
AB		360°
AD	731	113° 48′
DE	467	81° 18′
EF	583	105° 57′

Compute the necessary length and bearing of FC

18 From a point *C* it is required to set out a line *CD* parallel to a given line *AB*, and such that *ABD* is a right angle *C* and *D* are invisible from *A* and *B*, and traversing is performed as follows —

Line	Length in feet	Bearing
BA		360°
BE	258·5	290° 57′
EF	307·0	352° 6′
FC	196 5	263° 27′

Calculate the required length and bearing of *CD*,

19. A and B are two of the stations used in setting out construction lines for harbour works The total latitude and departure of A, referred to the origin of the system, are respectively +542 7 and −331 2, and those of B are +713 0 and +587 8 feet (north latitude and east departure being reckoned as positive) A point C is fixed by measuring from A a distance of 432 ft on a bearing of 346° 14′, and from it a line CD, 1,152 ft in length, is set out parallel to AB. It is required to check the position of D by a sight from B Calculate the bearing of D from B (Inst C E , 1925)

20 A mistake has been made in the tabulation of the numerical values of the co-ordinates of the following closed survey Eliminate the mistake, and compute the closing error

Line	Length in feet	Bearing	Latitude	Departure
AB	732	360° 0′	732 0	0
BC	328	291° 11′	118 5	305 8
CD	607	253° 39′	170 9	582 5
DE	523	158° 51′	487 8	188 7
EF	441	209° 30′	483 8	217 2
FG	505	67° 22′	194 3	466 1
GA	452	90° 12′	1 6	452 0

21 The following results are obtained on working out the co-ordinates of a closed survey in which the bearings have been satisfactorily checked

Line	Latitude in feet	Departure in feet
AB	+663 5	−379 0
BC	+383 0	+402 5
CD	−476 1	+493 6
DE	−877 2	+241 7
EA	+404 0	−785 8

By inspection, estimate where a mistake has probably occurred in the chaining

22 A and B are two stations of a location traverse, their total co-ordinates in feet being

	Total Latitude	Total Departure
A	34,321	7,509
B	33,670	9,652

A straight reach of railway is to run from C, roughly south of A, to D, invisible from C and roughly north of B, the offsets perpendicular to the railway being AC = 130 ft. and BD = 72 ft

Calculate the bearing of CD. (R T C , 1919.)

23 From a common point A, traverses are conducted on either side of a harbour as follows

(1)			(2)		
Line	Length in feet	Bearing	Line	Length in feet	Bearing
AB	875	74°	AE	348	192°
BC	320	109°	EF	436	160°
CD	1,064	82°	FG	521	97°
			GH	1,683	89°

Calculate the distance from H to a point K on GH due south of D and the distance DK.

24. The table below shows the positions of three stations in wooded country. It is required to set out the straight line from A to B, and to peg the point on it at which a perpendicular from C meets the line. Calculate the bearing of AB and the distance from A to the foot of the perpendicular

Station	Total Latitude in feet	Total Departure in feet
A	635 3 S	1,764 2 W
B	238 7 S	4,021 8 W
C	426 0 N.	3,122 5 W

25 The following co-ordinates are those of a shore traverse round a bay, of which soundings are to be taken in lines 200 ft apart

Line	Latitude in feet	Departure in feet
AB	−1,040	− 892
BC	− 925	+ 608
CD	− 378	− 413
DE	− 582	+ 779
EF	−1,233	+ 545
FG	+ 150	+1,231
GH	+ 762	+ 813
HI	+1,106	+ 879
IJ	+1,283	− 174

AJ is one of the lines of soundings, and the other lines are to be laid out parallel to *AJ* Compute, to the nearest foot, the positions on *AB* and *IJ* of the signals marking the line adjacent to *AJ* (T C D , 1930)

26 The following are the co-ordinates, in feet, of a series of survey lines

Line	Latitude	Departure
AB	326 S	978 W
BC	641 S	230 E
CD	508 S	392 E
DE	137 N	425 E

A line is to be set out from E on a bearing of 342°. Calculate at what distance from A it intersects the line AB.

27 M and N are adjacent stations of a large traverse survey, the length and bearing of MN being 2,364 ft and 136° 48′ respectively From station M an unclosed traverse, M*abcde*, referred to the meridian of the survey, was run out for tracing a stream, and the following are the co-ordinates, in feet, of this subsidiary traverse

Line	Latitude	Departure
M*a*	293 S	245 W
a b	506 S.	184 E
b c	455 S	217 W.
c d	612 S	206 E
d e	307 S	424 E

From station N a similar traverse N*1234* was run, and the co ordinates are :

Line	Latitude	Departure
N*1*	356 S.	161 E
1 2	418 S.	323 W
2 3	605 S.	172 E
3 4	431 S.	503 W

It was subsequently desired to obtain a check closure on this part of the survey by measuring the distance and bearing from *e* to *4* Compute, to the nearest foot and minute respectively, the values of these quantities from the original measurements. (Inst C.E , 1931.)

28 A compass traverse is run between two stations, X and Y, of a theodolite survey, of which the total co-ordinates, in feet, are

X, 20,374 N , 15,896 E , Y, 22,053 N , 15,140 E

The notes of the compass route are as follows

Line	Length in feet	Compass Bearing.
X1	426	352° 0′
1 2	270	12° 30′
2 3	588	311° 30′
3 4	403	346° 0′
4Y	316	6° 30′

Magnetic north is 8° 30′ west of the meridian of the theodolite survey

Taking the theodolite work as correct, calculate by how much the compass traverse fails to close on Y (T C D , 1928)

29 In a certain traverse the direction of the first line, AB, was adopted as the meridian, and the adjusted values of the co-ordinates of the first two lines were, in feet,

Line	Latitude	Departure
AB	1,326 N	0
BC	689 N	1,273 W

At a later date a larger survey, referred to a true meridian, was extended to the station A, and the total co-ordinates of that point were found to be 23,416 ft N and 38,572 ft W , the true bearing of AB being 286° 12′ Calculate, to the nearest foot, the total co-ordinates of points B and C from the origin of the large survey

30 The following traverse is run from a station P, of which the total latitude and departure are respectively +19,430 and −4,607 ft , to a station Q of total latitude and departure +15,106 and −4,488 ft respectively

Line	Length in feet	Bearing
P1	473	164° 12′
1 2	1,386	183° 54′
2 3	721	158° 24′
3 4	1,207	190° 39′
4Q	634	176° 18′

Taking the given positions of P and Q as correct, calculate the amount by which the traverse fails to close on Q

31 In the course of setting out a straight portion of a railway, pegs have been driven in the centre line at A and B on either side of a wood A traverse is carried from A to B along the side of the wood, as follows

Line	Length in chains	Bearing
AC	6 43	187° 30′
CD	3 88	110° 48′
DE	5 21	159° 42′
EB	4 06	78° 18′

If A is 393 21 chains from the commencement of the railway, find the corresponding distance of B, which is farther along the railway

From the traverse station D a line DF can be carried into the wood on a bearing of 61° 38′ Compute the length of DF so that F may be pegged as an intermediate point on AB, 32 A series of traverse lines was run by theodolite at the top of a wooded gorge A compass traverse was carried along the stream, and was connected to the theodolite survey at both ends, as shown in the following notes Assuming that the theodolite traverse is correct, find the closing error of the compass work.

Theodolite Traverse

Line	Length in feet	Bearing
AB	361	98° 20′
BC	514	136° 31′
CD	482	100° 9′
DE	737	84° 42′

Compass Traverse

Line	Length in feet	Magnetic Bearing
AB		112°
A1	186	74°
1 2	225	88°
2 3	380	125°
3 4	155	196°
4 5	476	139°
5 6	303	81°
6 E	540	119°

33 From a traverse station P, the total co-ordinates of which are 486 ft S and 12,374 ft E, a line is set out on a bearing of 49° 12′ From a station Q, having total co-ordinates 1,026 ft S and 15,188 ft E, a line is set out on a bearing of 325° 36′ Calculate the total co-ordinates of the point of intersection of these lines

34 It is required to ascertain the distance from A to an inaccessible point B invisible from A A straight line CAD is run, of which AC = 240 ft and AD = 190 ft, and angles ACB and ADB are found to be 64° and 72° respectively Calculate the distance AB

35 The notes of part of a traverse survey are

Line	Length in feet	Bearing
AB	4,050	N 12° 25′ E.
BC	3,763	N 4° 20′ W
CD	5,300	N 2° 40′ E
DE	2,911	N 7° 15′ W

The bearing from A to an inaccessible point O is N 35° 10′ W, and from E its bearing is S 43° 55′ W Calculate the distance of O from C (R T.C, 1919)

36 From the initial station A of an unclosed traverse the bearing of a distant point X is observed to be 62° 18′ From station K, which is 6,838 ft. N and 3,016 ft W of A, the bearing to X is 91° 37′ Calculate the bearing of X from T, of which the total co-ordinates referred to A are 11,273 ft N and 1,419 ft E

37 It is required to determine the distance between two inaccessible points A and B by observations from stations C and D 1,000 ft apart The angular measurements give ACB = 47°, BCD = 58°, BDA = 49°, ADC = 59°. Calculate the distance AB

38 The readings in the annexed table were obtained in a closed compass-traverse on an area subject to irregular local magnetic attraction at each station

Line	Bearing Forward	Bearing Back	Length line.
AB	85° 24′	268° 42′	536
BC	164° 54′	340° 0′	316
CD	261° 6′	78° 42′	634
DE	344° 24′	170° 36′	140
EA	8° 36′	186° 30′	200

Show how to check the accuracy of the readings, and tabulate the bearings you would use for plotting the different lines, in order to obtain as accurate a plan as is possible from these readings (The plan need not be drawn)

As the exact direction of magnetic North is of no importance, you may take the bearings observed at any one station—say *A*—as correct (Univ. of Lond , 1916)

39 Distribute the angular error of the following theodolite traverse, and write out the corrected bearings to the nearest 10″

AM	205° 22′ 40″	GH	332° 17′ 30″
AL	178° 36′ 00″	HE	49° 07′ 00″
AB	93° 14′ 10″	HI	272° 48′ 20″
BC	161° 49′ 40″	IJ	229° 27′ 00″
CL	251° 43′ 00″	JK	341° 06′ 40″
CD	230° 51′ 10″	KL	52° 58′ 10″
DE	206° 10′ 20″	LC	71° 44′ 20″
EH	229° 07′ 00″	LA	358° 37′ 20″
EF	147° 56′ 50″	LM	343° 13′ 40″
FG	238° 36′ 00″	MA	25° 24′ 00″

40 The distance and bearing between two points A and B have been carefully determined, and the observed values, AB = 3,236 3 ft , and bearing AB = 102° 36′ 0″, may be taken as correct. A theodolite traverse, referred to the same meridian, was afterwards run from A to B, and the following co-ordinates were obtained .

Line	Latitude in feet	Departure in feet
AC	621 2 S	183 6 W
CD	512 7 S	924 0 E
DE	1,029 3 S	728 6 E
EF	632 8 N.	1,566 4 E
FB	821 2 N.	118 7 E.

Adjust the co-ordinates to close on B

41. The following are the co ordinates of the lines of a theodolite traverse, the bearings of which checked satisfactorily

Line	Latitude in feet	Departure in feet
AB	1,426 3 N	0
BC	503 4 N	712 6 E
CD	617 0 S.	638 5 E
DE	920 8 S.	219 0 W.
EF	589·2 S	427 1 W
FA	200 1 N.	703 2 W.

Balance the survey The line AB was measured more carefully than the others, and lay over easy ground. It may be assumed that the probability of error per chain length in its measurement was one-third of that in each of the other lines. (T.C.D., 1928)

42 Balance the survey of Example (1), page 186, by assigning to the lines the following weights proportional to the relative difficulties of chaining—3, 3, 2, 1, 1 5, 1, 1 5, 2, 1 5

CHAPTER IV

ORDINARY LEVELLING

LEVELLING, or the determination of the relative altitudes of points on the earth's surface, is an operation of prime importance to the engineer, both in acquiring data for the design of all classes of works and during construction

Definitions.—A *Level Surface* is one which is at all points normal to the direction of gravity as indicated by a plumb line Owing to the form of the earth, a level surface is not a plane, nor has it a regular form because of local deviations of the plumb line caused by irregular distribution of the mass of the earth's crust. The surface of a still lake exemplifies a level surface

A *Level Line* is a line lying throughout on one level surface, and is therefore normal to the direction of gravity at all points

The *Horizontal Plane* passing through a point is the plane normal to the direction of gravity at the point It is therefore tangential to the level surface at the point, and sensibly coincides with it within ordinary limits of sighting (page 231).

A *Horizontal Line* passing through a point is one lying in the horizontal plane, and is tangent to a level line through the point and having the same direction

A *Datum Surface* (*Line*) is any arbitrary level surface (line) to which the elevations of points may be referred.

The *Reduced Level* of a point is its elevation above the datum adopted

PRINCIPLES

Difference of Level of Two Points.—The simplest operation with the level (page 101) is to determine the difference of level between two points so situated that, from one position of the instrument, readings can be obtained on a staff held successively upon them. The precise situation of the instrument is immaterial, but to eliminate the effects of possible instrumental error (page 227) the two sights should be as nearly equal in length as can be judged. Having set the instrument on firm ground, it is levelled by the foot-screws, and the eyepiece is focussed. The staffman is then instructed to hold the staff vertically upon the first point. The observer, having directed the telescope towards it and focussed, notes the reading (page 114), exercising care to ensure that the bubble is central during the observation The staff is then held upon the second point, and the telescope is pointed to it. Before sighting, the leveller should examine the bubble to see that it preserves its central position; if it does not, it must be returned to the centre by manipulation of the levelling screws or the tilting screw. The second staff reading is then obtained

Calculation of Difference of Level.—Let the difference of level between A and B (Fig 188) be required. The respective staff readings are 7·24 and 2·01 ft, so that A is 7·24 ft below the first line of sight, and B is 2·01 ft below the second But the two lines of sight lie in the same horizontal plane, any want of coincidence between their planes, due to relevelling the instrument for the second observation, being insignificant The staff readings are therefore measurements made vertically downwards from a horizontal plane, and this horizontal plane practically coincides with the level surface through the telescope axis The difference of 5·23 ft between the readings is therefore the difference of level between A and B, the smaller reading being observed on the higher point.

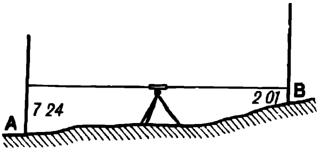


Fig 188

Now, let it be assumed that the reduced level of A is known to be 100 ft above a particular datum, then that of B is obtained as 105·23 ft above the same datum by application of the difference of level, adding a rise and subtracting a fall. Alternatively, the reduced level of B may be found by referring to that of the lines of sight. By adding the first reading to the reduced level of A, the instrument height, as it is termed, is found to be 107·24. But B is 2·01 ft. below this level, so that its reduced level is obtained as 105·23 by subtraction

Series Levelling.—The more general case occurs when the two points to be compared are so situated, by reason of their distance apart, their difference of level, or the intervention of obstacles, that it is impossible from any one instrument station to read a staff held successively upon them. In these circumstances, the work is performed in a series of stages, to each of which the previous method is applied. Thus, in Fig. 189, the difference of level between A and D is determined by observing that from A to a convenient point B, and then proceeding similarly from B to C, and from C to D.

The instrument is therefore first set up in such a position as 1, from which a staff held on A can be read and a clear forward view obtained When the sight on A has been taken, the staffman, proceeding up the slope, selects a firm point B on which to hold the staff. The first stage is completed by noting the reading on

B, and the instrument is then transferred to position 2, the staff meantime being held on B When the instrument is levelled, the staff on B is again sighted, so that the level of B may be compared

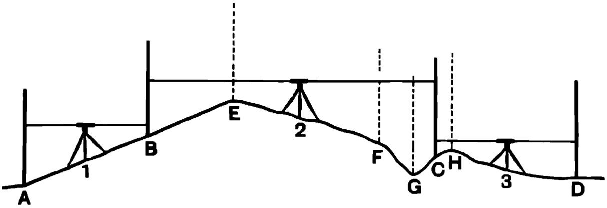


FIG 189

with that of a convenient point C in the same manner as before. A third step suffices to reach the point D

It will be evident that the essential feature of the system lies in the observation of two staff readings on each of the points B and C, one before, and the other after, moving the instrument When the instrument is being shifted, the staff must not be moved, and, while the staff is being carried forward, the instrument must remain stationary. Points such as B and C, on which the staff is held to permit the transfer of the instrument, are called change points or turning points It is unnecessary that they should lie in the line AD. The essential requirement is that they should be unyielding points, to avoid any risk of settlement occurring between the two observations (page 229)

Backsights and Foresights.—The word " sight " is used to denote either an observation or the resulting reading.

A *Backsight* is the first sight taken after setting up the instrument in any position.

A *Foresight* is the last sight taken before moving the instrument.

The first sight on each change point is therefore a foresight, and the second a backsight : every line of levels must commence with a backsight, and finish with a foresight. In Fig. 189 the sights 1A, 2B, and 3C are backsights, while 1B, 2C, and 3D are foresights. The two sets of sights are distinguished because they are differently applied in the calculation of the levels. A backsight is always taken on a point the reduced level of which is known or can be computed. By addition of the backsight to that level, the instrument height can be obtained A foresight is always taken on a point of unknown level with the object of ascertaining its level by subtraction of the foresight from the known instrument height

The terms, backsight and foresight, are unfortunate because they imply direction. The sights may be taken in any direction, the terms having no other significance than that contained in the above definitions

Intermediate Sights.—When the backsight has been taken, the observer is in a position to determine the level not only of the next change point, but of any number of points within range Thus, in the course of the operations of Fig 189, it may be desired to obtain the elevations of certain points E, F, G, and H The staff-man on his way from B to C holds the staff on E, F, and G successively, and the leveller takes a single sight on each The point H is observed from the next instrument station Sights taken between a backsight and a foresight to ascertain the levels of points are called *Intermediate Sights*

Datum.—To define the relative altitudes of a series of points, it is sufficient to ascertain their elevations above any one datum surface. A reduced level for one of the points may therefore be assumed arbitrarily, and those of the others are deduced from it Many advantages, however, accrue from the adoption throughout a country of a standard datum of reduction Mean sea level, as ascertained by prolonged observation, affords a universal datum, and is that most generally chosen

The standard datum of Great Britain is that of the Ordnance Survey This was originally the assumed mean level of the sea at Liverpool It was obtained from a short series of observations in March 1844, and is believed to be about 0 65 ft below the general mean level of the sea round the British coast With the object of establishing mean sea level with greater precision, hourly observations were made at Newlyn, in Cornwall, during the six years, 1915–21 Mean sea level at Newlyn is the new Ordnance datum, and a revision of the levelling of Great Britain has been undertaken with respect to this datum At present, 1931, both the Liverpool datum and that of Newlyn are in use, but the levels of each county are being referred to the new datum as it comes under revision *

The datum for Ireland was fixed on April 8th, 1837, as the level of a particular low tide in Dublin Bay, and is about 8 5 ft below Ordnance datum in Great Britain

Bench Marks.—A bench mark is a fixed point of reference, the elevation of which is known. In mapped countries, bench marks are established at intervals throughout the country by the State, and their positions and elevations above the standard datum are published Surveyors have thus a ready means of expressing reduced levels in terms of standard datum by commencing a series of observations with a reading of the staff held on a bench mark.

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* "A Description of the Ordnance Survey Large Scale Maps," Ordnance Survey Office, Southampton, 1930.

Standard bench marks take various forms In Britain, Ordnance bench marks (O.B M.) are chiselled on buildings, milestones, etc., in the form of an arrow and a horizontal groove (Fig 190), the centre line of which defines the elevation The new Ordnance bench marks are cut on bronze tablets let into walls, etc. The position of bench marks and their elevations in feet above Ordnance datum are shown on the 1/2,500 and 1/10,560 Ordnance maps, to the nearest 0·1 ft in the case of maps which refer to the Liverpool datum and those based on Newlyn which were published before April 1929. Maps published after that date, and referred to the Newlyn datum, give the elevations of bench marks to ·01 ft On these sheets the average difference between the old and new elevations of bench marks is stated to the nearest 0·1 ft This difference varies throughout the country, since it is due not only to the change of datum, but also to the correction of the old work.

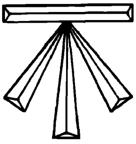

FIG 190. ORDNANCE BENCH MARK.

The surveyor may have occasion to establish private bench marks for his own use Thus, in the construction of engineering works, repeated levelling between the nearest Ordnance bench mark and the site of the work may be obviated by carefully determining at the outset the elevations of a number of permanent points at the site, these being referred to as required When levelling in unmapped country, the surveyor should leave marks on rocks, etc , at intervals, for convenience in connecting on to his assumed datum at a future date

Level Book.– The level book is ruled on the left-hand pages for the entry, in tabular form, of the field observations and for their reduction, the opposite pages being reserved for descriptions and remarks. Various forms of level book, differing in the number and arrangement of the columns, have been proposed with a view to facilitating the entry of the observations and the figures required in reducing, as well as the extraction of quantities to be plotted

Staff readings are entered in the different forms in one, two, or three columns The single column method is not recommended, as it necessitates the use of notes or symbols to distinguish backsights from foresights In using two columns, the first is reserved for backsights, and in the second are entered intermediate sights and foresights, these being similarly applied in reducing the levels. With three columns, the intermediate sights are separated from the foresights, and checking of the reduction is thereby facilitated, so that this system is most generally followed

Booking and Reduction of Levels.—The observations shown in Fig 189 may be used to illustrate the method of entering staff readings and reducing the levels In form *A* the first three columns are reserved for staff readings, which are entered in the appropriate columns, in the order of their observation, as shown.

LEVEL BOOK *A*

Back-sight	Inter Sight	Fore-sight	Rise	Fall	Reduced Level	Distance	Remarks
6 38					50 00		A
5 97		1 17	5 21		55 21		B
	2 10		3 87		59 08		E
	6 35			4 25	54 83		F
	10 20			3 85	50·98		G
1 53		8 22	1 98		52 96		C
	0 90		0 63		53 59		H
		3 76		2 86	50 73		D
13 88		13 15	11 69	10 96	50 00		
13 15			10 96				
0 73 Rise			0 73 Rise		0 73 Rise		

Each point on the ground is represented by a horizontal line, and, since the staff is read twice on a change point, the backsight to it is entered in the same horizontal line as the previous foresight

The next three columns are used in the reduction of the levels by the " rise and fall " system or method of differences, as outlined on page 208. Each point on the ground, after the first, is compared with that preceding it, and the difference of level, deduced from a comparison of the staff readings, is entered as a rise or a fall. Thus, from A to B we have a difference of 5·21, which is entered as a rise, since the second reading is smaller than the first, A being farther below the line of sight than B The rise of 3·87 from B to E is obtained by comparing 2·10 with 5·97, since these readings were observed from the same instrument station. No comparison is made of the figures 1·17 and 5·97, since they are common to one ground point and merely show the change of elevation of the instrument from 1 to 2.

On completing the tabulation of rises and falls, the accuracy of the arithmetical work should be checked The difference between the sum of all the backsights and that of the foresights represents the difference of level between the first and last points, and should equal the difference between the sum of the rises and that of the falls.

To compute the reduced levels, let it be supposed that A is known, or assumed, to be 50·00 ft above datum. Since the difference of level between consecutive points has been ascertained, it is only necessary to apply these successively, each reduced level being obtained from that of the preceding point by adding the rise or subtracting the fall between them. The calculation of the reduced levels is verified if the difference between the first and last equals the previously obtained total difference.

In book *B* the readings are entered as before, but the reduction is intended to be performed by the instrument height or collimation method, the principle of which was stated on page 208 A column is provided for noting each instrument height or elevation of the plane of sight, from which the reduced levels are to be computed.

LEVEL BOOK *B*

Back-sight	Inter Sight	Fore-sight	Inst Height	Reduced Level	Distance	Remarks
6 38			56 38	50 00		A
5 97		1 17	61 18	55 21		B
	2 10			59 08		E
	6 35			54 83		F
	10 20			50 98		G
1 53		8 22	54 49	52 96		C
	0 90			53 59		H
		3 76		50 73		D

13 88
13 15
——
0 73 Rise

13 15

50 00
——
0 73 Rise

The level of the first plane of sight, being 6 38 ft above A, is evidently obtained by adding the backsight to 50·00, the known elevation of A , and the sum, 56·38, is entered in the instrument height column horizontally opposite the backsight The reduced levels of all other points on which the staff was read from the first instrument station can be obtained from this elevation by subtracting their readings, so that the reduced level of B is found to be 55·21 by deducting the foresight 1·17. The next instrument height, 61·18, is then obtained by adding the new backsight, 5·97, to 55 21, the level of the change point, and the succeeding intermediate readings and foresight are subtracted as before Each backsight falls to be added to an already known reduced level to obtain an instrument height, from which intermediates and foresights are subtracted to give reduced levels.

As before, the difference between the sum of the backsights and that of the foresights equals the difference between the first and last reduced levels, but this check verifies the calculation of the instrument heights and levels of change points only. Each reduced level is not obtained from the preceding one, as in the previous method, so that an error made in the reduction of an intermediate sight is not carried forward. Intermediate levels may be checked either by repeating the subtractions or by applying the following rule : The sum of all reduced levels except the first =(each instrument height) × (the number of intermediate and foresight observations made from it)—(the total sum of the foresight and intermediate readings) In the example, $377{\cdot}38 = (56{\cdot}38 \times 1 + 61{\cdot}18 \times 4 + 54{\cdot}49 \times 2) - (13{\cdot}15 + 19{\cdot}55)$.

LEVEL BOOK C

Back-sight	Fore-sight	Inst Height	Reduced Level	Distance	Remarks
6 38		56 38	50 00		A
5 97	1 17	61 18	55 21		B
	2 10		59 08		E
	6 35		54 83		F
	10 20		50 98		G
1 53	8 22	54 49	52 96		C
	0 90		53 59		H
	3 76		50 73		D

13 88
13 15
— —
0 73 Rise

50 00
— — —
0 73 Rise

The two-column system of booking is illustrated in book *C* Reduction may be performed by either the rise and fall or instrument height method, care being taken, in checking, to add the appropriate figures from the second column

Notes—(1) Rapidity in reducing is acquired by practice In finding rises and falls, the beginner should keep in mind that the second of two figures being compared lies in the lower line, and is either directly below or to the right of the first

(2) Checking is essential, as slips are easily made In a long series of levels, each page should be checked before starting with the next It is best to check from the last entry on the previous page, and this reading, whether a backsight or an intermediate, is included in the backsight summation If the last reading on the page being checked is an intermediate, it is summed with the foresights, but if a backsight, it is not included

(3) To minimise the waste of time occasioned by an arithmetical slip, rises and falls should be checked before reduced levels are filled in With the same object, instrument heights and levels of change points may be worked out and checked before reducing intermediate readings

(4) It is to be observed that the field work is in no way verified by these checks, which are applied simply to detect and eliminate arithmetical errors in reduction.

Comparative Merits of Reduction Methods.—The instrument height system is economical of figuring, and proves the more rapid method It is well adapted for reduction in the field, particularly in setting out levels for constructional work. If, as is usually the case, the somewhat tedious check on intermediate levels is neglected, it is open to the objection that a mistake in an intermediate reduction may pass unnoticed This disadvantage is entirely absent in the rise and fall method, which affords a complete check, only vitiated if mistakes balance By reason of its greater certainty, the use of the latter method is sometimes insisted upon in important work.

FIELD WORK

The operations of ordinary levelling are

1 The determination of the difference of level of two points.
2 The running of sections
3. The setting out or giving of levels for constructional purposes.
4 The running of contour lines (Chap VI)

Determination of the Difference of Level of Two Points.—Ordinary levelling for the ultimate purpose of comparing the levels of two points only has sometimes to be performed by the engineer in the manner already described If the points are in the same vertical line, the use of a level and staff is often preferred to taping When both points cannot be seen from one set-up of the instrument, it is necessary to run a line of levels between them, observing backsights and foresights only

Sectioning.—The running of a section is the most common levelling operation, and consists in obtaining a record of the undulations of the ground surface along a particular line, straight or curved, so that they may be represented to scale This involves observing not only the elevations of a number of points on the line, but also their distances along it It is important that the points selected should be those at which the inclination of the ground sensibly changes, so that, having plotted them, it is justifiable to assume a uniform slope between each consecutive pair Thus, in Fig 191, *a* represents the actual undulations of a piece of ground · if the levels and chainages of the salient points numbered are observed and plotted, a good representation of the slopes is obtained even although the points are joined by straight lines as in *b* Fig *c* shows the serious misrepresentation produced by omission to observe points 3 and 5 On the other hand, if, as in running a section along a road or a railway, the points of change of gradient are not evident, it is permissible to observe equidistant points, say a chain length apart.

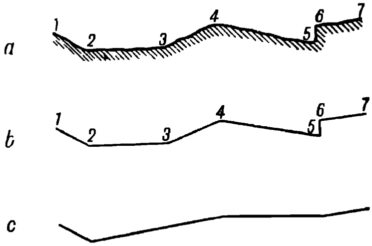

FIG. 191.

Longitudinal and Cross Sections.—Sections are of two kinds : (*a*) Longitudinal Sections, or Profiles , (*b*) Cross Sections.

A longitudinal section is one which follows some predetermined line, usually the centre line of proposed work, *e.g.* a road, railway, canal, or pipe-line By means of such sections the engineer is enabled to study the relationship between the existing ground surface and the levels of the new work in the direction of its length In the design of works, it is frequently necessary to run longitudinal sections along various proposed centre lines and compare the costs of the several schemes

Since a plotted profile cannot exhibit any particulars as to the character of the ground on either side of the centre line, it does not convey sufficient information for the complete design of works. In the case of those which occupy a very narrow strip of ground, *e g.* a pipe-line, it will, however, be sufficient to assume that the ground within the limits of the width of the work is level in a direction normal to the centre line Otherwise, additional information is required, and this is conveniently obtained by means of cross sections, which are sections taken at right angles to the centre line and of sufficient length to embrace the limits of the work on either side.

LONGITUDINAL SECTIONS

Field Party.—The work is most expeditiously performed by a party of four, *viz*

The Leveller, who directs the ranging of the line, sees that the staff is being held on suitable points, uses the instrument, and books the staff readings and distances

The Staffman, who holds the staff and calls out the chainages.

Two Chainmen, who manipulate the chain.

Commonly one of the chainmen holds the staff, the leveller having to wait while the chain is being pulled forward If only one assistant is available, much time is wasted by the surveyor having to leave the instrument in order to take part in the chaining.

Equipment.—*Essential* —Level, Staff, Ranging Poles, Chain or Band, Arrows, Chalk, Level Book, and Pencil

Optional.—Tape and Plumb Line for stepping.

Running the Section.—If necessary, the line of the section must first be set out by locating a sufficient number of points to define the straights and curves, intermediate poles being ranged by eye. In a long section only a sufficient length should be ranged in advance as will occupy, say, half a day to level.

It frequently happens that the section is run along the courses of a traverse survey. If the traversing has been completed previously, it is necessary to recover the stations by reference to the notes describing their positions, and the lines must be ranged afresh. In mapped country it may not be necessary to run a traverse, and the line of the required section is drawn on a map, from which it has to be set out by the levelling party. Its position relatively to buildings, fences, and other definite features represented on the map is obtained by scaling, and the scaled dimensions are reproduced on the ground. In cases where a number of points so located are intended to lie on a straight line, it frequently happens that they fail to do so exactly on account of errors of scaling and of measurement, but it is usually quite sufficient to range a straight line by eye in such a position as will average out the irregularities. In districts where there is a scarcity of well-defined detail the transfer of the section line from the map to the ground is sometimes troublesome, and it becomes necessary to set it out by theodolite.

To refer the levels to a standard datum, levelling should start from an Ordnance bench mark, and the position and value of that nearest the beginning of the section should be extracted from an Ordnance map before proceeding to the ground Levelling is conducted from the bench mark, by observing backsights and foresights only, until the instrument can be placed in a position from which the first part of the section can be commanded.

Chaining is now commenced, and the staffman proceeds along the chain, and holds the staff at all points of change of slope. After each reading is taken, he calls out the chainage of the point to the leveller, or he may enter them in a notebook, from which they are transcribed into the " distance " column of the level book at frequent intervals When the length of sight reaches the maximum permissible for good reading (page 230), or when inequalities of the ground prevent the taking of further intermediate sights, the leveller signals the staffman that he wishes a foresight The latter selects a firm change point, either on or off the chain line, marks it with chalk, and holds the staff upon it, taking due care as to verticality. Having entered the reading, the leveller carries the instrument forward He selects his next instrument station on firm ground, and in such a position that he can sight back on the staff, and also obtain an unobstructed forward view along the section. When the backsight observation is completed, the staffman proceeds to give intermediate sights as before, and this routine is followed to the end of the section

As the various features lying in the section line are reached, such information regarding them as is likely to prove useful in the design of the proposed works should be acquired. Such items include the levels of the beds of streams, flood water levels, if indications of them can be traced, the chainage at which fences, etc , intersect the line, the names and levels of roads and railways crossed, the headroom of bridges, etc When the section line passes below a bridge, readings are taken with the staff held inverted against the underside of the girders or arch, these being distinguished in booking by being marked with a plus sign or otherwise. The elevation of the road or rail surface on the top of the bridge is ascertained by levelling up the bank.

Notes —(1) The chainage should be continuous from start to finish of the section

(2) The use of salient points *on* the line of section as change points saves time, but accuracy should not be sacrificed by using an unstable point on the line in preference to a firm point on one side.

(3) Time may be wasted by excessive refinement in reading intermediate sights In ordinary cross-country work it usually suffices to read intermediates to the nearest tenth of a foot only, but the relative importance of the staff points should always be kept in view

(4) See notes on expedition in levelling (page 233)

Checking.— It is impracticable to check every elevation in a line of levels except by duplicating the whole of the observations In the great majority of cases, however, it is sufficient to test the quality of the work in so far as the change points only are concerned, or, in other words, to ascertain whether the various instrument heights are correct within allowable limits of error Elevations obtained by means of intermediate sights from these instrument heights are affected by errors, and possibly by serious mistakes in reading or booking, but the effects are quite local, and cannot be carried forward (page 226)

In a country unprovided with standard bench marks the levelling is checked

(*a*) By adopting a special routine at the change points to render the work self-checking

(*b*) By levelling back to the starting-point

In the former system, the most convenient method of limiting the propagation of error and of detecting gross mistakes is to use double change points At each set-up of the instrument two backsight readings are obtained on the respective change points, so that each instrument height is determined twice If the discrepancy between them at any stage is no more than is expected to arise from accidental errors, the mean is accepted Otherwise, repeat sights are taken on the change points The method is economical of time as compared with the running of a check line of levels, but does not provide quite such a thorough check (Vol II, Chap V)

By the latter method the error of closure is exactly known if this exceeds the permissible error (page 232), the work must be repeated A long section should be divided into parts, each to be checked back before the next is levelled Check levelling may be conducted along the easiest return route or along the original change points, so that mistakes may be discovered and rectified expeditiously The more prominent change points should be referenced as temporary bench marks for possible future use

If, however, numerous standard bench marks are available, the work is completed by levelling from the end of the section to an adjacent bench mark, the value of which has been ascertained from the map. Any discrepancy between the known elevation of the bench mark and its value as obtained by reducing the levels may be accepted as the closing error, except in mining districts, where the possibility of irregular subsidence of bench marks makes any check other than levelling back to the starting-point of little or no value Opportunities of suitably dividing up the section by checking on to the bench marks near the line should not be neglected, and the positions and values of adjacent bench marks should be noted before proceeding to the field

Note While it is unnecessary to reduce all the levels as the section progresses, backsights and foresights should be kept summed for the purpose of checking on to bench marks

Plotting the Profile.—On reduction of the levels in terms of Ordnance or other datum, the reduced levels are available for plotting Having drawn a datum line, and marked off along it the chainages of the points to be plotted, perpendiculars are erected, and the appropriate reduced level is set off on each A continuous line joining the points so obtained represents the profile of the ground

Since the horizontal distances involved are in general very much greater than the variations in level, it is usual to plot vertical dimensions to a larger scale than horizontal distances In this way the irregularities of the ground are made more apparent The steepness of slopes is exaggerated, and artificial features intersected suffer corresponding distortion Exaggeration is of value in enabling the relationship between the original surface and the proposed levels of new work to be clearly and accurately shown The ratio of exaggeration adopted runs from 5 to 15 times and upwards, depending upon

(*a*) *The Character of the Ground* —A greater exaggeration is required to exhibit the irregularities of flat ground than those of rough country

(*b*) *The Horizontal Scale* —The horizontal scale may be chosen arbitrarily, but it is commonly that of the plan upon which the section line is drawn The smaller the horizontal scale, the greater should be the ratio of exaggeration

(*c*) *The Purpose of the Section* The vertical scale should be increased in cases where a highly accurate representation of vertical dimensions is required

If the section has been run in a district at a considerable elevation above sea level, the reduced levels in terms of Ordnance or a similar datum have large values, and to plot them to a sufficiently large scale to exhibit the surface irregularities would require an inconveniently large depth of paper In such a case, the base line should be assumed to have an elevation of a convenient number of feet above the datum of reduction The value of the base line may have to be changed several times in a long section, as it is desirable to maintain the lengths of ordinates within the limits of about 1½ and 6 in.

In finishing the profile, the datum and ground lines are drawn in black ink, and the ordinates are shown as thin blue lines If plotted points are joined by a continuous curve, regard should be had to the character of the ground and the ratio of exaggeration adopted It is usually safer to join the points with straight lines, rounding off sharp angles unless the exaggeration is great. The value of the datum line should be given, the reduced levels written against the ordinates, and descriptions entered where necessary.

Notes—(1) Scaling is obviated by the use of profile paper having printed horizontal and vertical lines Various rulings are sold, that having 20 horizontal and 4 vertical lines to the inch being most used

(2) Only points on the line of section are plotted, and caution must be exercised against including any other observations inadvertently

Working Profile.—When the location of an engineering scheme has been decided, and the design made, a working section is prepared for the use of the resident engineer This profile incorporates the features of the original ground surface, as well as the levels of the new work, and must exhibit definite information regarding the relationship between the new levels and those of the original ground All further information likely to be required during construction, and which can be clearly shown, is also included

The character of this information depends upon the nature of the work Part of a working profile for a railway is shown in Fig 192 The new work is represented by two parallel lines, the lower, in red, denoting formation or sub-grade level, *i e* the surface level of the earthwork, and the upper, usually in blue, representing rail level Ordinates are drawn, in this case at 1 chain intervals, and the datum line is figured to show distances from the commencement of the railway in miles, furlongs, and chains The figures written against the ordinates represent original ground level, formation level, and depth of cutting or bank Original levels are written in black, formation levels in red, and depths of earthwork in red or blue, according as they refer to excavation or embankment The gradients of the new work are figured boldly, and the limits of each clearly shown by arrows against ordinates drawn in red The positions of bridges, culverts, level crossings, etc , as well as brief

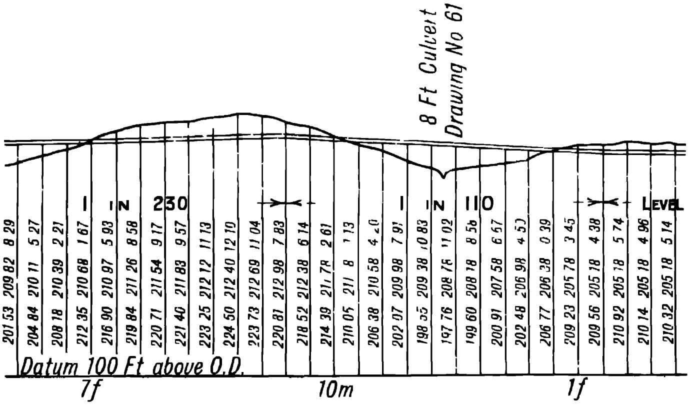

FIG 192 —WORKING PROFILE OF RAILWAY

particulars of existing features crossed by the line of section, are also entered.

CROSS SECTIONS

These are usually taken during the progress of the longitudinal section If the best results are desired, the observations may be made with level and staff, particularly if the cross sections are long More rapid methods by clinometer or hand level are sufficiently accurate for many purposes, and are specially suitable for short sections, the length of which precludes the accumulation of serious error.

Interval between Sections.—The purpose of cross sectioning is to furnish the engineer with sufficient information regarding the levels of the ground on either side of the longitudinal section to enable him to design the intended works and compute the quantities of earthwork, etc , involved To facilitate estimation of the character of the ground between cross sections, they should be taken at every marked change of slope transverse to the longitudinal section, so that it is valid to assume that the ground surface changes uniformly from one section to the next This desideratum is often neglected in practice, cross sections being commonly taken at constant intervals of 1 chain or 100 ft

Setting Out.—The lines of cross sections are in general perpendicular to the longitudinal section line and radial on curves When for any reason a cross section is run in another direction, the angle it makes with the longitudinal section must be measured, so that its position may be shown in plan In sectioning a wide area, *e g* for a reservoir or a dock, the cross sections may have a considerable length, and their lines should be set out by theodolite, box-sextant, or optical square In the case of a narrow strip of ground, *e g.* for a road or a railway, their perpendicularity is usually judged by eye Since cross sections must be long enough to include the width of the new work, a sufficient margin must be given in cases where the latter is not known exactly

Cross Sectioning by Level and Staff.—The procedure is similar to that followed in longitudinal sectioning, but in the case of short sections the distances are taped A chainman remains with the tape box at the point on the centre line from which the section is projected, while the staffman takes the ring and proceeds along the section The chainman guides the staff holder into the perpendicular, and calls out the measurements of the points observed If the leveller has only one assistant, he should first mark by arrows or twigs the salient points to be levelled, and note the distances on sketches of the sections

The levelling of cross sections on flat ground is performed from the instrument stations used for the longitudinal section, but on sidelong ground it may be impossible to complete any cross section from one instrument station In these circumstances, the repeated shifting of the instrument renders the progress very slow, particularly if each cross section is finished before the next is begun The number of instrument stations may be greatly reduced by sighting from each the several points within range on a number of sections Thus, in Fig 193, the points *a* may be observed from the instrument station selected for the sighting of the points a of the longitudinal section On shifting the instrument up the slope, points *b* are levelled, their distances out being measured from marks left on the centre line Finally, points *c* are observed from a lower position of the instrument

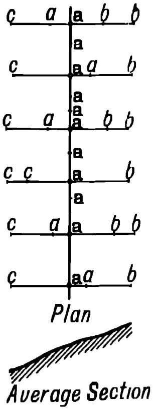

FIG 193

In booking cross section observations, the staff readings are entered in the appropriate columns in the usual manner Distances along the cross sections must be clearly distinguished from those of the longitudinal section This is best secured by having three distance columns, the centre one being reserved for distances on the longitudinal section, and the others for the cross section measurements to right and left respectively If only one column is provided for distance, the cross section measurements may be entered in it with *R* or *L* written after each to signify whether the measurement is to the right or left of the centre line It is, however, preferable to reserve the single distance column for longitudinal section measurements only, and to enter transverse distances in the remarks column Care must be exercised in booking cross sections which are observed in groups, as in Fig 193, and explanatory sketches are often necessary

The plotting of cross sections observed as above is similar to that of profiles, except that, for the purposes of showing and measuring new work, it is more useful in this case to have vertical and horizontal measurements plotted to the same scale The scale is commonly that used for the vertical dimensions of the profile It is usual to arrange the cross sections on a sheet in rows on a series of vertical centre lines. The elevations represented by their datum lines may be frequently altered to keep the ordinates reasonably short To economise room, datum lines and ordinates are sometimes omitted in inking in, and the reduced level at the centre is written horizontally on the ground line

Cross Sectioning by Hand Level.—The hand level is held at a constant known height above the ground either by supporting it against a pole or by standing erect when sighting, the height of the eye having been measured The surveyor stands over the centre point, and observes and notes the readings on a levelling staff or a ranging rod, and books the taped distances Alternatively, the staff may first be held on the centre point and a backsight taken on it from a convenient position It may be necessary to move the instrument in running a section on steep ground, and this is effected by sighting on a change point as in ordinary levelling Since the level of the centre point is obtained from the longitudinal section, the levels may be reduced, but it is more usual to plot the section by setting down the staff readings from the line, or lines, of sight

Cross Sectioning by Theodolite.—On steep ground the labour of frequently changing the instrument is obviated by the use of an inclined line of sight roughly parallel to the ground and of known inclination This is most accurately given by the theodolite

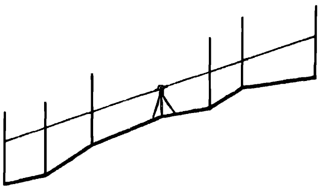

Fig 194

In Fig 194, the theodolite is set over the centre line mark, the level of which is known Having levelled up, the height of the horizontal axis above the ground is taped and noted The line of sight is set roughly parallel to the ground, and the vertical circle is clamped at the nearest whole degree, the angle being booked The inclinations of the line of sight on either side of the instrument need not be the same, and may differ considerably on rough ground Where the general slope of the ground is fairly uniform, however, the value of the angle of elevation on the uphill side is reproduced on the circle as an angle of depression on the downhill side in order to simplify the plotting a little The readings of the staff held vertically at the various salient points are observed Distances are taped along the line of sight, the staffman holding the ring of the tape against the staff as nearly at the reading as he can judge, while the measurements are read by the chainman at the horizontal axis of the instrument In the case of long sections, two tapes should be tied together

The results of the observations are usually booked in columns, but may be noted on sketches In the tabular arrangement, the middle column should show the chainage at which the cross section is taken, the height of the telescope axis above the ground, and the elevation of the instrument station, if it is already known The adjacent columns to right and left are reserved for the vertical angles observed on the corresponding sides of the section, the entries being marked plus or minus, or *e* or *d*, to distinguish elevations and depressions In the remaining columns on either side are booked the staff readings, with the corresponding distances written below If sketches are preferred, they may be made on the lines of Fig. 194, with the dimensions entered in place Otherwise, use is made of a survey field book with two rulings down the middle of the page, and in the space between them the chainage, height of axis, etc , are entered as in the tabular method The vertical angles are figured prominently on the sketches on either side

To plot the section, the reduced level of the centre point is set up from the datum, and from it is scaled the height of the instrument axis above the ground From a horizontal line through the point obtained, the line of sight is plotted by protractor The several distances are then marked off along the line of sight, and from those points verticals equivalent to the staff readings are measured down A continuous line through their lower ends represents the ground line.

Cross Sectioning by Clinometer. -The Abney or other pattern of clinometer may be used in place of the theodolite This is the most rapid means of cross sectioning, and the results are sufficiently accurate for many purposes The clinometer is much used in location surveys

Three systems of observation are shown in Fig 195

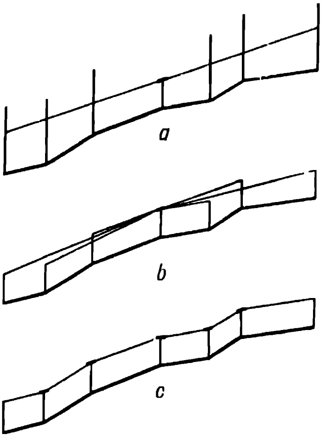

Fig. 195

(*a*) The observer, standing over the centre point, holds the clinometer at a known height above the ground with the line of sight roughly parallel to the ground The angle on each side is noted, readings are taken on a levelling staff or ranging pole, and slope measurements are made as in using the theodolite

(*b*) The observer at the centre sights a mark on a ranging pole at the same height as the instrument is held above the ground The various angles and slope distances are noted

(*c*) The angle and length of each slope are separately measured, the observer using method (*b*) and proceeding along the section.

Of these methods, the first is the most rapid in observation, booking, and plotting, and is to be preferred unless the ground is so irregular that the required staff readings cannot be obtained from two lines of sight In this case the other methods are preferable Method (*c*) is likely to prove slower than (*b*), but it is useful if the sections are long, as it avoids lengthy tape measurements If, in the course of using method (*a*) or (*b*), a hump occurs on a section, so that the sighting cannot be completed from the centre point, it becomes necessary to move the instrument to a point on which an observation has already been taken in order to continue the section beyond the summit Circumstances may warrant the combination of two or more methods on one section.

The booking is performed in columns or with the aid of sketches, as described for theodolite sectioning Sketches are useful in cases where the instrument has to be moved from the centre station, since the points from which the angles are observed can be indicated clearly The plotting in the case of method (*a*) has been described above In the other methods it is, of course, unnecessary to draw the lines of sight, the points on the section being plotted directly from the angles and distances.

Notes — (1) If the clinometer is supported against a ranging pole, it is well to hold the latter upside down, as otherwise the pointed shoe will enter the ground a variable distance An Abney clinometer may be attached to the pole by means of a rubber band Alternatively, a wooden rod is sawn to such a length that, on placing the instrument on top of it, the line of sight is 5 ft or other convenient distance above the ground The sighted rod, used in methods (*b*) and (*c*), is cut to this dimension, so that it is a little longer than the other

(2) The observer may prefer not to have to carry a support for the clinometer, and it is only necessary for him to know the height of his eye when standing erect

(3) If a levelling staff is used in method (*a*), it should have an open, bold graduation If readings are estimated on a long ranging pole, care is required to avoid mistakes of a whole foot.

Setting Out Levels.—The levels to which work is to be built may be shown by driving a peg or making a mark either at the desired level or at a stated distance above or below it In the former case, the required staff reading when the foot of the staff is at the correct level must first be deduced, and the staff is raised or lowered until this reading is obtained Instrument height booking will be found the more convenient for such work In the latter method, an arbitrary point is established, and the staff is read upon it, the difference between its level and that of the construction being communicated to the foreman There is less likelihood of mistakes on the part of workmen if this distance is an exact number of feet.

Notes —(1) The leveller must be constantly on his guard against mistakes and errors, as these may have very serious consequences in setting out. The work must always be well checked against mistakes.

(2) In setting pegs to a required level, time is wasted by driving a peg too far, as it may have to be removed and driven afresh The latter stages of the driving must be performed with caution, and the reading observed at frequent intervals The staffman should be told after each reading by how much the peg still requires to be lowered If, however, a peg is only a small fraction of an inch too low, a nail may be driven in it to the correct level to save time.

Example —An engineer on works is required to give a number of levels. He observes a backsight on a private bench mark (P B M) of elevation 297 34, and reads 4 06 He then takes a reading of 5 62 at a change point, and transfers the instrument to a position from which he can see the work. His new backsight is 3 81. He is required (*a*) to give a mark at elevation 295·60, (*b*) to correct a peg which is roughly indicating a height of 5 ft. above the bottom of an excavation to be taken out to level 291 00, (*c*) to check the finished level of different points of a concrete foundation which is intended to be at 293 75 In the second case he reads 3 45 when the staff is held on the peg, and in the last his readings are 5 88, 5 86, 5 85, and 5 89

The height of instrument from which the required observations are taken is 297 34+4 06−5 62+3 81=299 59

(*a*) To set out a level of 295 60, the staff must be adjusted until the reading obtained is 299 59−295 60= 3 99, the mark being made at the bottom of the staff

(*b*) The elevation of the peg is 299 59−3 45=296 14 As the peg is intended to be at 296 00, it must be lowered by 0 14 ft, or until a reading of 3 59 is obtained with the staff held on it

(*c*) If the foundation were at the correct level of 293 75, each staff reading would be 299 59−293 75=5 84 The work is therefore too low by ½ in, ¼ in, ⅛ in, and ⅝ in at the respective points

Levels given in this way can be transferred within a limited distance by the workmen by the use of a straight edge and small spirit level This is also accomplished by means of T-shaped crossheads or boning rods, which furnish a line of sight whereby, from two given pegs, points at the same level or on the same gradient may be established in their line In Fig 196, A and B are pegs set on a particular gradient and, say, 100 ft apart The foreman, holding a boning rod on A and looking along the top of it at the top of a similar rod held on B, can direct the adjustment of points such as C and D into the gradient by judging when the upper surfaces of the boning rods held upon them are in the line of sight

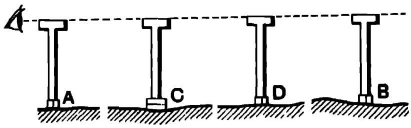

Fig 196

Setting Slope Stakes.—See page 355.

SOURCES OF ERROR IN ORDINARY LEVELLING

Numerous sources of error may affect the accuracy of a line of levels, but the precautions against them are of a simple nature, so that it is not difficult to obtain good results without delaying progress It is to be understood that the precautions detailed below refer more particularly to the observation of backsights and foresights, since an error introduced at a change point is carried forward throughout the subsequent work An error in an intermediate sight affects the recorded level of that point only, and may not prove of much consequence, although it should be recognised that some intermediate sights may be of great importance, especially in engineering work The various errors and mistakes may be classified as ·

(1) Instrumental Errors.
(2) Errors and Mistakes in Manipulation.
(3) Errors due to Displacement of Level and Staff
(4) Errors and Mistakes in Reading.
(5) Mistakes in Booking
(6) Errors due to Natural Causes

1. Instrumental Errors.—*The Level* —The testing and adjustment of the instrument have been discussed in Chapter I The important desideratum is that the line of sight should be exactly parallel to the level tube axis, so that the line of sight is horizontal when the bubble is at the centre of its run The error introduced by non-adjustment is proportional to the length of sight, and is entirely eliminated between change points by equality of the backsight and foresight distances, but intermediate sights, being usually of various lengths, will be thrown into error by different amounts Errors arising from imperfect estimation of the equality of backsight and foresight distances are compensating, but if the backsights are consistently longer or shorter than the foresights—a tendency to be guarded against on steep slopes—the error becomes cumulative

A defective level tube may have a considerable influence If under-sensitive, the bubble may apparently come to rest in the central position although the tube axis is not horizontal, this giving rise to a compensating error On the other hand, over-sensitiveness in an instrument for ordinary use leads to waste of time in levelling up Irregularity of curvature of the tube is a serious defect, the influence of which will, however, tend to compensation with equal sights

The tripod should be examined and loose joints tightened, as instability of the instrument causes waste of time, and leads to erroneous readings

The Staff —It is advisable to test the graduation of a new staff by a steel tape or foot-rule, but the error is likely to be negligible in ordinary work There is greater probability of error through wear of the staff at the joints or by dirt adhering to hinges or sockets A telescopic staff should be let down gently to minimise wear, and in using the Scotch staff, the ends of the separate pieces should be kept clean to enable them to be pushed firmly home in the sockets It is also important that the separate parts should all belong to one staff, otherwise very serious errors of graduation may occur at the joints

It may be noticed that wear at the bottom of a staff is of no consequence, since it is unnecessary that the zero of the graduation should be placed at the foot of the staff in order that differences of staff readings may represent differences of level An exception occurs, however, in obtaining the difference of level between a point below the plane of sight and one above, the staff being held inverted on the latter point In this case, the difference of level is the sum of the two staff readings, and an error would be produced equal to twice the distance between the zero of the graduation and the foot of the staff

2. Errors and Mistakes in Manipulation.—*The Level* —The most serious and common mistake in observing is the omission to have the bubble central at the instant of sighting In the instruments illustrated in Figs. 91, 93, 100, and 102 the observer obtains, by reflection, a view of the bubble while he is reading the staff In using a level without a reflector, he should examine the bubble before sighting, and bring it central if necessary After reading, he should again glance at the bubble, and, if correct, it may be assumed that it remained so during the observation After the instrument is levelled, it should not be handled unnecessarily The tripod should not be grasped, and, in turning the focussing screw or wheeling the telescope, the application of vertical pressure should be avoided.

The Staff —The staff should be held quite vertical If held off the plumb, it will be intersected by the line of sight farther from the foot than it should be, and the reading will be too great (Fig 197) As the errors caused by a given deviation from the vertical are proportional to the readings, special care must be taken with large readings Errors are avoided (*a*) by having a spirit level (Fig 304) or a pendulum plumb bob attached to the staff, to facilitate holding it plumb, or (*b*) by swinging or waving the staff.

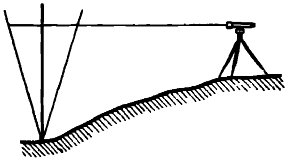

FIG 197

The latter is a most useful and simple method The staffman, holding the staff on the point in the ordinary manner, inclines it slowly towards and away from the instrument, on both sides of the vertical The observer sees the reading vary against the horizontal hair, but the smallest reading corresponds to the vertical position, and is that noted Waving should be performed in the direction of the line of sight only, as the leveller can detect lateral non-verticality by means of the vertical hair or hairs It is unnecessary and inadvisable to swing the staff if the reading is below about 3 ft , since the bottom of the graduated face is raised appreciably off the ground when the staff is leaning away from the instrument This difficulty could be overcome by fitting the bottom of the staff with a knife edge or pin placed in the plane of the graduated face, but such a design, although convenient for a precise levelling staff supported on a peg or a plate, is unsuitable for use in ordinary levelling

Errors due to non-verticality of the staff tend to compensate at change points, but, if the backsight readings are consistently greater or smaller than the foresights, the error becomes cumulative Thus, in levelling up a slope the observer will read well up the staff in taking backsights and near the bottom for foresights. Careless staff holding increases the former without appreciably affecting the latter, and, in consequence, too great a rise is recorded between change points, and the slope appears steeper than it really is. In levelling downhill, the foresights are the larger readings, and their increase makes the fall between change points appear too great, so that the slope is again exaggerated In levelling over a hill, therefore, the error accumulated in working up one side is more or less completely neutralised in descending the other side, and the levels may check at the finish, but the observed elevation of the hill is too great

A further precaution to be observed by the staffman is to see that dirt does not accumulate on the foot of the staff, as this would cause a variable relationship between the zero of the graduation and the foot of the staff

3. Errors due to Displacement of Level and Staff.—*The Level* —If the instrument is set up on soft ground, it may gradually settle from the moment of the backsight observation to that of the foresight This will always make the foresight reading smaller than it should be, giving too great a rise or too small a fall between change points The error is cumulative, as every settlement of the instrument increases the reduced level of all subsequently observed points by the amount of the sinkage It follows that the level should, as far as possible, be placed upon solid ground with the legs thrust firmly into the ground, and that time should not be wasted between the backsight and foresight observations If the engineer must plant the instrument upon staging, he should avoid treading on the planks which support it

It sometimes happens that the level is disturbed by the tripod being accidentally kicked, but, as the mishap will be noticed, no error need result If the positions of change points have been marked, it is only necessary to relevel the instrument and again backsight on the last change point, substituting the new reading for the previous one Any intermediate points taken prior to the dislevelment must have their readings correspondingly altered, or may be observed anew If change points are not marked, it is necessary to return to the start or to the first change point which can be identified with certainty

The Staff —A serious and common error is that occasioned by change of level of the staff at a change point while the instrument is being carried forward It is commonly caused by choosing unsuitable turning points Soft ground should be avoided owing to the probability of the staff sinking between the foresight and backsight observations A flat stone embedded firmly in the ground makes a good support If only irregular or rounded boulders are available, the staff should be held on the highest point as at *a* (Fig 198) if held as at *b*, it is difficult to maintain the foot on the point while turning the staff to face the new instrument station The use of a peg or a foot-plate as a support in soft ground prevents sinkage Having selected a suitable point, the staffman should first mark the spot with chalk, and should keep holding the staff upon it until the backsight observation is completed

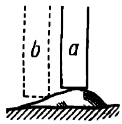

Fig 198

Since any change of level of the staff will nearly always be in the direction of sinkage, the error is cumulative The backsight reading on the settled staff will be too great, and the reduced levels of all subsequently observed points made too high

4. Errors and Mistakes in Reading.—Small compensating errors occur in the estimation of the decimal part of the readings The increased size of the image makes estimation easier with short sights than with long ones, and it is desirable that important sights should not exceed about 300 ft , but this limit depends upon the quality of the telescope as regards resolving power, and also upon the character of the staff graduation and the clearness of the atmosphere Focussing must be carefully performed to eliminate parallax (page 25) The observer should keep moving his head up and down while sighting, and should adjust the focussing screw until no apparent movement of the horizontal hair relatively to the staff can be detected

In sighting an openly graduated staff, it is sometimes difficult to choose between two possible readings differing by ·01 ft The smaller should be preferred owing to the possibility of non-verticality, especially if the staff is not swung

Common mistakes made by beginners are (*a*) reading upwards, instead of downwards , (*b*) reading against a stadia hair , (*c*) concentrating the attention on the decimal part of the reading, and noting the whole feet wrongly , (*d*) omitting the zero from decimals under ·10 , (*e*) reading downwards, instead of upwards, when the staff is inverted

5. Mistakes in Booking.—These include (*a*) entering a reading in the wrong column , (*b*) omitting an entry , (*c*) noting a reading with the digits interchanged , (*d*) entering the wrong distance or remarks opposite a reading

A fruitful source of erroneous booking occurs when the end of a line is reached and check levelling is to be carried back along it, as the last change point may be used twice Thus, in Fig 199, from A, the last position of the instrument for the forward levelling, the reading on the change point B is first observed, and entered as a backsight When the intermediate readings on C, D, and E have been noted, the reading on B is again observed, and must be entered as a foresight Uncertainty is avoided by keeping in mind the definitions of the terms, backsight and foresight (page 209)

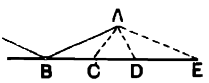

FIG 199

To detect mistakes in writing down readings, the best method is to read the staff, book the reading, and then sight the staff again to see that the figure booked is the correct reading

6. Errors due to Natural Causes.—*Wind and Sun.*—(See page 172) If levelling must be performed in a high wind, an endeavour should be made to shelter the instrument, and high readings should be avoided owing to the difficulty of holding a long staff sufficiently steady and plumb

On hot sunny days the apparent vibration of the staff caused by irregular refraction makes close reading impossible, and, as a partial remedy, the lengths of important sights should be reduced. Distortion of the instrument by unequal heating and expansion or contraction of the staff produce errors which are negligible in ordinary work

Curvature and Refraction —In consequence of the curvature of the earth, the point read on the staff is not strictly at the same level as the horizontal hair of the reticule, since the line of sight is not a level line In the observation from A to a staff BC (Fig 200), a difference CD is developed between the horizontal and level lines through the instrument If the line of sight coincides with AC, the graduation C observed is the distance CD above the instrument height, and the reduced level of B is made out to be lower than it really is by this amount

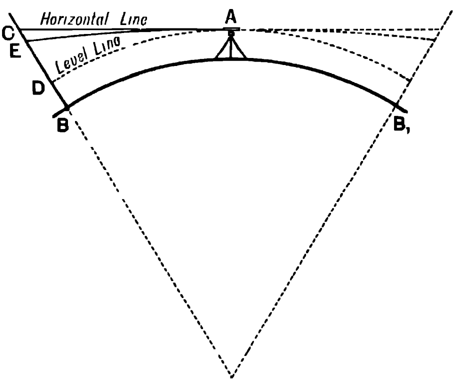

FIG 200

Denoting the length of sight by L, then, by geometry,

$$CD = \frac{L^2}{\text{Diameter of Earth}}$$

If L is in miles, $CD = 667\ L^2$ ft

Actually, the line of sight is not straight, but, in consequence of the refraction of light in passing through layers of air of different densities, is, in general, a curve concave towards the earth It is represented by AE, so that the graduation at E is that actually read Under normal atmospheric conditions, arc AE may be taken as circular and of radius seven times that of the earth The effect of refraction is therefore $\frac{1}{7}$th that of curvature, but is of opposite sign, so that the combined error,

$$ED = \cdot 57\ L^2 \text{ ft },$$

by which amount the point sighted is made out lower than it really is

For ordinary lengths of sights the error is very small It is eliminated by equality of backsights and foresights, *e g* between B and B_1 (Fig 200), assuming the atmospheric conditions the same for both sights, but will accumulate if the backsights are consistently longer or shorter than the foresights. The error is also eliminated by the method of reciprocal levelling (see below).

Allowable Closing Error.—The total error developed in a line of levels is composed partly of cumulative and partly of compensating error, but the more refined the procedure becomes, the more nearly will the former be eliminated. In ordinary levelling, adherence to the usual precautions, especially as regards equality of backsight and foresight distances and stability of change points, prevents the propagation of serious cumulative error, and experience shows that the closing error of ordinary careful work may be taken as obeying the law of accidental error. The total error is therefore proportional to the square root of the number of instrument stations. As the number of stations per mile will not vary greatly for a particular kind of country, the error E, in feet, developed in a distance of M miles may be expressed as

$$E = C\sqrt{M},$$

where C is a constant depending upon such circumstances as the observer's experience, the quality of the instruments, the character of the ground, and the atmospheric conditions.

For ordinary levelling on moderately flat ground, $E = \cdot05\sqrt{M}$ represents good work, and is not difficult to attain under favourable conditions. $E = \cdot10\sqrt{M}$ represents fair accuracy under the same conditions, but is regarded as satisfactory on steep ground, and is quite sufficient for many purposes.

Reciprocal Levelling.—When the difference of level between two points has to be determined under conditions necessitating considerable inequality between the sights, the effects of collimation error, as well as of curvature and refraction, may be eliminated by reciprocal levelling. The routine involves two sets of observations yielding two erroneous differences of level, the mean of which is the true result.

Thus, to ascertain the difference of level between A and B (Fig. 201), on opposite banks of a wide river and remote from a bridge,

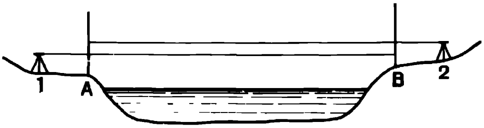

Fig. 201

readings on a staff held on each point are taken from instrument station 1. The instrument is then transferred to position 2, so that 2B is equal to 1A, and the staff is again observed on A and B. If, due to the combined effect of instrumental error, curvature, and refraction on the long and short sights, the difference of level between

A and B, as determined from one of the instrument stations, is made too great, the other determination evidently makes it too small by the same amount. The mean of the two differences of level so obtained is therefore the required difference

Note —In levelling of the highest precision 'it would be recognised that, while the elimination of instrumental errors and the error of curvature is complete, that of the refraction error is not so, owing to the possibility of change of atmospheric conditions during the transfer of the instrument (see Vol II, Chap V)

Expedition in Levelling.—The rate of progress possible depends greatly upon the character of the ground If a line of levels intersects deep wooded gorges, the necessity for numerous instrument stations on the slopes, combined with difficulty of sighting, makes progress much slower than in open country. Attention to the following items prevents unnecessary delay

(1) The surveyor should endeavour to select instrument stations from which he will be able to command as much ground as possible, and in particular should avoid setting the instrument too high or too low to read the backsight

Note To test whether the staff can be sighted, plant two of the legs, point the telescope, and bring the bubble roughly to the centre of its run by moving the third leg, retained in the hand A glance through the telescope will then show if the instrument is at a suitable height It is better if the surveyor, while looking through the telescope, is warned by a chainman when the bubble is central

(2) To avoid delays arising from mistakes on the part of an inexperienced staffman, he should be warned of the importance of his share of the work, particularly with regard to change points, and should be instructed not to remove the staff from a point until signalled to do so

(3) Misunderstandings are largely obviated by using a code of signals, such as

A quick upward movement of the right hand—Observation completed

Both hands above the head—Hold on a change point

Right hand waved up and down—Swing staff

Right (left) hand up and moved to left (right), and left (right) hand down and moved to right (left)—Plumb staff as indicated

Right (left) arm extended—Move staff to right (left)

(4) It is sometimes permissible to adopt expedients which are not allowable when the best results are required Thus, when the staff, held for an intermediate sight, is a little below the line of sight, it can be raised a foot or two off the ground against the divisions of a ranging pole, the reading booked exceeding the staff length If a high wall has to be crossed, a circuitous route may be avoided by continuing the levels across with the aid of taping Levelling may be carried across a sheet of water by taking advantage of the fact that the surface of still water is level. The results

are quite good if the obstruction is a still pond or lake, but appreciable error may be introduced in the case of a river, unless the points on the water surface are directly opposite and on a straight reach with a symmetrical channel

EXAMPLES

1 The following staff readings were observed successively with a level, the instrument having been moved forward after the second, fourth, and eighth readings

1 67, 8 32, 4 05, 1 39, 9 83, 4 70, 5 45, 1 23, 7 36, 2 87, 5 43

The first reading was taken with the staff held upon a bench mark of elevation 75 40. Enter the readings in level book form, and reduce the levels

2 A line of levels has been run from a bench mark of elevation 63 47, and ends on one of elevation 63 50 The sum of the backsights is 49 26, and that of the foresights is 49 29 What is the closing error of the work ?

3 Starting from a bench mark of elevation 123 46, a surveyor runs a longitudinal section in the course of which he takes an intermediate sight of 4 42 with the staff held on a bench mark of elevation 158 60 The sum of the backsights from the start to this point is 120 74, and that of the foresights is 81 13 What is the error of closure on the second bench mark ?

4 It was required to ascertain the elevations of two points, A and B, and a line of levels was run from A to B The levelling was then continued to an Ordnance bench mark of elevation 127 30, the readings obtained being as shown Obtain the reduced levels of A and B

B S	I S	F S	R L	Remarks
3 92				A
1 46		7 78		
7 05		3 27		
	2 36			B
4 81		0 85		
8 63		2 97		
7 02		3 19		
		4 28	127 30	O B M

5 Starting from a point A of elevation 303 46, levels were taken for a section which extended to a point X, the reduced level of which was found to be 322 00 Check levels were carried back along the shortest route from X to A, the readings being given below Find the error of closure on the starting point

B S	F S	Remarks
3 68		X
2 43	10 02	
5 90	9 17	
8 16	1 23	
2 39	4 05	
5 97	11 12	
2 36	6 28	
	7 51	A

6 The following staff readings were observed at a road bridge over a railway for the purpose of measuring the clearance between the rails and the girder on the side at which rail level is the higher The readings marked + were taken with the staff held inverted on the underside of the girder Reduce the levels, and state the minimum vertical distance between the girder and a rail.

B.S.	I.S.	F.S.	R.L.	Remarks
1.22			68.40	O.B.M.
0.54		11.45		
	7.92			Rail *a*
	+5.99			Girder above *a*
	8.07			Rail *b*
	+5.94			Girder above *b*
	7.94			Rail *c*
	+5.88			Girder above *c*
	8.06			Rail *d*
	+5.83			Girder above *d*
12.16		0.32		
		2.15		O.B.M. 68.40

7. Find the error of reading of a level staff if the observed reading is 12.00 ft. and at the point sighted the staff is 6 in. off the vertical through the bottom.

If the bubble tube of the level has a sensitiveness of 20 seconds per 2 mm. division, find the error in the staff reading at a distance of 300 ft. caused by the bubble being one division out of centre.

8. The following level book figures show the readings observed on the first few of a series of pegs 100 ft. apart from which the levels of a sewer are to be worked. The reduced level of the invert of the sewer at A will be 90.50, and it will fall with a gradient of 1 in 300 from A. Draw up a list of the depths from the several pegs to invert level, giving the measurements to the nearest $\frac{1}{8}$ in.

B.S.	I.S.	F.S.	R.L.	Remarks
5.71			89.43	O.B.M.
7.16		2.85		
3.09		1.93		
	4.21			Peg A
	5.63			B
	6.07			C
	4.83			D
	5.78			E
3.51		4.62		F
	5.58			G

9. A gradient of 1 in 120 falling from elevation 202.34 was set out by driving pegs at 100 ft. intervals with the tops of the pegs on the required gradient. After a time it was suspected that some of the pegs had been disturbed, and the following observations were taken in checking their levels. Draw up a list of the errors of the pegs.

B.S.	I.S.	F.S.	R.L.	Remarks
5.28			192.40	O.B.M.
7.93		2.17		
5.89		4.25		
	2.80			Peg 1
	3.59			2
	4.49			3
	5.27			4
	6.08			5
	6.91			6
2.07		7.78		7
	2.86			8
	3.70			9
	4.55			10
1.84		3.64		
		5.17		O.B.M. 192.40

10 *A*, *B*, and *C* are successive points of change of gradient on a portion of an existing railway, the positions and rail levels of these points being

	Distance in chains	Rail Level
A	2,032 00	386 34
B	2,095 00	439 65
C	2,171 00	462 45

In a regrading scheme it is proposed to raise rail level at *A* by 4 50 ft, and to improve the existing 1 in 78 gradient from *A* to 1 in 100 Calculate the greatest change of rail level which will be involved and the chainage of the point where the new gradient meets the existing gradient between *B* and *C* (T C D., 1928)

11 The following readings were taken along the centre line of a length of road

B S	I S	F S	R L	Distance in feet
12 32			252 10	0
	12 80			100
	7 95			200
11 67		0 68		300
	5 80			400
5 06		1 55		500
	2 90			600
	2 15			700
	3 05			800
	3 30			900
12 35		0 42		1,000
9 77		3 15		1,100
	4 20			1,200
	1 05			1,300
8 49		1 40		1,400
		3 45		1,500

It is proposed to improve this portion of road by regrading it to a 1 in 30 gradient passing through the existing surface at 1,100 ft on the section Calculate the greatest difference of level between the new and existing surfaces which will occur in excavation and in embankment

12 Two pegs, A and B, are driven into the ground about 300 ft apart A dumpy level is set up near A and levelled, and observations are taken upon a staff held on A and then on B, giving the readings

on A, 5 18, on B, 5 53.

The level is then placed near B, and sights of the same lengths as before are taken with the staff held on the same points The readings are now

on A, 5 31, on B, 5 26

State whether the instrument is in adjustment or not If the reduced level of peg A is 100 00, what is that of B ?

CHAPTER V

PLANE TABLE SURVEYING

PLANE tabling is a method of surveying the peculiar feature of which is that the field observations and plotting proceed simultaneously. It is admirably adapted for the survey of detail between theodolite stations, and is extensively employed for recording topography in government and engineering surveys

The plane table and the various ways in which it is used in surveying are dealt with in this chapter Special features of plane table surveying when applied to the small scale mapping of extensive areas are described in Vol II, Chap. VI

THE PLANE TABLE

The plane table (Fig 202) consists of a drawing board, which carries the sheet, and is mounted on a tripod in such a way that the board can be (*a*) levelled, (*b*) rotated about a vertical axis and clamped in any position A sight rule, or alidade, which is simply a straight-edge carrying a line of sight, is used for sighting objects to be located and for recording on the paper the directions in which they lie. The parts may be considered under four heads :

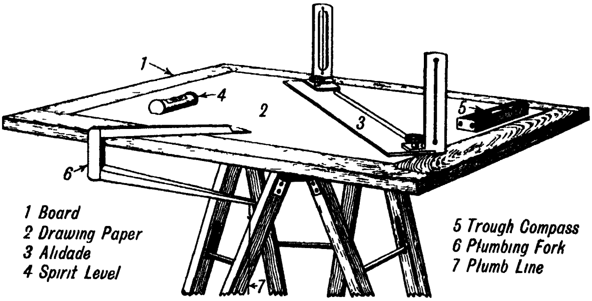

FIG 202 —PLANE TABLE

(1) The Board.
(2) The Tripod and Horizontal Movement.
(3) The Alidade.
(4) Accessories.

The Board.—The board, ranging in size from about 15 in × 15 in to 30 in × 24 in, is made of thoroughly seasoned wood, and is constructed in such a manner as to counteract the effects of warping as much as possible. The upper surface must be plane, and parallel to the surface on which the horizontal movement is made

The Tripod and Movement.—The tripod is generally of the open frame type, combining rigidity with lightness, and may be made to fold for convenience of transportation

In the simplest and lightest forms of plane table, levelling of the board is effected simply by manipulation of the tripod legs Otherwise a levelling screw head or ball and socket joint is fitted Heads differ in detail as regards the manner of permitting horizontal movement of the board about a vertical axis of rotation, but the fitting must afford plane motion without play, and should be as light as is consistent with rigidity A simple form with levelling screws is illustrated in Figs 203 and 204 The board is rotated about the axis *1*, and can be clamped in any desired position by the nut *2* The underside of the board either bears directly upon the triangular frame or is fitted with a flat ring of brass to give a smoother motion

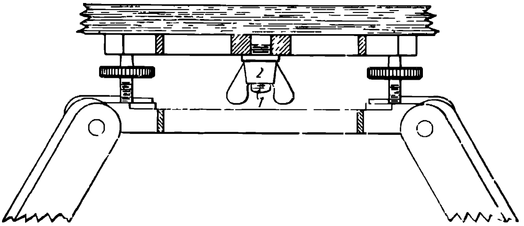

FIG 203 — SIMPLE MOVEMENT—SECTION

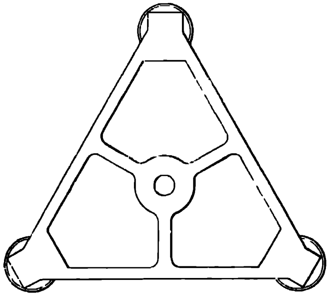

FIG 204 —SIMPLE MOVEMENT—PLAN

Fig. 205 shows the Gurley ball and socket movement The ball *1*, supporting the board, fits into the socket *2*, which is screwed to the head of the tripod The two spherical surfaces are held together by the spiral spring *3* acting on the clamp piece *4*. When the board is levelled, it is clamped by the nut *5*, but can be rotated horizontally on the flange of *1*, and, when oriented, is secured in position by the screw *6*

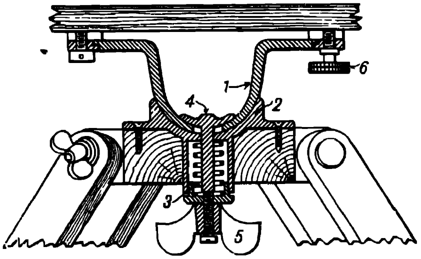

Fig 205 Gurley Movement

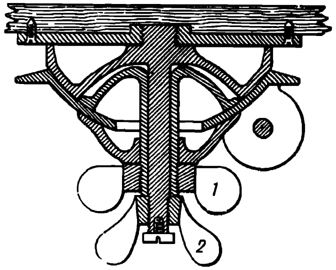

Fig 206 —Johnson Movement

The Johnson movement is illustrated in Fig 206 Levelling is performed as in the last case, and the table is clamped in the horizontal position by the nut *1* On releasing the nut *2*, the board can be rotated about the now vertical axis, and, when oriented, is fixed by tightening the same nut

Fig. 207 shows the United States Coast and Geodetic Survey movement Levelling is performed by means of three foot screws, and stability is promoted by supporting the table on a wide V-shaped annular surface A clamp and tangent screw are fitted

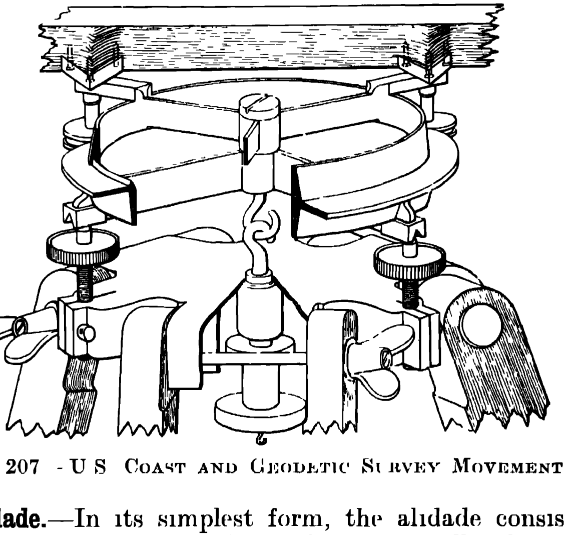

Fig 207 - U S Coast and Geodetic Survey Movement

The Alidade.—In its simplest form, the alidade consists of a wooden or brass ruler of length equal to the smaller dimension of the board and furnished with a pair of sights (Fig 202) One of the vanes is provided with a narrow slit, while the other is open and carries a hair or wire It is essential that the plane of the sights should be perpendicular to the under surface of the ruler, but it is unnecessary that the line of sight should be in the same vertical plane as, or even parallel to, the fiducial or ruling edge of the alidade, provided the horizontal angle between the two lines remains constant The sights are commonly placed in the vertical plane of the fiducial edge, but they are sometimes mounted midway across the breadth of the ruler, and either edge can be used in drawing The alidade plate is sometimes constructed as a parallel ruler with the straight-edge used in drawing connected by two links to the base plate The ruler may be moved out from and parallel to the latter, and, in consequence, rays can be drawn through a plotted point without the alidade being set exactly against it

The alidade with plain vanes provides a sufficiently definite line of sight for many purposes, but, as ordinarily constructed, it is somewhat inconvenient for work in hilly country owing to the limited range of inclination possible in the line of sight By stretching a thread tightly between the centres of the tops of the vanes, highly inclined sights can be taken

The accuracy and range of sighting are considerably increased by the use of the telescopic alidade, and, if the telescope is adapted for tacheometry by the provision of stadia hairs and a vertical circle or arc, the scope and utility of the plane table are greatly enhanced The telescope is usually equal to that of a small theodolite, and is mounted on a short horizontal axis on which it can be rotated, in some forms completely, and in others only partially, but no lateral motion relatively to the ruler is possible. The details of construction of telescopic alidades vary considerably The horizontal axis may be overhung from one support (Fig. 208), so that the telescope can be transited, or the axis may be mounted on two short standards at the top of a solid column, which serves as a convenient handle.

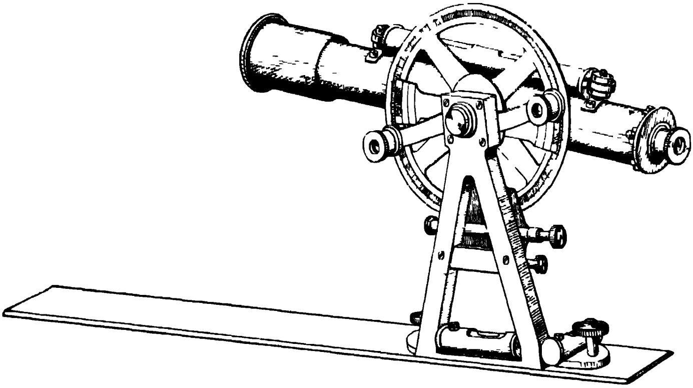

FIG. 208 —TELESCOPIC ALIDADE

The requirements of the telescopic alidade with respect to the levels, the line of sight, and the horizontal axis are similar to those of the theodolite

Accessories.—The remaining features of the instrument are · (1) a spirit level (if not fitted to the alidade) , (2) a trough compass , (3) a plumbing fork , (4) a waterproof cover

The spirit level need not be very sensitive The table is levelled by placing it on the board in two positions at right angles and getting the bubble central in both positions A circular level is quite suitable

The trough compass is commonly about 6 in long, and is used when orienting the table to magnetic meridian and to facilitate orienting to any other meridian

The plumbing fork (Fig 202) is used in large scale work for setting the table so that the point on the paper, representing the instrument station being occupied, may be brought vertically over the mark on the ground When the table is level, the point of the leg resting on the board is vertically over the centre of the plumb bob The point of the leg is therefore placed at the plotted station point, and the position of the table is adjusted until the plumb bob hangs over the ground mark Since the plane table is seldom employed for large scale work, the use of a plumbing fork is unnecessary in the great majority of plane table surveys

The cover is used to protect the drawing from the effects of a shower

Paper.—For other than rough work, the quality of the paper should receive attention Paper is very sensitive to changes in the humidity of the atmosphere, which produce expansion and contraction of different amounts in different directions, and not only alter the scale, but distort the map It has been found that if two sheets are mounted with their grains at right angles and with a sheet of muslin between, the effects of a change of atmospheric conditions are practically negligible Such double-mounted sheets are sold by some dealers If a single-mounted or an unmounted sheet is to be used, it may previously be seasoned for about a week by exposing it alternately to a damp and a dry atmosphere, such treatment reducing its tendency to distort For work in damp climates, sheets of celluloid and of zinc have been used

Attachment of Paper to Board.—The paper must be held very firmly on the board to prevent the possibility of any displacement by the friction of the alidade Ordinary drawing pins may work loose, and the projecting heads interfere with the placing of the alidade The principal devices used are .

(1) Pasting down the edges of the paper, or having the linen sheet, on which the paper is mounted, larger than the board, and pasting the margin to the underside of the board.

(2) A wooden frame fitting tightly round the board, the upper surface being flush with that of the board when the paper is clamped (Fig 202)

(3) Two rollers carrying the paper, one on either side of, and below, the board

(4) Screw clamps round the sides of the table

(5) Screw tacks entering hollow brass screws in the board When the tacks are screwed in, and hold the paper, their heads are flush with, or a little below, its surface

(6) Spring clips round the sides of the board

Of these methods, the first is probably the best

TESTING AND ADJUSTMENT OF THE PLANE TABLE

The Board.--(1) *The upper surface of the board should be a perfect plane*

Test --Apply a straight-edge in all directions

Adjustment —If necessary, reduce high parts by planing or sand-papering

(2) *The upper surface of the board should be perpendicular to the vertical axis of the instrument*

Test —Place a spirit level on the table, and bring the bubble to the centre of its run Wheel the table through 180° Place the level at 90° to its previous position, and repeat If the bubble remains central on reversal, the adjustment is correct

Adjustment —If not, correct half the apparent error by rubbing down or packing between the underside of the table and its support

The Alidade.—(1) *The fiducial edge of the ruler should be a straight line*

Test —Draw a fine line along the edge Reverse the ruler, place it against the ends of the line, and again draw a line If the two lines coincide, the ruler is true, except in the very improbable case when one part is convex and the part equidistant from the other end has a concavity of exactly the same size and shape

Adjustment —If the two lines do not coincide, bring the edge true by repeated rubbing and testing

(2) *The alidade spirit levels should have their axes parallel to the base of the ruler*

Test.—Place the alidade on the table, and bring the bubble of either level central by the levelling screws of the table Mark the position of the ruler, lift and reverse it through 180°, and replace it within the marks If the bubble remains central, the adjustment is correct

Adjustment --If not, eliminate half the error by the level adjusting screws and the remainder by the levelling screws, and repeat until the test is satisfied Examine and adjust the second level tube in the same way

If the spirit level is not attached to the alidade, the test by reversal is performed in the above manner, but there may be no provision for adjustment

(3) *The sights of the plain alidade should be perpendicular to the base of the ruler*

Test—Having levelled the table, observe whether the sighting slit and hair appear parallel to a plumb line suspended in front of the instrument Alternatively, test them with a set-square held on the board

Adjustment—If wrong, adjust by filing or packing the base of the sights

The Telescope.—(1) *The line of sight should be perpendicular to the horizontal axis of the telescope*

Proceed as in adjustment 2 of the transit or wye theodolite, according to the manner in which the telescope is mounted (pages 86 and 93)

(2) *The horizontal axis should be parallel to the base of the ruler*

Proceed as for adjustment 3 of the transit theodolite (page 88), if there is provision for making the adjustment

(3) *The axis of the telescope level should be parallel to the line of sight*

Proceed as in the "two peg" adjustment of the dumpy level (page 116)

The Index Frame.—*The vertical circle should read zero when the line of sight is horizontal*

Proceed as for adjustment 5 of the transit theodolite (page 90)

FIELD WORK

Advantages and Disadvantages of Plane Tabling.—*Advantages.*—

(1) Since the map is plotted in the field, there is no danger of omitting necessary measurements

(2) The correctness of the plotted work can be readily verified by check observations

(3) Notes of measurements are seldom required, and the possibility of mistakes in booking is eliminated

(4) By virtue of the foregoing, no great skill is required to produce a satisfactory map, and the work may be entrusted to a subordinate

(5) The method is particularly useful and rapid when detail can be sketched in by estimation, as the surveyor has the ground before him, and need take only such observations as are strictly necessary, while the double work of booking sketches and reproducing them on the plan is avoided

(6) The subsequent office work is confined to finishing up the

drawing, so that, in ordinary circumstances, plane tabling proves one of the most rapid methods of surveying.

Disadvantages—(1) The advantages of the plane table are most evident in open country. It is inferior to the compass in densely wooded ground.

(2) It is unsuitable for work in a wet climate, and is awkward in high wind

(3) The plotting is trying to the eyes in a bright sun, unless the paper is shaded by an umbrella, or tinted paper or tinted eye-glasses are used

(4) The instrument is heavy and cumbersome, and the various accessories, being loose, may get lost

(5) The absence of field notes is sometimes inconvenient if quantities have to be calculated, or if the survey has to be re-plotted to a different scale

Setting Up the Table.—In setting up the table at a station, three requirements have to be met

(1) The table must be levelled

(2) The table must be oriented

(3) The point on the paper representing the station being occupied should be vertically over the point on the ground

Levelling—The legs should first be spread to bring the table approximately level and the board at a convenient height for working—preferably not above the elbow The table is then placed over the station to fulfil requirements 2 and 3 approximately, and the levelling is completed by means of the levelling screws, by tilting the board by hand if the instrument has a ball and socket head, or simply by adjusting the legs if there is no levelling head

Orientation -- The table is said to be oriented when it is so placed, with respect to its vertical axis, that all lines on the paper are parallel to the corresponding lines on the ground This is obviously an essential condition when more than one instrument station has to be occupied, as otherwise the board would not be kept parallel to itself at the various stations, and the result would be equivalent to using a different meridian at each

The manner of orienting is analogous to that of theodolite surveying in making use of a backsight Thus, if the table be set over a station B, represented on the paper by a point *b* which has been plotted by means of a line *ab* drawn from a previous station A, the orientation will consist in bringing *ba* on the paper over BA on the ground The edge of the alidade is therefore placed along *ba*, and the board is turned in azimuth until the line of sight bisects the signal A, when the horizontal movement is clamped

Orientation may also be effected (independently of a backsight) by the employment of the trough compass. At the first station, after the table has been levelled and clamped, the compass box is placed on the board—preferably outside the limits of plotting—

so that the needle floats centrally, and a fine pencil line is ruled against the long side of the box At any subsequent station the compass is placed against this line, and the table is oriented by turning it until the needle again floats centrally. The accuracy of compass orientation is dependent upon the absence of local attraction, but is suitable for work in which speed is of greater importance than accuracy The compass, however, often proves a valuable adjunct in enabling a rapid approximate orientation to be made prior to the final adjustment

Further methods of orienting the table will be considered later (page 248)

Centering—It has been assumed that *b* is set vertically over B by use of the plumbing fork, so that *ba* is brought into the same vertical plane with BA If *b* happened to lie in the vertical axis of the instrument, its position would be unaffected by the movement of the board in orienting, but otherwise *b* will be shifted relatively to the mark on the ground The operations of orienting and centering are therefore interrelated, and, if circumstances require that the plotted station point shall be exactly over the ground point, repeated orienting and shifting of the whole table are necessary Commonly, however, accurate centering is a needless refinement (see page 259)

Systems of Plane Tabling.—Methods of surveying with the plane table may be classed under four distinct heads, *viz* Radiation, Traversing, Intersection, and Resection In the figures illustrating these, points on the ground are indicated by capital letters, and the corresponding points on the sheet by the corresponding small letters, the size of the board being shown greatly exaggerated

Radiation.—Select an instrument station, O, from which all points to be surveyed are visible (Fig 209) Set up and level the

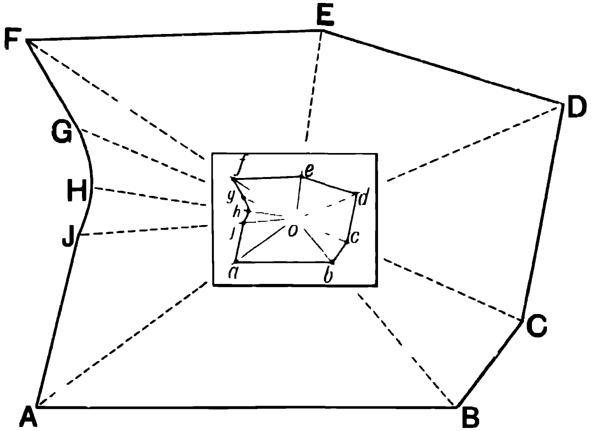

FIG 209

table, and clamp the horizontal movement. Select a point *o* on the sheet to represent the instrument station, and, with the alidade touching *o*, sight the various points A, B, etc , to be located, drawing radial lines towards them Measure distances OA, OB, etc , set them off to scale, and join the points *a*, *b*, etc , so obtained

Note – This method is of very limited application for making a complete survey, but in large scale work is useful, in combination with other methods, for surveying detail within a tape length from a station The method has a wider scope if the distances are obtained tacheometrically The work can be satisfactorily checked only by comparing distances such as AB with the corresponding plotted lengths *ab*, etc

Traversing.—This method is used for laying down the survey lines of a closed or unclosed traverse, and corresponds to theodolite traversing The detail may be located by offsets in the usual manner

Having selected a system of stations A, B, C, D, E (Fig 210), set up over one of them, say A, and, having selected *a* on the paper,

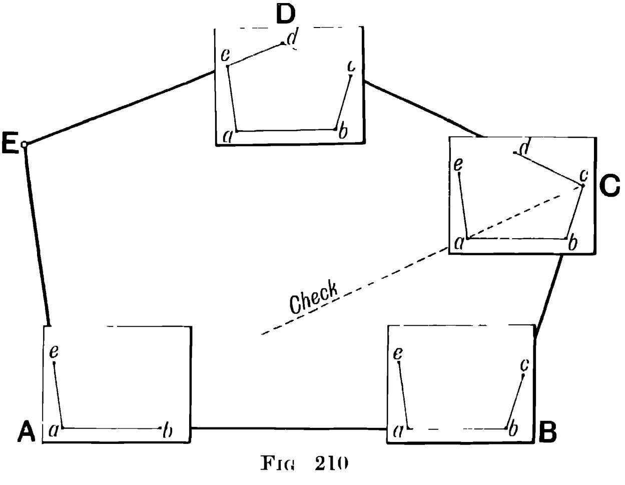

FIG 210

bring it over A Clamp the board, and, with the alidade touching *a*, sight E and B, and draw rays *ae* and *ab* Measure AE and AB, and scale off *ae* and *ab* Set up at B, with *b* over B, and orient by laying the alidade along *ba*, turning the table until the line of sight strikes A, and then clamping With the ruler against *b*, sight C, and draw *bc* to scale Proceed in this manner at the other stations, in each case orienting by a backsight before taking the forward sight

Notes —(1) The error of closure at *e* is determined at D in the case described, and E need not be occupied. As in theodolite surveying, intermediate checks should be taken whenever possible Thus, if A is visible from C, the work up to C may be checked there by sighting A with the ruler against *c* and noting if the edge touches *a*

(2) If E is not sighted from A on starting, either on account of its not being

marked, or because the traverse is unclosed, A need not be occupied by the table, *ab* simply being drawn in a convenient position on the sheet If this is done for the traverse illustrated, the closing error is found on setting up and orienting at E.

(3) Plane table traversing becomes analogous to compass traversing if each orientation is made entirely by compass This is less accurate than back sight orientation, especially if there is a possibility of local attraction Since compass errors tend to compensate, however, the method is useful for a lengthy traverse of short courses, and proves rapid when the table is set up at alternate stations only A small and light form of table with a trough compass recessed into the board is sometimes used for such work, and is known as a traverse plane table.

Intersection or Triangulation.—This method is largely used for mapping detail, but is also available for plotting the positions of points to be used as subsequent instrument stations The only linear measurement required is that of a base line

Lay out and measure a base line AB (Fig 211) Plot *ab* in a convenient position on the sheet Set up at A with *a* over A, and orient by laying the alidade along *ab* and turning the table until the line of sight cuts B Clamp, and, with the ruler touching *a*, sight the various points defining the surrounding detail and points selected as future instrument stations, drawing a ray from *a* towards each Proceed to B, set up with *b* over B, and orient by back-sighting on A. Through *b* draw rays towards the points previously sighted Each point is located by the intersection of the two rays drawn towards it Before leaving B, draw a series of first rays towards other points not sighted from A, and then proceed to C, orient on A or B, and obtain a new series of intersections

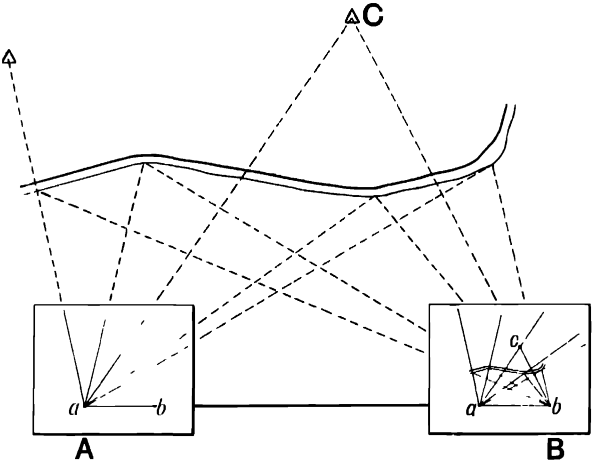

FIG 211

Notes —(1) An extended survey should, whenever possible, be based upon a system of points whose relative positions have been obtained by theodolite triangulation or traversing, and plotted on the sheet No base line is then required, and not only can such stations be occupied by the table, but, on setting up at other sites, the orientation can be verified by reference to them, and accumulation of error is avoided An intersection survey uncontrolled by theodolite points is termed a graphic triangulation, and, if extensive, demands constant precaution against the propagation of error

(2) In any case, greater refinement is called for in plotting points to be used as plane table stations than in fixing detail No station can be considered as satisfactorily located unless it has been intersected from three others Triangles should be well-conditioned in order to yield definite intersections. it is desirable, particularly in graphic triangulation, that intersection angles should not be less than 45°

(3) Since the length of the base line influences only the scale of plotting, it is evident that a graphic triangulation can proceed without the preliminary measurement of a base The scale can then be determined when the survey includes two points whose distance apart has been determined astronomically, trigonometrically, or by direct measurement Such procedure is useful in exploratory and reconnaissance surveys

Resection.—This method can be used for the location of station points only As in the preceding system, one linear measurement is required The simplest case is as follows

Measure a base line AB (Fig 212), and plot *ab* in a convenient position Set up at A with *a* over A, orient on B, and through *a* draw a ray of indefinite length towards D Set up at D with the estimated position of *d* over D, and orient by backsighting on A Place the alidade against *b*, and sight B The point in which the edge of the ruler cuts the line previously drawn from *a* towards D is the required point *d* From *d* draw a ray towards C Set up at C with the estimated position of *c* over C, and orient by backsighting on D. Place the alidade against *b* and sight B, obtaining the intersection *c* Check *c* by sighting A with the ruler touching *a*.

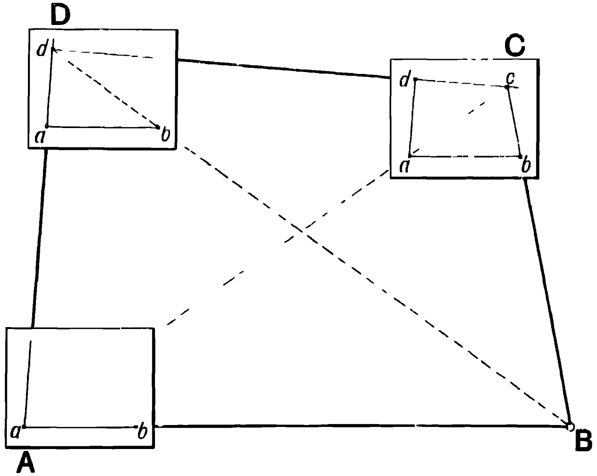

FIG 212

Notes.—(1) Errors of centering are inevitable, but, since resection is usually confined to small scale work, the accuracy of the plotting is not appreciably affected thereby.

(2) The characteristic feature of resection is that the observations at any station after the first are directed to plotting the point occupied by the instrument, the surrounding detail then being located by radiation or intersection. The back ray method of orienting, as described, necessitates a ray being drawn from a preceding station to that being occupied, and therefore involves the previous selection of the instrument station. This is only a particular case of resection. Solutions for cases where no ray has been drawn to the instrument station are given below.

Of the various cases where resection, or the plotting of the station occupied, can be performed, that in which the data consist of two visible stations, their plotted positions, and a ray from one to the station occupied has just been described. In the more usual case in which no such ray has been drawn, the data must consist of either

(*a*) Three visible points and their plotted positions. (The Three-point Problem.)

(*b*) Two visible points and their plotted positions. (The Two-point Problem.)

Of these, the former is the more important and useful case.

The Three-point Problem.—The problem may be stated: Given three points A, B, and C, visible from a station P over which the table is set, and *a*, *b*, and *c*, their plotted positions. Required to plot *p*.

If the table were oriented at P, so that *ab* and *bc* are respectively parallel to AB and BC, it would only be necessary to draw rays through *a* and *b* with the alidade directed towards A and B respectively. These would intersect at *p*, the position of which could be checked by seeing whether a ray through *c* towards C also passes through *p*. The problem therefore resolves itself into that of orienting the table at P, so that A*a*, B*b*, and C*c* will pass through one point *p*.

Strength of Fix.—The relative position of A, B, C, and P has an important influence on the accuracy with which the position of *p* can be determined.

If P lies anywhere on the circumference of the circumscribing circle through A, B, and C, its position is indeterminate, since *p* may lie anywhere on one of the segments of the circumference of the circle through *a*, *b*, and *c*, and still subtend the correct angles APB and BPC with *ab* and *bc* respectively. No matter how faulty the orientation is in such a case, the three rays, A*a*, B*b*, and C*c*, will always meet in a single point. For other situations of P they will not pass through one point unless the table is oriented, but will either form a triangle, called the triangle of error, or two of them may be parallel and intersected by the third. In such cases, the accuracy with which P is fixed varies with different positions of P relatively to A, B, and C

The fix is good when

(*a*) The middle station B is much nearer than the others

(*b*) P is within the triangle ABC

(*c*) One angle is small (becoming zero when two points are in range) and the other is large, provided the points subtending the small angle are not too near each other

The fix is bad when

(*a*) Both angles subtended at P are small

(*b*) P is near the circumference of the circumscribing circle

Trial and Error Solution.—It is evident that, unless *p* is indeterminate, its position can be obtained by repeated trial of the orientation until the table is so placed that the rays A*a*, B*b*, and C*c* all pass through one point Unnecessary expenditure of time is avoided by making a preliminary orientation by compass or by a range when the line of two well-defined objects already plotted passes through or near P If this is done, the triangle of error will be small, and will be eliminated after one or two trials The adjustment in orientation is, however, facilitated by the application of Lehmann's rules for estimating the position of *p* from the triangle of error (Fig 213)

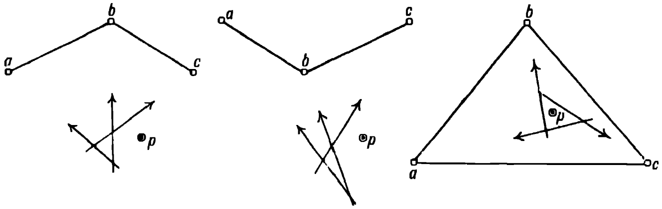

FIG 213 EXAMPLES OF TRIANGLE OF ERROR

Lehmann's Rules—(1) The point *p* is always distant from each of the three rays A*a*, B*b*, and C*c* in proportion to the distance of A, B, and C from P, and, when looking in the direction of each of the distant points, it will be found on the same side, right or left, of each of the rays

Note—It follows from this rule that *p* lies within the triangle of error only when P lies within the triangle ABC

Although rule (1) suffices for the solution of the problem, two subsidiary rules are of assistance

(2) When P is within the circumscribing circle ABC, *p* is always on the same side of the ray to the most distant point as the intersection of the other two rays

(3) When P falls within either of the three segments of the circumscribing circle ABC, formed by the sides of the triangle ABC, the

ray towards the middle point lies between *p* and the intersection of the other two rays.

Having estimated the position of *p* by applying the rules, the table is oriented by laying the alidade touching *p* and one of the points *a*, *b*, or *c*, and turning the table to sight the corresponding object. If the three rays now intersect in one point, the estimate has been correct. If not, a new position of *p* must be assumed with reference to the new triangle of error, and the orientation is repeated.

Mechanical Solution.—(1) Clamp the table, fasten a sheet of tracing paper on the board, and on it mark a point *p′* approximately over P.

(2) With the alidade touching *p′*, sight A, B, and C successively, and draw a ray of indefinite length towards each. These rays will not pass through *a*, *b*, and *c*, except in the unlikely event of the table being correctly oriented with *p′* coinciding with *p*.

(3) Unfasten the tracing paper, and move it over the drawing until the three rays simultaneously pass through *a*, *b*, and *c*, and then prick through the position of *p′*. The point so obtained is the required point *p*.

(4) Remove the tracing paper, unclamp, and, with the ruler along *pa*, orient the table by turning it until the line of sight strikes A. B*b* and C*c* should now also pass through *p*. If, owing to careless manipulation or unequal stretching of the tracing paper, a small triangle of error is formed, it can be eliminated by the trial and error method.

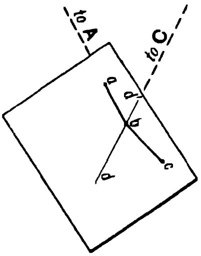

FIG 214

Graphical Solution.—Of the various graphical methods which have been proposed, Bessel's solution by the inscribed quadrilateral is the simplest, and it only will be described. The table is set up and levelled at P, and the process of orienting it may be divided into three steps.

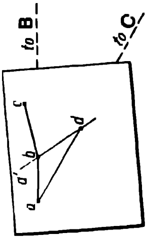

FIG 215

(1) (Fig 214) With the alidade along *ba*, turn the table until A is sighted, *a* being towards A. Clamp, and through *b* draw a ray *dbd′* towards C.

(2) (Fig 215) Unclamp, and, with the alidade along *ab*, sight B, *b* being towards B. Clamp, and through *a* draw a ray *ad* towards C, meeting the previously drawn ray at *d*.

(3) (Fig 216) Unclamp, and, with the alidade along *dc*, turn the table to sight C, and clamp. The table is now oriented, and *p* must lie on *dc*, produced if necessary. But *p* must also lie on A*a* and B*b*. Therefore, with the edge of the alidade on *a*, sight A, and draw *ap* meeting

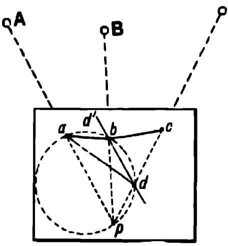

FIG 216

cd in *p*, and check the orientation by testing whether B*b* also passes through *p*

Proof—Assuming that the distance of P from A, B, and C is sufficient to render the effect of centering error negligible, then—

In Fig 214, $abd' = \text{APC}$
In Fig 215, $bad = \text{BPC}$,
and $bda = abd' - bad = \text{APB}$.
In Fig 216, $bpc = \text{BPC}$,
$bpc = bad$, and consequently *abdp* can be circumscribed by a circle
$bda = apb$, since both are subtended by chord *ab*,
so that $apb = \text{APB}$
p simultaneously subtends with *a*, *b*, and *c* the required angles APB and BPC

Notes—(1) If *d* comes off the paper, *ab* and *bc* may, for the purpose of orienting, be decreased proportionally by drawing through them a parallel to *ac*. If *d* comes very near *c*, *ab* and *bc* should be increased. The point *d* obtained is used with the new *c* in effecting the final orientation

(2) In the first two steps, instead of sighting through *a* and *b* and drawing rays towards C, any two of the points may be used and the rays drawn towards the signal corresponding to the third, which is then sighted in the final step.

The Two-point Problem.—The problem may be stated Given two points A and B, visible from a station P over which the table is set, and *a* and *b*, their plotted positions Required to plot *p*

This problem appears indeterminate unless the orientation is effected by compass, and a solution cannot be obtained, without the use of the compass, by occupying P alone. Two instrument stations are required. The most convenient solution is as follows (Fig. 217)

(1) Select a fourth point C, such that angles PAC and PBC are not too small for good intersections at A and B. Set the table at C, and orient approximately by estimation or by compass Sight A and B, and draw rays through *a* and *b* respectively to intersect at *c′*. C is only approximately represented by *c′*, since the orientation was only approximate Through *c′* draw a ray towards P

(2) Set up at P, and assume a point *p′* on the last drawn ray to represent P by estimating the distance PC. Bring the table to the same orientation as at C by placing the alidade along *p′c′* and sighting C. Now, with the alidade against *p′*, sight A and B, and draw rays intersecting *c′a* and *c′b* in *a′* and *b′* respectively.

(3) These points have been obtained by ordinary intersection from C and P, so that *a′b′* is parallel to AB But *a′b′* does not coincide with *ab* because: (1) the orientations at C and P, although consistent, are only approximate; (2) *p′c′* represents PC by estimation only. In other words, *a′b′c′p′* is similar to ABCP, but is of erroneous size, and is wrongly placed on the sheet. The error of orientation is the angle between *ab* and *a′b′*.

(4) To orient the table, it must be turned through this angle until *ab* becomes parallel to the present direction of *a′b′* The best method is to place the alidade along *a′b′*, and erect a distant pole in that line Transfer the alidade to *ab*, and turn the table until the pole is again sighted, clamp, and the orientation is made

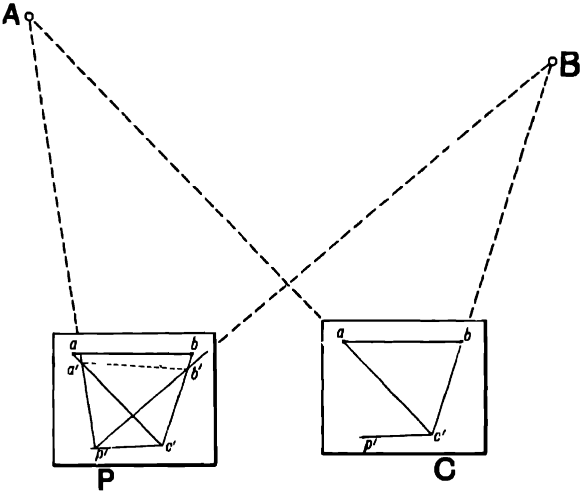

FIG 217.

(5) P is now plotted by sighting through A*a* and B*b* and obtaining the intersection *p*

Note —If C can be selected in line with AB, the problem is simplified, since the table can be oriented at C by bringing *ab* in line with AB Having done so, draw a ray towards P, and, on setting up at P, orient by means of this ray and a signal left at C

Application of Resection.—The practical utility of the three-point and two-point problems, particularly the former, is greater than may appear at first sight These methods of resection have the great merit that the topographer is saved the necessity of selecting forward stations in advance He has entire liberty to set up solely at points favourable to the taking of detail, and is dependent only upon the visibility of suitable trigonometrical stations and not upon his previous or succeeding plane table fixings The advantages of resection are most apparent, and the method is greatly used, in the small scale mapping of open country, when detail is taken entirely by sketching with reference to the plane table stations.

Field Party.—For small surveys, the surveyor does not require more than two men to perform chaining, mark stations, etc. In large surveys, the work may be expedited by the surveyor having a qualified assistant, who can proceed with the plotting while the chief reconnoitres the forward ground The number of men required depends on the nature of the ground as well as the method of surveying In tacheometrical radiation, it will usually be possible to keep two staffmen fully occupied, and more if the points sighted are widely spread, as in small scale work in open country. It may be necessary to have a man to carry the table between stations, hold the umbrella, etc

Equipment.—The amount of apparatus to be carried depends upon the nature and magnitude of the survey A full equipment consists of

Plane table, tripod, and alidade

Spirit level, trough compass, plumb-bob, fork, cover, and field glass

Scales, one or two set-squares, pencils, rubber, sand-paper, ink, colours, drawing pen, and small notebook

Portfolio or cylindrical case for sheets

Poles and flags

For tacheometry Staff and stadia reduction tables or diagram

For traversing, etc Chain or band and tape

For trigonometrical levelling Curvature and refraction table

For barometric levelling Aneroid

Vertical Control.—The widest application of plane table surveying has been in the preparation of contour maps, for which the instrument is admirably adapted, principally on account of the great advantage that the topographer has the terrain in view while representing its form

With Telescopic Alidade—If a telescopic alidade with vertical arc is available, the necessary elevations may be obtained either by tacheometric or trigonometric levelling Tacheometry (Chap IX) is well adapted to close contouring when the sights do not exceed 1,000 ft Trigonometrical levelling is most suitable for small scale work involving long sights The methods are discussed in Vol II, Chap V The vertical angle to a located point, of which the elevation is required, is observed, and from the known distance the difference of level is computed In determining the elevations of points to be used as future instrument stations, observations are necessary from at least two stations whose elevations have been found with corresponding care, and account must be taken of the effects of curvature and refraction In the resection method, the elevation of the instrument is determined from vertical angles to two or more plotted points of known elevation

With Plain Alidade—Since a plain alidade can be used for horizontal control only, elevations must be determined by ordinary level, hand level, or one of the various forms of clinometer Direct levelling is very much slower than clinometric levelling on account of the large number of points of observation required, and its use is justified only in the close contouring of small areas

The Indian clinometer (Fig 218) is specially adapted to plane tabling, and is extensively used. The instrument is placed upon the board, and, when the table has been set level by estimation only, the clinometer is levelled by the levelling screw and spirit level shown The folding sight vanes are 8 in apart The front one is graduated in degrees and natural tangents, and the eye-hole of the rear vane is horizontally opposite the zero of the scales when the instrument is levelled With the object of facilitating the reading of the scales, the front vane is sometimes fitted with a small frame carrying a horizontal wire, which is moved along the scales by means of a rack and pinion It is, however, preferable in point of accuracy to use a plain front vane The level tube is provided with adjusting screws, the testing and adjustment of the instrument being performed by means of reciprocal observations, as in the case of the Abney clinometer (page 122)

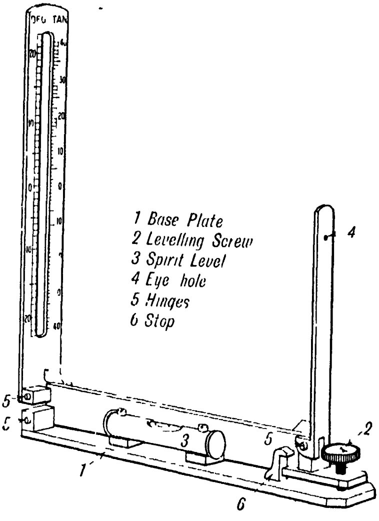

FIG 218 —INDIAN CLINOMETER

The elevation of the station occupied can be determined by observation to a point of known level already plotted The surveyor places the clinometer in the direction of the distant point, levels it, and, with the eye a few inches from the sighting hole, observes the graduation on the tangent scale opposite the point sighted The difference of level is the tangent times the distance as scaled from the map, and to this result will be applied the height of the line of sight from the ground under the table It is preferable to make the determination from two or more points and adopt the mean result From the now known elevation of the station occupied, those of surrounding points can be obtained in the same manner

The allowable length of sight depends upon the degree of accuracy required the Survey of India Handbook limits it to 3 or 4 miles The available precision does not warrant the application of curvature and refraction corrections

Field Methods.—In the execution of a plane table survey, the general system and the details of routine to be adopted both for horizontal and vertical control depend on several factors, such as · (*a*) Scale of plotting ; (*b*), Degree of accuracy required ; (*c*) Extent of survey , (*d*) Character of ground , (*e*) Time at disposal

The following outlines of field methods will serve to suggest the routine suitable for any particular case.

Scales greater than 1/2,500.—*Horizontal Control*—Traversing, Radiation, and Intersection

Plane table stations in large scale work should be located by traversing If the survey is extensive, and a high degree of accuracy is required, a controlling theodolite traverse or triangulation may be undertaken so that a number of the plane table stations may be plotted beforehand The detail is fixed by offsetting and by radiation, the distances being taped Points so situated with respect to the stations as to yield good intersections may be located by intersection As the importance of the detail and the scale decrease, tacheometric radiation and pacing may be employed, and the same attention need not be paid to the quality of intersection fixes

Vertical Control—Direct Levelling, Tacheometry, Indian Clinometer

If the contour interval is as small as 2 ft , and the contours have to be located with sufficient accuracy for earthwork measurements, the best results are obtained by locating points *on* the contours by ordinary levelling and surveying them by plane table The method of using the telescopic alidade for direct levelling is slow and inconvenient owing to the necessity for frequent shifting of the table. In ordinary contouring for location surveys, the tacheometric method is most suitable

Scales of 1/2,500 to 1/10,000. –*Horizontal Control*—Intersection, Resection, and Traversing

The survey may either be made by plane table throughout, or may be based upon a system of points established by theodolite triangulation or traversing. The theodolite stations and their plotted positions are used in four ways (*a*) Some or all of them are occupied by the plane table , (*b*) They serve to verify the orientation of the table at any station, and are used in locating stations by resection , (*c*) The distances between them serve as so many base lines, as required in the intersection system , (*d*) From their known altitudes the elevations of all other stations are determined

The detail in the larger scales is commonly located by tacheometric radiation As the scale decreases, the amount of sketching allowable increases, and the controlling points are most expeditiously fixed by intersection

Vertical Control—Tacheometry, Indian Clinometer, Alidade Vertical Angles

Tacheometric levelling may still be employed where good contour delineation is required, particularly in flat country. When the distances become too great for tacheometric reading, elevations of intersected points are obtained by clinometer or by measurement of vertical angles by the alidade In all cases, points surveyed should be characteristic points on ridge and valley lines

Scales of 1/10,000 to 1/50,000.—*Horizontal Control* —As above. The framework is generally fixed by theodolite triangulation For plotting plane table stations, resection is now used where practicable, in preference to intersection, to avoid the marking of stations and much of the reconnoitring necessary in the intersection method Where traversing is adopted owing to the presence of woods or the flatness of the country, the distances may be obtained with sufficient accuracy tacheometrically or by wheel (Vol II, Chap VI) When long traverse lines are possible, it is an advantage to have an alidade adapted for movable hair tacheometry, so that observations may be made on conspicuous targets fixed on the staff Points taken to guide the sketching of detail are usually intersected, but few are required Since $\frac{1}{100}$ in on the sheet corresponds to from about 8 to 42 ft on the scales under consideration, much of the detail may safely be sketched by estimation

Vertical Control —Indian Clinometer, Vertical Angles

The number of observations, other than those required to fix the elevation of the instrument stations, is reduced as the scale decreases, and increasing care is necessary in the selection of the points to be located As a guide in the sketching of contours, the topographer should estimate the surface slopes

Scales less than 1/50,000.—*Horizontal Control* —Resection and Intersection

The plane table fixings are almost exclusively made by resection from triangulation points Traversing may still be found necessary in confined situations, but should be reduced to a minimum. The detail is commonly sketched entirely by estimation, particularly on the smaller scales, and the surveyor is constantly called upon to decide what features should be shown and what must be omitted because of the smallness of the scale

Vertical Control —Vertical Angles, Aneroid, Indian Clinometer

On the smaller scales, the elevations of the plane table stations only are determined, and the contours are sketched entirely by estimation The production of a satisfactory map in the least time calls for a high order of skill on the part of the topographer, who must have a good eye for country.

Preparation of Field Sheets.—When plane tabling is employed for the mapping of detail between control stations, the surveyor must plot these on the sheets before proceeding to the field. In surveys of moderate extent, the plotting is performed by rectangular co-ordinates, as described on page 191 In extensive small scale surveys, however, the points are plotted by geographical coordinates, and it is then necessary to construct a graticule of meridians and parallels (Vol. II, Chap. VII), on which the stations are plotted in terms of their computed latitudes and longitudes.

The detail surveyor is furnished with a description of the stations, and, before commencing to map, he should test the accuracy of their plotted positions. Having set up over one of them, and oriented by sighting another, he will be able, by successively sighting the other visible stations, to verify the accuracy of the corresponding plotted points.

Junction of Sheets.—When detailing between plotted stations, no trouble is presented in changing from one field sheet to another, since each has its system of points from which fixings are made. The graticules on adjacent sheets are, however, plotted to afford a marginal overlap by means of which a satisfactory junction can be effected between the features on adjoining sheets.

In large scale work, especially if the survey is executed entirely by plane table, very little overlap may be required. When a new sheet is commenced, the manner of securing continuity with the finished one will depend upon the system of observation being employed. The method of using a long joining line, common to both sheets, and pricking through sufficient points on to the new sheet will suggest itself in any case.

General Suggestions for Plotting.—(1) Lines must be drawn as fine as possible. The pencil should be 4H or harder, and the point should be kept very sharp by frequently touching it up on sandpaper. A chisel point may be used for drawing rays, but for marking points a conical point or a needle pricker is required.

(2) Lines must be drawn close to the edge of the ruler. When drawing a ray from a point, the pencil should be held upright and exactly on the prick mark, and the ruler is laid against the pencil. The pencil should then be maintained at a constant inclination while drawing the ray.

(3) Unnecessary complication of rays must be avoided in using the intersection method. The possibility of confusion is lessened by adopting the following suggestions :

(*a*) Instead of making the first rays from a station point continuous lines, draw only a short length of each ray sufficient to contain the point sought. Draw also a reference mark, or *repère*, which is a short length of the ray at the margin of the sheet. A brief description of the point written against the reference mark serves to identify it. In the case of rays likely to be used for subsequent orientations, reference marks at opposite margins of the paper facilitate accurate setting of the alidade.

(*b*) Intersections should be obtained, without actually drawing the second rays, by marking the points at which the edge of the ruler cuts the appropriate first rays, but reference marks should be made for rays to be used for orientation.

(*c*) Do not take more sights than are really necessary. In particular, avoid drawing rays to distant objects which can be located with shorter rays from other stations

(*d*) Erase rays and reference marks, with soft rubber, as soon as they can be dispensed with

(4) The drawing should be kept as clean as possible by paying attention to the cleanliness of the underside of the ruler, the plumbing fork, and the compass, by lifting, rather than sliding, the alidade into position, and by keeping the paper free from moisture and dust Do not erase lines from damp paper In small scale work, the part of the sheet not being used should be covered with a piece of tracing paper Ink in each day's work the same evening

Errors in Plane Tabling.—It is impossible to state generally the degree of precision to be expected of the plotted map, because it depends not only upon the character of the survey, the quality of the instrument, and the system adopted, but also to a considerable extent upon the degree to which accuracy is deliberately sacrificed for speed Sources of error may be classified as

(1) Instrumental Errors
(2) Errors of Manipulation and Sighting
(3) Errors of Plotting

(1) The influence of residual errors of adjustment may be regarded as quite negligible

(2) Errors of manipulation and sighting include :

(*a*) *Non-horizontality of Board* —As in theodolite observations, the effect of dislevelment is most marked when there is a considerable difference of level between the points sighted. The discussion of the error caused by defective levelling in theodolite work (page 171) applies here whether the alidade is telescopic or plain In the latter case dislevelment causes both vanes to be inclined to the vertical If the table is not horizontal in a direction at right angles to the alidade, the line of sight is parallel to the fiducial edge only for horizontal sights, and makes an increasing angle with the fiducial edge the more inclined is the sight

(*b*) *Error of Centering.*—Since the essential requirement of correct orientation necessitates repeated trial in accurately centering a point not on the vertical axis of the table, the surveyor may frequently avoid unnecessary expenditure of time in setting up by having a proper conception of the extent of error introduced by inexact centering.

In Fig 219, let P represent the instrument station, and *p* the corresponding plotted point The angle APB is erroneously plotted as *apb*, and the angular error (APB$-$*apb*) will depend upon the distances PA and PB as well as upon the error of centering

If the angles PA*p* and PB*p* are denoted by α and β respectively, evidently

$$(\mathrm{APB}-apb) = \alpha+\beta = \sin^{-1}\frac{\mathrm{PC}}{\mathrm{PA}} + \sin^{-1}\frac{\mathrm{PD}}{\mathrm{PB}}$$

Assuming 1 ft as the greatest value of PC and PD at all likely to occur in practice, the extent of the resulting angular error for equal lengths of sights is indicated by the following figures ·

Distance in Feet.	Angular Error
10	11° 29′
100	1° 9′
500	14′
1,000	7′
5,000	1½′

Of more practical consequence is the error in the positions of a and b Point a should be at a' to the left of a, where $aa' = ap \times \alpha$ approximately . similarly for b These displacements will not affect the accuracy of the plan if they are too small to be plotted, and this evidently depends upon the scale

If s = the fractional scale,
and D = the distance PA in ft ,
then plotted length $pa = Ds$ ft.
Actual displacement $aa' = Dsa$,
but $\alpha = e/D$, where e = PC in ft (Fig. 219),
. $aa' = se$ ft

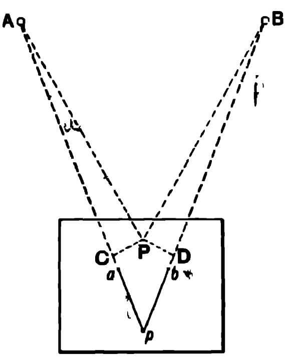

FIG 219

If aa' is not to exceed $\frac{1}{100}$ in , assumed as the smallest quantity which can be shown, then e must not exceed $1/1{,}200s$. Centering must therefore be performed with care in large scale work For a scale of 10 ft = 1 in , the horizontal distance pP should be kept under one inch, but for scales smaller than about 200 ft. = 1 in , it is sufficient to have the ground point within the limits of the board. Greater care is desirable in the case of very short rays, to obviate serious error in direction, particularly if such rays are to be used, with the aid of marginal reference marks, for orientation

(*c*) *Defective Sighting*—The plain alidade with open sights is much inferior to the telescopic alidade in the definition of the line of sight. The resulting errors will tend to compensate, and are, in general, relatively unimportant.

(*d*) *Movement of Board between Sights.*—This should not occur if the clamp has been firmly applied, but it is always advisable to check the orientation at the conclusion of the observations from a station.

(*e*) *Defective Orientation.*—The above errors contribute towards erroneous orientations and consequent distortion of the survey. To limit the propagation of error, the orientation of the table should be checked at as many stations as possible by reference to a distant prominent object already plotted.

(3) Errors of plotting are a common source of inaccuracy, and can only be minimised by constant care in drawing and in the use of scales, and by using a paper so mounted as to eliminate distortion errors as far as possible

Survey Adjustment.—*Traversing* —A considerable length of traverse may have to be run out from a known point without the surveyor having an opportunity of checking his orientation by observation on a triangulation point If, in addition, the measurement of distance is not very refined, there is every probability of appreciable error being generated Most commonly the error is determined, not by a return to the starting point, but by making a resection fix whenever the opportunity occurs The adjustment of the traverse between these fixed points is then made by the graphical method described on page 188

Triangulation —Considerable error may likewise be propagated in graphic triangulation, but if it is conducted between points of known position, the results may be adjusted A rigorous adjustment is uncalled for, and the following arbitrary method will suffice

In Fig 220, let *Abc* . . . *m* be the plotted result of a triangulation conducted between stations A and M and originating from a base line *Ab* If M is the correct position of the terminal point, the error *m*M may be distributed by treating *Abdgknm* as a traverse and adjusting the points by the previous method The route *Acfhlm* is similarly treated Lines *ce* and *ef* are not included, since they do not influence the position of *m*, and *e* is replotted from the adjusted positions of *c* and *f*

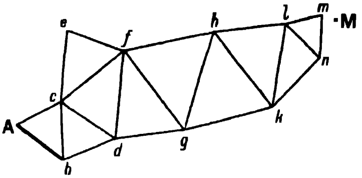

Fig 220

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CHAPTER VI

CONTOURS AND CONTOURING

Representation of Three Dimensions.—The value of a plan or map, whether of large or small scale, is greatly enhanced if the altitudes of the surface features are represented as well as their relative positions in plan One way in which the conformation of the ground may be presented on the plan is by delineation of the surface slopes on some conventional system of shading, intended merely to convey an impression of relative relief, but without indicating actual elevations These systems of representation (Vol II, Chap VII) are sometimes used in geographical mapping on small scales, but they do not impart sufficiently definite information to be of service for engineering purposes

In plans for location or construction work, vertical dimensions should be represented at least as accurately and clearly as horizontal distances, and this points to two requirements in delineation

(*a*) The reduced levels of numerous points in terms of a known datum should be shown

(*b*) These should be arranged in such a manner that the form of the surface can readily be interpreted

The first requirement may be met by the plotting of spot levels, the elevation of each being written over a dot representing the position of the point The utility of spot levels is, however, limited, as they can convey only a vague idea of the form of the ground Both conditions are fulfilled by contour lines, a method of representation of great value, as it is equally suitable in flat as in mountainous country, and for large as well as small scale plotting.

Contour Lines.—A contour line, or contour, may be defined as the line in which the surface of the ground is intersected by a level surface It follows that every point on a contour has the same elevation—that of the assumed intersecting surface If the contour lines determined in this way by several equidistant level surfaces are imagined to be traced out on the surface of the ground and surveyed, the resulting plan will exhibit the contours in their proper relative positions, and will portray the character of the ground Thus, in Fig 221, *a* represents a hill which is shown intersected by a series of level surfaces at elevations of 100, 150, etc., ft above datum. The contours may run as shown on the plan *b*, which serves to depict the shape of the hill.

It follows from the definition that the water mark of still water is a contour line, and the student may at first find it helpful to conceive contours as shore lines of a water surface which can be adjusted to any desired level. Regarded in this way, Fig. 221*b* shows the varying outline of an island when the level of the water stands at 100, 150, etc., ft. In the case of a valley or other depression, the contours may similarly be considered as indicating shore lines of a lake.

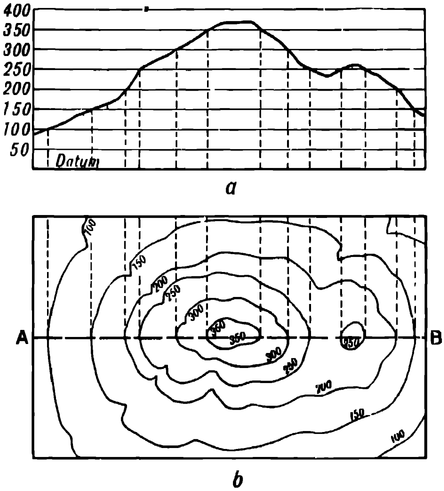


Fig. 221

The Contour Interval.—On the assumption that the contours of Fig. 221*b* are correctly located and plotted, full information is available regarding the surface along each, but not about the intervening ground. Considerable irregularities may occur between adjacent contours, but, having escaped intersection, they are not represented on the plan. The number of available level surfaces is, however, infinite, and the representation of the ground can be carried to any degree of refinement by sufficiently increasing the number of contours. The constant vertical distance between the contours in any case is called the *Contour Interval*. The most suitable interval to adopt on a survey is a matter to be decided at the outset from consideration of the following items:

(*a*) *Time and Expense of Field and Office Work.*—The smaller the interval, the greater is the amount of field work, reduction, and plotting required in the preparation of the plan.

(*b*) *The Purpose and Extent of the Survey.*—Close contouring is required in cases where the plan is intended to be utilised for the detailed design of works or for the measurement of earthwork quantities. In general, the area included in such a plan will be comparatively small, so that it may be quite practicable to locate contours with an interval as small as one foot. In location surveys for lines of communication, reservoirs and their drainage areas, etc., since a rather large area may be involved, a wider interval must be made to suffice. The requirements of geographical mapping are met by the use of still greater intervals.

(*c*) *The Nature of the Country.*—An interval which would be sufficient to show the configuration of mountainous country would be quite inadequate to portray the undulations of comparatively flat ground The representation by contour lines of a stretch of very flat ground is made possible only by the adoption of a very small interval.

(*d*) *The Scale of the Plan or Map*—The interval should, *ceteris paribus*, be in inverse ratio to the scale of the map By using a close interval on a small scale map, the detail is obscured, and, unless the ground is very flat, the result is confusing

Values of the contour interval adopted for various purposes are as follows

For building sites 1 or 2 ft

For reservoirs, landscape gardening, and town planning schemes : 2 to 5 ft

For location surveys 5 to 10 ft

For general topographical work from 10 ft upwards, depending upon the scale and the character of the ground , for average country a common rule is

Contour interval = 50 ft ÷ no of inches per mile

Whatever interval is adopted, it should be constant within the limits of a map If the regularity of the intervals is interrupted by the interpolation of additional contours for the better definition of flat ground, these should be distinguished in drawing by dotting or otherwise Unless this is done, the general appearance of the map is misleading, the closely contoured ground appearing steeper than it really is

Characteristics of Contours.—By virtue of the fundamental property of a contour, that every point on it has the same elevation, a contour map with constant interval portrays elements of configuration in a characteristic manner A knowledge of the more important attributes of contours facilitates the interpretation of a map, and, in plotting, enables the draughtsman to avoid possible misrepresentations false to nature The following items should be kept in view in map reading and plotting

1 Since the rise or fall from a point on one contour to any point on the next above or below is the constant contour interval, the direction of steepest slope is that of the shortest distance between the contours The direction of steepest slope at a point on a contour is therefore at right angles to the contour

2 Steep ground is indicated where the contours run close together , flat ground where they are widely separated Uniform distance between contours indicates a uniform slope, and a series of straight, parallel, and equally spaced contours represents a plane surface

3 Contour lines of different elevations can unite to form one line, only in the case of a vertical cliff

4 Two contour lines having the same elevation cannot unite

and continue as one line, nor can a single contour split into two lines This is evident from the conception of contours as water marks The single line would indicate an impossible knife-edge ridge or hollow

5 There are conceivable circumstances in which two contours of the same elevation may touch at a point, giving them the appearance of crossing, but this occurs very rarely Contours of different elevations can cross each other only in the case of an overhanging cliff or a cave penetrating a hillside At the intersections, two or more points of different levels on the ground are represented by one point in plan

6 A contour cannot end anywhere, but must ultimately close on itself, although not necessarily within the limits of the map This property is obvious from the idea of water marks A series of closed contours indicates either a hill or a depression without an outlet, according as their elevations increase or decrease towards the centre of the series

7 The same contour appears on either side of a ridge or valley, for the highest level surface which intersects the ridge and the lowest one which intersects the valley do so on both sides It is impossible for a single higher or lower contour to intervene between two of equal value

8 Contour lines cross a watershed or ridge line at right angles They curve round it with the concave side of the curve towards the higher ground

9 In valleys and ravines, the contour lines run up the valley and turn at the stream so that the convexity is next the higher ground. They are intersected at right angles by the stream If the scale allows of both banks being shown, the contours should be stopped at the edge of the water unless they have been located across the bed of the stream

METHODS OF CONTOURING

Since in the location of contours both vertical and horizontal measurements are involved, the field work may be executed in various ways according to the instruments used Field methods may, however, be divided into two classes—*Direct*, and *Indirect.*

Direct methods comprise those in which the contours to be plotted are actually traced out in the field by the location and marking of a series of points on each These points are surveyed and plotted, and the appropriate contours are drawn through them

Indirect methods are those in which the points located as regards position and elevation are not necessarily situated on the contours to be shown, but serve, on being plotted, as a basis for the interpolation of the required contours This system is used in all kinds of survey, and proves less laborious than the first

In both methods, but particularly in the case of the latter, the accuracy of the resulting map will greatly depend upon the number and disposition of the selected points The careful location of numerous points may be out of the question owing to the time required in the field, and for some purposes sufficient accuracy is attained by interpolating between widely spaced points. In such work, horizontal positions may be determined by intersection, and elevations by barometric or trigonometric levelling, but this class of field work belongs more properly to the subject of topographical and geographical surveying, and is dealt with in Vol II, Chaps V and VI The field methods to be considered here are those directed to the production of a map suitable for location or construction work Variation in procedure will depend upon which of the following instruments are used

Vertical Control Dumpy or Wye Level, Hand Level or Clinometer

Horizontal Control Chain and Tape, Theodolite or Tacheometer, Compass, Plane Table

Combined Control Tacheometer, Plane Table with Tacheometric Alidade or Clinometer

Direct Methods.—Direct contouring is necessarily very slow, and on this account it is comparatively seldom adopted on large surveys It has the merit of superior accuracy, and is suitable for the close contouring of small areas where considerable precision is required. The field work consists of two steps (1) the location of points on the contours ; (2) the survey of those points These operations may be conducted nearly simultaneously if performed by two parties, one levelling and the other surveying, but in dealing with a small area the pegging out of the contours may be completed before their survey is commenced

Although the field routine is largely dependent upon the instruments employed, two principles of general applicability are to be observed

1. The degree of accuracy required in the location of contours should depend upon the scale to which they are to be plotted, and, for a given scale, upon the use to which the plan is to be put.
2. Since the contours are to be drawn through the plotted points, only such points should be located that the contours are nearly straight between them. At places of sharp curvature more locations are necessary than elsewhere Salient points on ridge and valley lines are of special importance, and should never be omitted

Direct Vertical Control.—1. *By Level and Staff* —Having levelled from the nearest bench mark to the site of the survey, set the instrument to command as much ground as possible From the known instrument height deduce the readings to be observed when the staff is held on the various contours within range of the level. Taking one contour at a time, direct the staffman up or down hill until the required reading is sighted. On receiving the signal to mark, the staffman should insert a lath or twig split to receive a piece of paper on which is noted the reduced level of the contour. Having located one contour over the length visible from the instrument, proceed in the same manner for the others with the new staff reading, until a fresh instrument station is required.

Fig. 222 shows a possible arrangement of the points fixed from one set-up, the arrows showing the route followed by the staffman. If the instrument height were 86·37, the staff readings would be 1·4, 6·4, and 11·4 in locating points on the 85, 80, and 75 contours respectively. It is unnecessary to read to two places of decimals

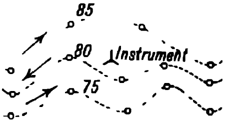


FIG. 222.

Notes —(1) It is advisable that the levelling from the B M to the first control station should be checked before contouring is commenced A temporary B M should therefore be established on the site of the survey, and checked first of all If, however, there are one or more Ordnance B M 's on or near the ground, this step is unnecessary

(2) In walking forward, the staffman should try to avoid going up or down hill It is an advantage to have the assistance of an experienced staffman, who, when remote from the instrument, can be relied upon to select suitable points, but the surveyor should nevertheless be on the lookout for possible omissions at salient points A special signal should be used to indicate that the staffman has to begin on a different contour

(3) Fixing adjacent points on several contours occasions loss of time and confusion with the marking tags , the method of working along each contour, as described, is much preferable

2 *By Hand Level*—The hand level may be of any type, an Abney clinometer clamped to zero proving suitable, and the instrument may be used in conjunction with a levelling staff or simply a ranging pole marked off in feet By levelling from a B M , first locate a point on one of the contours, preferably one having an elevation about the mean of those to be traced This contour is to be pegged out first in order to provide points of known elevation from which the remaining contours can be located To trace the contour, stand on the initial point, and direct the staffman until the point on the staff or pole corresponding to the height of the instrument above the ground is in the line of sight The level should be held against a pole, so that the reading is a whole number, say 5 ft. Locate as many points on the contour as can be conveniently sighted from the instrument station, and then move forward to the last point marked

When a sufficient length has been set out to form the basis of the day's work, the fixing of points on the contours on either side can be commenced Let it be assumed that the contour interval is 2 ft , and that the instrument is held 5 ft above the ground. Send the staffman uphill until a reading of (5—2) = 3 ft is obtained, and fix a few suitable points with that reading The next series gives a reading of 1 ft., but to locate the higher contours the surveyor must take up a position over one of these points and start anew. In setting stakes on the downhill side of the reference contour, the readings from it will be 7, 9, etc., ft. If a sufficiently long rod is not available, the staffman can hold his pole on the reference contour, while the surveyor places himself approximately on the lower contour and shifts about until he sights 3 ft. as before. Having in this manner located portions of the several contours with respect to the initial point, the other points on the reference contour are utilised in the same manner.

Notes.—(1) Greater accuracy is attained if the preliminary work of levelling from the B.M. and setting out the reference contour is performed by dumpy level. A reference contour may be dispensed with by establishing a number of temporary B.M.'s over the area by means of ordinary levelling.

(2) If the contour interval exceeds the height of the instrument above the ground, the contour above the reference contour cannot be located directly from it, but the levelling must be carried up from the reference points in a series of stages.

Direct Horizontal Control.—The survey of the positions of the stakes defining the contours is a somewhat tedious operation. The system to be adopted must be decided upon from consideration of the size and shape of the area involved and of the accuracy required. For small areas, chain surveying may prove suitable, but in general it is necessary to execute a traverse, either by theodolite, compass, or plane table. If contouring is confined to a narrow strip of ground, a single traverse can be run approximately along the centre line, and offsets taken to the stakes on either side. Some of these offsets may be longer than is generally desirable, but this is not a serious objection, and one or two 100 ft. tapes should be carried. When the width is too great for effective control by a single traverse, a network must be laid out.

The survey should be conducted as soon as possible after the location to avoid possible errors due to displacement of the stakes. If only one surveyor is in the field, he should therefore locate and survey on alternate days, or employ the forenoons for locating and the afternoons for surveying. A considerable saving of time may be effected by the use of the tacheometer, which, although best adapted for indirect contouring, can be utilised in the direct system for horizontal control only. As soon as the staffman has been placed on a contour by the leveller, an observation is taken for the bearing and distance of the staff from the tacheometer set over a traverse station. In this way the need for marking located points is eliminated.

However the points are surveyed, a note must be taken of the contours on which they are situated, this information being obtained by reading the tag on the stake when offsetting, or from the leveller in the above tacheometric system. When the points are plotted, the contours are drawn through them as curved lines, due attention being paid that the fundamental characteristics of contours are nowhere violated.

Indirect Methods.—In these methods, the points to be completely located may be either (1) situated along a series of straight lines set out over the area, or (2) scattered representative points In (1), the straight lines simply constitute lines of sections, and in general no cognisance is taken of the ground between these sections. In (2), more particular attention can be paid to the salient features upon which the topography depends.

(1*a*) *By Cross Sections*—Suitably spaced sections are projected from traverse lines, the observations being made in the usual manner by level, clinometer, or theodolite The sections should be spaced more closely than usual at places where the contours curve abruptly, as on spurs and in ravines, and in the latter case it is expedient to run a section approximately along the line of the stream, taking its direction by compass The configuration may in places suggest the running of a number of sections radiating from a point

The sections need not be plotted if an ordinary level has been used, so that the reduced levels of the various points are known To draw the contours, the points levelled are first marked off along the section lines, and the elevation of each point is written against it On the assumption of uniform slope between adjacent points, the contours are then interpolated by estimation as shown in Fig 223

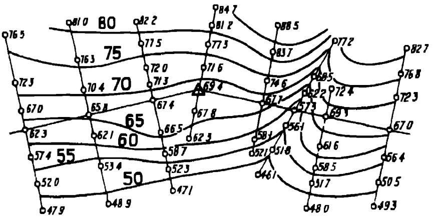


FIG 223.

If the sections are plotted, the interpolation may be performed more quickly and mechanically by the following method, which must be used when the sectioning is performed by clinometer Rule on a sheet of tracing paper a series of equidistant parallel straight lines, the distance between which represents the contour interval to the scale of the sections Place this sheet over a section in such a position relatively to its datum or its ground line that the ruled lines coincide with the intersecting planes defining the required contours. In Fig 224, the lines on the tracing paper are shown dotted, the interval between them being 5 ft. The tracing has been placed so that one of the lines coincides with the datum line of the section, and, reckoning upwards, it will be observed that the ground surface is intersected by lines of elevations 65, 70, 75, and 80. The distances from the centre line to those intersections on right and left are transferred to the section line on the plan, and the contours are then drawn as in the direct method.

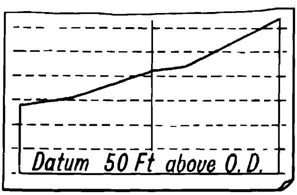


FIG. 224.

Notes.—(1) If the base lines of the cross sections have not been drawn, it is necessary to subdivide a portion of the vertical centre line on the tracing paper into feet. The tracing paper can then be placed with the horizontal lines in their proper position relatively to the centre line level marked on the cross section

(2) The transfer of the right and left distances from the section to the plan is expeditiously performed if the work is shared by two men, one using the sections and calling out the distances and elevations of the various contours, the other plotting the points on the plan and drawing the contours through them

The cross section method of contouring does not fall far short of the direct method in point of accuracy, provided always that additional sections are run where called for, and that the ground is fairly uniform in slope between the points located It has the merit that the sections can be employed in connection with estimates and construction

(1*b*) *By Squares* —If the area to be contoured is not very extensive, it may be divided up into a series of squares, the corners of which define the points to be levelled Bounding lines at right angles to each other are set out to enclose the area, and ranging poles are placed at regular intervals along them If the ground is not too rough, the pegging out of each separate square may be obviated by providing sufficient poles (as in Fig 225) to enable the staffman to place himself at each point To prevent misunderstanding, every fifth line may be flagged, and, for referencing, one set of lines should be lettered and the other numbered If the site is one where it is proposed to carry out earthwork, the enclosing lines should be permanently pegged for convenience in running levels for progress measurements

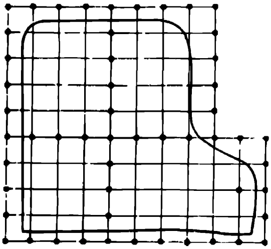


Fig 225.

The interpolation is similar to that of the previous case, and is exhibited in Fig 226 It is to be observed that the assumption of uniform slope between the located points is not so justifiable here as in the cross section method, but the smaller the squares the more valid will be this hypothesis. The sizes employed range from 10 ft to 100 ft. side, small squares being used for close contouring or on rough ground. In ground of varying character, the squares need not be of the same dimensions

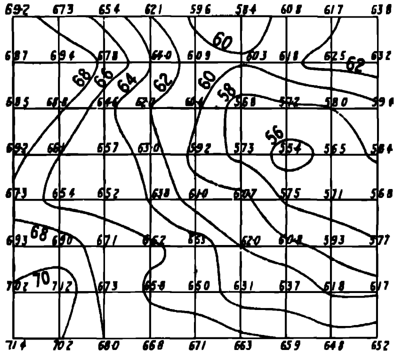


Fig. 226.

throughout, but can be reduced on rugged parts, or, alternatively, a few additional observations on points within the squares can be located on the diagonals or by other measurements from the corners

(2) *By Representative Points* —If representative points only are observed, the number required for contour location is considerably reduced, but the facility of horizontal control possessed by the previous two methods does not obtain Notwithstanding, this system is the most popular, particularly on large surveys, because of its suitability for tacheometrical methods, which, since they are designed to furnish both horizontal and vertical control, are specially adapted for contouring This branch of surveying is described in Chapter IX

Contour Drawing.—The contour lines, having been drawn in pencil, are inked in, either in black or brown, the latter being preferred as being less likely to lead to confusion where roads, buildings, etc , appear on the plan A drawing pen gives a better line than a writing pen, and French curves should be used as much as possible The elevations of the contours must be written against them in a uniform manner, either on the higher side or in a gap left in the line The figures should be normal to the contours, and should lie either in a straight line or along a curve intersecting the contours at right angles Several rows of figures are required on a large sheet In small scale mapping it is sufficient to figure every fifth contour, which should be distinguished by a bolder line, but the values of intermediate contours must be shown where there is a possibility of misinterpretation.

Interpolation of Contours.—When contours have been directly located with a rather wide interval, it is a common practice to draw intermediate contours by interpolation These should be distinguished by dotted lines It is usually amply sufficient to interpolate by estimation On the supposition of uniform slopes, interpolation between plotted contours, or between given points, can also be performed :

(1) By calculation of the intermediate positions. This method is unwarrantably tedious

(2) Mechanically.

(*a*) By superimposing a piece of tracing paper on which are ruled a number of equally spaced radiating lines (Fig. 227), and moving it about until an imaginary line as nearly as possible normal to both contours, or the line between the points, is suitably divided by the rays. The points of division are then pricked through.

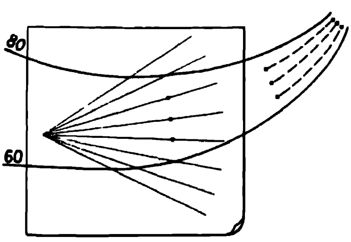


FIG. 227.

(*b*) By marking off a number of small equal divisions on a flat rubber band, which can then be placed on the drawing and stretched so that the appropriate number of divisions is intercepted between the given contours or points The intermediate points are then ticked off

Contour Gradients.—A line lying throughout on the surface of the ground, and preserving a constant inclination to the horizontal, is called a contour gradient The path of such a line can be traced in the field or on a map (page 274), if its inclination is known, and it is specified to pass through a given point

In the field, the location is most quickly performed by means of a clinometer, theodolite, or gradienter (page 393) The instrument is first set up at the given point, and the line of sight is laid at the given inclination A man, carrying a level staff, or simply a graduated ranging pole, proceeds in the estimated direction of the contour gradient to a greater or less distance from the instrument, according as the ground is of uniform slope or undulating He holds up the rod, and is directed by the instrument man to move up or down hill until the rod reading equals the height of the instrument above the ground The line from the instrument station to the point on which the rod is held is then parallel to the line of sight, and forms a part of the required contour gradient The point so obtained is pegged, and is used as the instrument station for the location of the next point in the same manner

Alternatively, a level may be used, and in this case the instrument need not necessarily be set up over the contour gradient Since the line of sight is horizontal, it is necessary to measure out from the starting point, or from the last point pegged, the distance, say 100 ft , to the point to be fixed The required staff reading is computed from this distance, the gradient, and the elevation of the instrument. Thus, if a down gradient of 1 in 75 is being traced, and the line of sight is at 4·2 ft above a peg on it, the reading on a staff 100 ft. from that peg should be 5 5 ft The staffman, holding the end of a 100-ft. tape or chain stretched from the peg, is moved up or down hill until the required reading is observed

Notes —(1) In the clinometric method, the instrument has to be moved forward after each observation, unless the ground surface is plane A hand clinometer is therefore the most convenient of the instruments available In the method of levelling, the same instrument station may be used for the pegging of several points

(2) From any point there will usually be two or more directions in which a given gradient may proceed, so that an infinite number of contour gradients may be laid out from a given origin There is, however, less difficulty than might be supposed in deciding which of the directions should be followed, since the gradient required in practical cases is that which is least winding.

(3) If the gradient as set out has to be surveyed, it is usually sufficient to take compass bearings from peg to peg and note the distances Otherwise, offsets may be taken to the pegs from a traverse with longer courses.

The tracing of contour gradients is a common field operation in the location of road and railway routes When the locating engineer has to find the best route for, say, a road over a range of hills, it is generally impossible to adhere, even roughly, to the straight line route between the controlling points on either side because of the steepness of the ground By laying off a contour gradient equal to the steepest slope which is considered allowable on this part of the road--the limiting or ruling gradient --he discovers a route over the difficult ground which, if it were followed, would enable the road to be constructed without excavation or embankment along its centre line

The contour gradient located is generally that passing through the lowest point on the ridge near the straight line route, and it is best set out from the ridge down both sides of the hill It affords a most useful guide to the location of the road, but its adoption as the actual centre line would result in a very tortuous route except where the ground is of uniform slope In crossing a gorge, for example, the contour gradient runs up the gorge, and crosses the stream in a manner similar to a contour line The road, when constructed, will necessarily be carried across the gorge at a point downstream from the contour gradient, as it must be above the stream The engineer, instead of locating the true contour gradient at such a place, therefore examines the ground for the most suitable site for the road crossing, and projects the contour gradient across in the air It is likewise unnecessary for him to trace the true gradient round small hollows and ridges, at which it would have a sharp curvature At every place, however, where the finally adopted route for the road deviates from the contour gradient of the same value as the road gradient, earthwork will be required on the centre line Deviations downhill necessitate embankments, while those to the uphill side involve excavations

USES OF CONTOUR PLANS AND MAPS

1. Drawing of Sections. A section along any line, straight or curved, on the map can be drawn by marking off on a datum line the distances along the line of section at which the various contours are intersected The respective contour elevations are then set up as ordinates from these points Thus, in Fig 221, *a* is the section AB of the hill represented in the plan *b*.

2. Determination of Intervisibility.—Whether two points of given position and elevation are intervisible or not can be ascertained from the map without having recourse to drawing the section between them. Let it be required to test the points A and B (Fig 228) in this respect Join AB From its length and the known levels of A and B, determine by calculation, or other method of interpolation, the position of the points on it which have the same elevation as the contours These are marked with sloping figures. Determine by inspection whether these points are above or below the ground Examining the point of 90 ft. elevation on AB, it is seen to fall on the lower side of the 90 contour, and therefore is above ground, since a vertical line through it would cut the ground at an elevation of less than 90 ft. Similarly, the points at 80 and 50 ft are clear, but those at 70 and 60 are below the surface, and in consequence A and B are not mutually visible C and D, the points at which AB and the surface are estimated to have the same levels, about 72 and 57 respectively, show the limits of the obstruction The determination of intervisibility, of importance in military work, is sometimes of service to the surveyor The above method of solution does not apply to very long sights on account of the effects of curvature and refraction

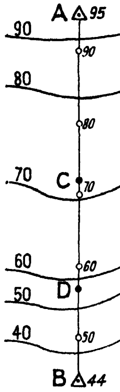


Fig 228

3. Tracing of Contour Gradients.—The method of locating on a map the route of a given contour gradient through a given point is illustrated in Fig 229 in which it is required to trace the upward course from the point A of a contour gradient of 1 in 30 It will be sufficient to locate the points at which the gradient intersects the given contours, and, since the contour interval is 10 ft , the horizontal distance between the successive required points must be

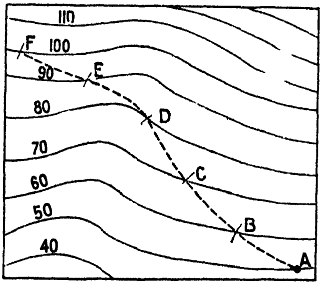


Fig 229

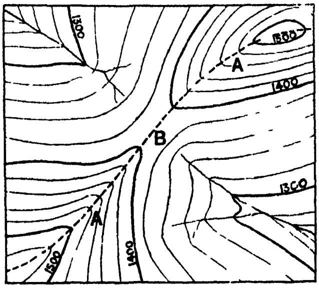


Fig. 230

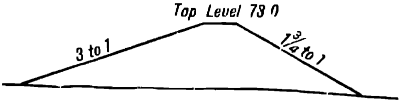
300 ft. With centre A on the 50 contour, describe an arc of this radius to cut the 60 contour at B, and, with centre B, describe a similar arc to cut the next contour, and so on. Join these points with a curve This line represents the path of the contour gradient with sufficient accuracy for most practical purposes notwithstanding that the lengths intercepted between adjacent contours are not now 300 ft. precisely. The actual line would be more tortuous than is shown.

Note.—Each of the arcs described in Fig. 229 will cut its appropriate contour at two points, so that the contour gradient drawn is only one of those which can be marked out from A. The others which could be shown within the limits of the diagram have a zigzag course.

4. Measurement of Drainage Areas.—For water supply and irrigation purposes the extent of these can be estimated by measurement of the area contained within the watershed line separating the basin from those adjoining. This line is to be traced on the map in such position that the ground slopes down on either side of it, *i.e.* it lies along ridges, as at A (Fig. 230), and on cols or passes, B. Care must be observed to ensure that the whole area draining into the valley under consideration is included.

5. Intersection of Surfaces.—If two intersecting surfaces are each represented in plan by a system of contours, then the points in which the contours of one surface cut those of equal elevation belonging to the other are situated on the line of intersection of the two surfaces. Such points are common to both surfaces, and the line of intersection can be drawn by joining up successive points. This construction is of practical utility in affording a rapid method of determining on a plan the boundary of proposed earthwork, which is simply the line in which the earth slopes cut the original surface of the ground.

Thus, in Fig. 231, let it be required to draw the plan of an earth



Section of Dam

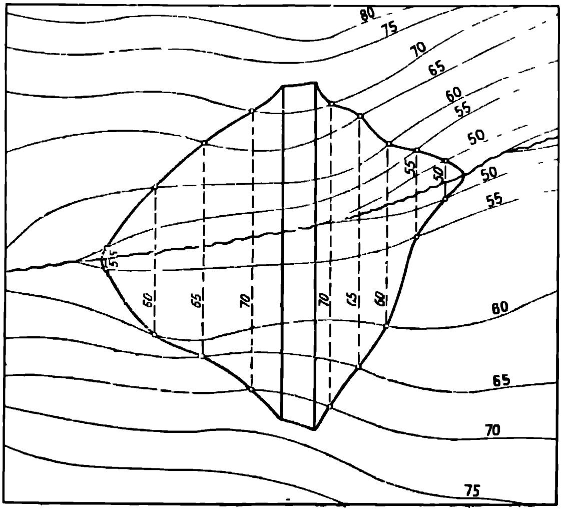


FIG. 231.

dam, of the given dimensions, to be built across a valley Having drawn the top surface in the proposed position, set off the contours of the sloping sides for the same elevations as the contours representing the original ground The new contours are shown dotted, their positions being obtained from the given side slopes and the top level of the dam The line joining the points of intersection of the equivalent contours of the new and original surfaces shows the extent of the earthwork in plan

In the above case, since the top of the dam is level, the contour lines along the side slopes are parallel to the top edges Fig 232

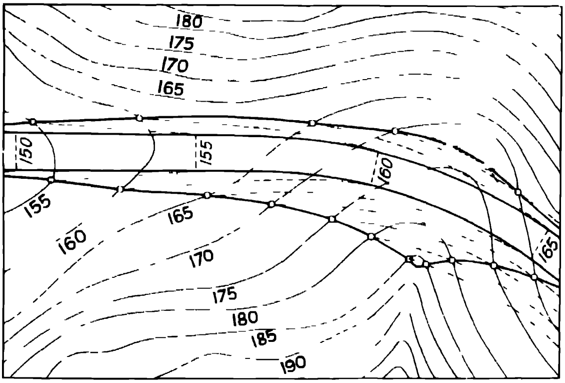


FIG 232

illustrates the method of finding the limits of an excavation for a road which has an upward gradient of 1 in 22 towards the right From a known formation level the positions of contours 150, 155, etc , 110 feet apart, are obtained on the formation surface, and, on the assumption that this surface is horizontal transversely, the contour lines are drawn straight across it Along the side slopes, which are to be $1\frac{1}{2}$ horizontal to 1 vertical, the contours will deviate from the sides of the formation surface at the rate of $7\frac{1}{2}$ ft in 110 ft. The intersection points of the two sets of contours are obtained and joined up as before

A similar application of the principle to earthwork occurs when it is desired to show where excavation and filling respectively will be necessitated in grading a piece of ground This problem is illustrated in Fig. 255, where the solid lines represent the contours of the original ground, and the dotted lines the altered surface The line *aaa*, obtained as above, marks the boundary between cutting and filling , the ground within it must be excavated, and that outside filled within the limits of the diagram

In tracing these intersection lines, there may at times be some doubt as to the direction of the line between the points located on it In such cases, the interpolation of a few intermediate contours on both series will yield additional intersections and fix the line more definitely Thus, in Fig 232, two points are obtained on the upper intersection line by the interpolation of small parts of the 172½ contours

6. Measurement of Earthwork.—See page 302

CHAPTER VII

MEASUREMENT OF AREAS AND VOLUMES

In this chapter are given the methods employed for the measurement of areas and volumes Needless to say, such measurements are commonly required in civil engineering practice, and form an important branch of office work.

Necessity for System in Calculation.—At the outset it is necessary to emphasise the advantage of a methodical arrangement of calculations The aim should be to set out the work in such a manner that the result of each step can be seen at a glance, and so that all the intermediate work can be followed and checked The calculations should be performed in books of about foolscap size, one being used to show the whole of the figuring, and the other to record the results of the various steps and the final quantities As in all other operations in connection with surveying, verification of the results is necessary before they can be employed for any purpose If the checking is performed by the person who made the original calculations, the most reliable check is afforded by working out the result by a different method, when possible If the method first adopted is considered preferable to any other, care should be taken to ensure that the checking embraces every part of the work involved in the first determination In office work, verification by another person is preferred, and the checker should adopt the same routine, and should work quite independently from beginning to end

Accuracy of Results.--Arithmetical slips being eliminated, the degree of accuracy of the final result will depend upon (*a*) the accuracy of the field work, (*b*) the accuracy of plotting, when the calculations are made from scaled measurements, (*c*) the method of calculation adopted It must be realised that the data employed in computation are subject to error and that the accuracy of the final result cannot be increased by needless refinement of figuring.

As there is considerable difficulty in assessing the precision of the data, it is not easy to judge the degree of refinement justifiable in calculation. One must, to a large extent, estimate the probable accuracy of the field and office work, and the final result should be expressed with no more significant figures than can be relied upon.

Note.—The beginner is apt to overlook this point, and, in computing areas for example, would find it helpful to form a mental picture of the extent of ground corresponding to the last significant figure in his result. Keeping in view the probable accuracy of the field methods and the total area in any case, he will thus acquire some conception of how to express results consistently with the value of the data.

MEASUREMENT OF AREA

By the area of a piece of ground is meant its area in *plan*. The British unit of land measurement is the imperial acre, 1/640th of a square mile, subdivided as follows:

1 acre = 4 roods = 160 poles (or perches),
= 10 sq. chains = 100,000 sq. links,
= 4,840 sq. yards = 43,560 sq. feet.

Fractional parts of an acre are expressed either in decimals or in roods and poles.

The decimal connection between the acre and the square chain facilitates reduction when the Gunter chain has been used in the field. For example, let the area of a piece of ground be calculated as 349,400 sq. links. By moving the decimal point five places to the left we have at once 3·49400 acres, which, having regard to the circumstances of the measurement, might be stated as 3·494 acres. To convert the fractional part to roods and poles, we have:

$$\begin{array}{r} 3\cdot494 \\ 4 \\ \hline 1\cdot976 \\ 40 \\ \hline 39\cdot040 \end{array}$$

or 3 acres 1 rood 39 poles

General Methods of Measurement.—Areas may be obtained (*a*) from the plotted plan, (*b*) by direct use of the field notes. The former is the more common and less troublesome method, but the latter is susceptible of greater accuracy since errors introduced in plotting and scaling are eliminated.

Measurement from Plan.—The area is found either (*a*) by dividing the plot into geometrical figures or by ordinates and computing from their scaled dimensions, or (*b*) mechanically by planimeter.

By Division into Geometrical Figures.—(*a*) The most convenient method is to divide up the survey into triangles, either by pencil lines on the plan or on superimposed tracing paper. The base length and perpendicular height of each triangle are scaled and noted, care being taken not to omit any nor to record the same one twice. When one or more of the boundaries of the plot of ground consist of irregular curves, these lines are replaced by straight ones such that the area contained within them is equal to that of the original figure This "equalising" of the boundaries is easily accomplished by the use of a piece of black thread or a transparent set-square, the straights being drawn so that, as nearly as can be judged, the areas contained between them and the irregular boundary are equally disposed on either side of the straight (Fig 233) No attempt should be made to equalise large irregularities, but small triangles should be introduced in such cases

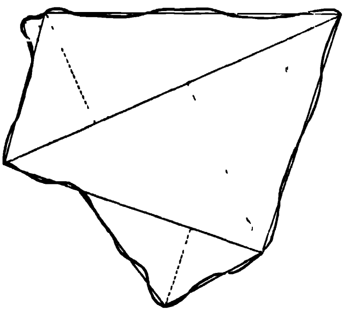


FIG 233

(b) A somewhat similar method consists in placing over the drawing a piece of tracing paper ruled into squares, which may conveniently be an exact fraction of an acre to the scale of the survey (Fig 234) The number of complete squares within the boundary is counted, but there are in addition a number of partial squares next the boundary, the areas of which have to be evaluated If a rough estimate only is required, and the squares are small, it may be sufficient to judge these fractional areas by eye in terms of a whole square, the resulting errors tending to a certain extent to compensate A better result is obtained, however, if each partial area is computed from its scaled dimensions, the irregularities being equalised where necessary

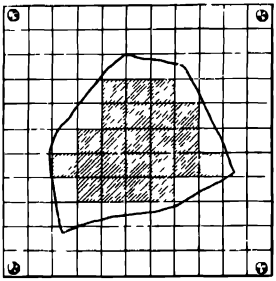


FIG 234.

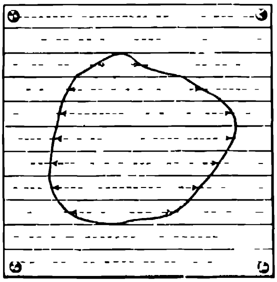


FIG 235

(c) A third method is to divide the area into a series of strips by equidistant parallel straight lines ruled on tracing paper, the constant distance between them being some round number of feet or links Midway between each pair of lines there is drawn another in a different colour or dotted (Fig 235) The tracing paper is placed over the drawing, and is moved about until the area to be measured lies between two of the full lines The figure is thus divided into strips of constant breadth, and, if it is assumed that the strips are either trapezoids or triangles, the area of each is expressed by the length of the mid-ordinate times the breadth The lengths of the dotted lines intercepted within the boundary are therefore scaled, and the total sum of their lengths times the constant breadth gives the required area The assumption that portions of the boundary between adjacent full lines are straight causes some of the partial areas to be over- and others under-estimated, the errors being compensating The smaller the constant breadth of the strips, the more nearly will the correct result be approached

By Ordinates.—Calculation by ordinates is similar to the last method, and is suitable for the case of a long narrow strip of ground, such as that occupied by a road or railway A line is described axially through the area, and at equal distances along it perpendicular ordinates are drawn, and their lengths scaled From these lengths and their common distance apart the area can be calculated by (*a*) the Trapezoidal Rule, (*b*) Simpson's Rule

Trapezoidal Rule —In this method it is assumed that the short lengths of the boundary between the ordinates are straight lines, so that the area is divided into a series of trapezoids (Fig 236)

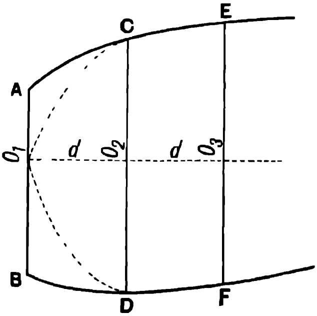


FIG 236

If d = the common distance between the ordinates,
$o_1, o_2, o_3,$ o_n = the lengths of the ordinates ,

$$\text{then area of first trapezoid} = \frac{o_1 + o_2}{2} \times d,$$

$$,, \quad \text{second} \quad ,, \quad = \frac{o_2 + o_3}{2} \times d,$$

$$,, \quad \text{last} \quad ,, \quad = \frac{o_{n-1} + o_n}{2} \times d$$

Summing up, we have the total area,

$$A = d\left[\frac{o_1}{2} + o_2 + o_3 \quad . + o_{n-1} + \frac{o_n}{2}\right]$$

The required area therefore equals the common distance apart of the ordinates multiplied by the sum of half the first and last ordinates plus all the others

Caution must be exercised in the case where there is an apex at one or both ends of the figure, as shown dotted, resulting in o_1 or o_n being zero These must not be omitted from the formula, a not uncommon mistake being to disregard them and to treat o_2 and o_{n-1} as the first and last ordinates.

Simpson's Rule—This rule assumes that the short lengths of boundary between the ordinates are parabolic arcs. With the previous notation, we have (Fig 236)

$$\text{Area of first two divisions} = \frac{o_1+o_3}{2}\times 2d+\frac{2}{3}\left[o_2-\left(\frac{o_1+o_3}{2}\right)\right]\times 2d,$$

$$=\frac{d}{3}\left(o_1+4o_2+o_3\right),$$

since they are equivalent to the trapezoid ABEF, obtained by joining AE and BF, and the parabolic segments ACE and BDF.

$$\text{Area of next two divisions} = \frac{o_3+o_5}{2}\times 2d+\frac{2}{3}\left[o_4-\left(\frac{o_3+o_5}{2}\right)\right]\times 2d,$$

$$=\frac{d}{3}\left(o_3+4o_4+o_5\right),$$

and so on to the last two divisions, the area of which is given by

$$\frac{d}{3}\left(o_{n-2}+4o_{n-1}+o_n\right).$$

Summing up, we have the total area,

$$A=\frac{d}{3}\left[o_1+4o_2+2o_3+4o_4 \quad \ldots +2o_{n-2}+4o_{n-1}+o_n\right].$$

Note—It is not necessary that the curves should be concave towards the axis, as in Fig 236 If, in the case of the first two divisions, o_2 is less than $\frac{o_1+o_3}{2}$,

$$\text{the area of the first two divisions} = \frac{o_1+o_3}{2}\times 2d-\frac{2}{3}\left[\left(\frac{o_1+o_3}{2}\right)-o_2\right]\times 2d,$$

$$=\frac{d}{3}\left(o_1+4o_2+o_3\right), \text{ as before}$$

This method necessitates an even number of divisions of the area, i e the total number of ordinates must be odd If the order of the multipliers, 1, 4, 2, 4, . . 2, 4, 1, is memorised, confusion will be avoided The remarks about the case where one or both of the end ordinates are zero again apply

Simpson's rule is likely to prove more accurate than the trapezoidal formula, but their relative accuracy depends upon the shape of the figure For the calculation of the area within a regular curve, Simpson's rule is correct not only in the case of $y=a+bx+cx^2$, but also for $y=a+bx+cx^2+dx^3$. In dealing with irregularly shaped figures, the degree of precision of either method can be increased by increasing the number of ordinates. Several other rules have been formulated, but the above two are sufficient for practical purposes.

By Planimeter.—The planimeter is a mechanical integrator used for the measurement of plotted areas. The various forms which have been devised possess the common feature that a point of the instrument is guided round the boundary of the area, and the resulting displacement of another part of the mechanism is such as to enable the area to be recorded The most commonly used form is Amsler's polar planimeter, and it only will be considered

Principle of the Planimeter.—The principle of the instrument may be followed from consideration of the area swept out by a moving line In Fig 237, let a straight line of length l, the ends of which move in given loci, be given a small displacement from AB to A′B′ The motion is equivalent to a normal translation to A″B′ and a rotation about B′ through $d\theta$ To the first order of infinitesimals, the area swept out is

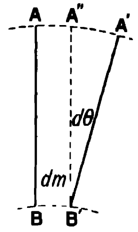


FIG 237.

$$dA = ldm + \tfrac{1}{2}l^2 d\theta,$$

where dm = the distance between the parallels AB and A″B′,
= the normal displacement of B

Let A describe a closed curve, B being constrained to move along a line and returning to its initial position thereon when A completes a circuit With the convention that areas swept out to the right are positive and to the left negative, the net area A swept out by AB during a circuit by A is the area round which A has travelled, and

$$A = l\int dm + \tfrac{1}{2}l^2\int d\theta,$$
$$= lm, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

since $\int d\theta = 0$ when AB returns to its initial position

If B also describes a closed figure of area B, outside or partially outside that traced by A, then, since the line returns to its original position,

$$A - B = lm \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

If B describes a closed curve of area B completely inside that traced by A, then

$$A - B = lm + \tfrac{1}{2}l^2 2\pi, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

since AB then makes a complete revolution, so that $\int d\theta = 2\pi$

The foregoing principle can be applied to the measurement of the area A round which A travels, if the area of the locus of B is known, and if m, the total normal displacement of B, can be determined. It may be more convenient to obtain the total normal displacement n of any other point W on AB If W is distant a from B, and if dn represents a small normal displacement of W, then

$$dn = dm + ad\theta,$$

a being taken as positive if W is between B and A or on BA produced beyond A, and negative if W is on AB produced beyond B. The results (1), (2), and (3) above now become ·

$$A = ln, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$
$$A - B = ln, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$
$$A - B = ln + (\tfrac{1}{2}l^2 - la)2\pi \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Amsler's Polar Planimeter.—This instrument (Fig 238) consists of two bars AB and BC hinged at B The first carries at A a tracing point, which is guided round the boundary of the area to be measured C is a stationary point or pole, the bar BC terminating at C with a needle point, which is fixed in the paper and held down by a weight B is therefore constrained to move along the circumference of a circle of radius CB while the tracing point travels round the area under measurement

The normal displacement is measured at W by means of a wheel, the plane of which is perpendicular to AB As AB moves, the wheel partly rotates, and partly slides, over the paper The amount of rotation measures the total normal displacement the axial component of the motion causes slip, and does not affect the record

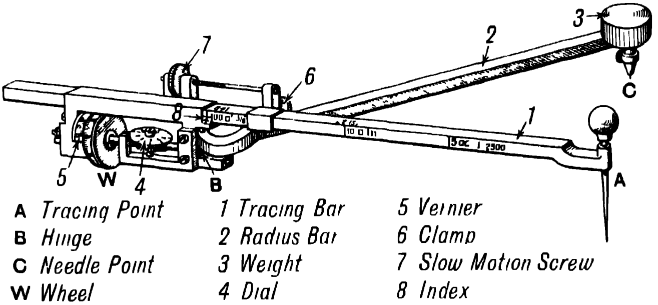


FIG 238 —AMSLER'S POLAR PLANIMETER

The wheel is geared to a dial *4*, which shows the number of revolutions made by the wheel, ten revolutions of the wheel corresponding to one of the dial The wheel carries a graduated drum divided into 100 parts, which are subdivided to tenths by a vernier In the instrument illustrated, the distance between A and B is adjustable, and, on setting the appropriate mark on the tracing arm opposite the index *8* by means of the clamp and tangent screw, readings are obtained in square inches, square centimetres, acres, etc

Use of the Planimeter.—To measure an area, first adjust the tracing arm so that the result may be given in the desired units Fix the needle point in the paper outside the area and in a position which will enable the tracing point to reach all parts of the boundary. Mark a point on the boundary from which to start, and place the tracing point there. Read the dial and wheel, or, alternatively, set these to zero Guide the tracing point in a clockwise direction along the boundary, and, on returning it exactly to the starting-point, again read the dial and wheel The final reading less the initial reading gives the required area

In this case, B oscillates along a circular arc, and returns to its initial position without sweeping out any area, so that, from (4) above,

$$A - ln = l \times 2\pi\rho \times n,$$

where ρ and n are the radius and number of revolutions respectively of the wheel The graduation of the recording dial is, however, such that the required area is read directly

If the area is too large to permit of the tracing point traversing the boundary with the pole outside the area, the figure may be divided into parts, which are separately measured Alternatively, the needle may be fixed inside the boundary The point B then describes a complete circle within the area traced by A, and, from (6) above,

$$A - B = ln + (\tfrac{1}{2}l^2 + la)2\pi,$$

the positive sign being used since a is negative

Denoting the length of the radius arm CB by b, then, since $B = \pi b^2$,

$$A = ln + (\tfrac{1}{2}l^2 + la + \tfrac{1}{2}b^2)2\pi,$$
$$ln + \pi r^2,$$

r being the distance between C and A when the plane of the wheel passes through C (Fig 239) If A were to travel round the circumference of the circle of radius r, no rotation of the wheel would be caused, and this circle is, in consequence, called the zero circle Its area forms a constant to be added to the recorded result when the needle is placed within the area to be measured The values of the constant for various units and scales are engraved on the tracing arm

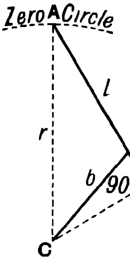


FIG 239

Notes (1) The accuracy of setting of the tracing arm for any particular unit may be verified by drawing a trial square of known size and measuring its area

(2) In moving the tracing point round the area under measurement, it should be guided as far as possible by a straight edge or a French curve Small errors arising from the tracing point not being maintained throughout on the boundary are compensating, and are rendered negligible by making two or three measurements and adopting the average result

(3) In the case of areas too large to be measured with one setting of the instrument with the pole outside, it is generally preferred to make the measurement in parts, as a precaution against error in the engraved value of the constant The constant can, however, be verified once for all by comparison of the result so obtained with that given with the pole within the area

(4) The plan should be in a horizontal position, not on an inclined drawing board The surface of the paper on which the wheel rolls must be smooth, but not too highly glazed Satisfactory results cannot be expected from drawings which have been folded

Measurement from Field Notes. –The area is divisible into (*a*) that enclosed by the outside survey lines, (*b*) the irregular strips between those lines and the boundary The method of calculating the former depends upon whether the survey has been executed by chain only or by traversing The areas of the strips are positive or negative, and, since they are divided up by offsets between which the boundary is sensibly straight, are computed as a series of trapezoids

Area from Chain Survey Notes.—The triangles composing the area within the bounding chain lines are calculated from the lengths of the three sides by the formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where a, b, c are the sides, and s is the semi-perimeter

In computing the areas of the irregular strips, the mean of each successive pair of adjacent offsets is multiplied by the chainage between them, and the results are summed, areas within the extreme survey lines being regarded as negative The field work should be arranged to obviate difficulty with the corner areas The best way is to take offsets, such as Aa, Aa' (Fig 240) at each corner station from both lines meeting there, with such additional measurements as will define the boundary between them Otherwise, the strips are taken as beginning with the offset from the initial station of each line, and the line is continued to cut the boundary beyond the end station The separate strips obtained are shown hatched Small overlaps, shown black, occur at corners where the internal angle exceeds 90°, and a part is omitted at D, where the angle is less than 90° These will not seriously affect the accuracy of the result, and can be scaled if necessary.

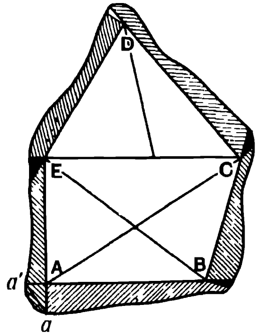


FIG 240

Area from Co-ordinates.—The figure contained by the survey lines of a closed traverse may be so divided up that its area can be computed from the latitudes and departures of the lines Two methods are used

1 *By Latitudes and Longitudes.*—The meridian passing through the most westerly station of the survey is to be termed the reference meridian

The *Longitude* of any survey line is the perpendicular distance of the mid-point of the line from the reference meridian. The longitude of either of the lines meeting at the most westerly station equals half the departure of those lines A sketch will show that the longitude of any other line may be expressed as the algebraic sum of the longitude of the preceding line plus half the departure of that line plus half its own departure

Rule . *The area within the lines of a closed traverse survey is the algebraic sum of the products of the latitude of each line by its longitude*

Since the reference meridian is that of the most westerly station, all longitudes are east or positive quantities Latitudes are reckoned positive if north, and negative if south, as before.

To test the truth of the rule, let the survey ABCDE (Fig. 241) be considered. Perpendiculars BB′, CC′, etc., drawn from each station to the reference meridian, form a number of figures, one side of each of which is a survey line, another side being a portion of the reference meridian equal to the latitude of that survey line. In general, these figures are trapezoids, but the trapezoids reduce to triangles in the case of the two lines meeting on the reference meridian. The area of each, whether a triangle or trapezoid, is given by the latitude of the line to which it belongs times the longitude L of that line. Now, if from the sum of the two areas CDD′C′ and DEE′D′, which belong to lines having positive latitudes, we subtract those pertaining to negative latitudes, *viz* ABB′, BCC′B′, and EAE′, the remainder is the required area ABCDE.

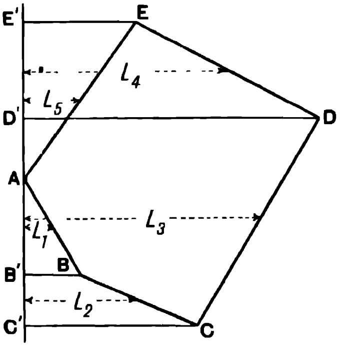


Fig. 241

Since the computation of the longitudes involves halving the departures, it is more convenient and expeditious to omit this step and to work with "double longitudes." The double longitude of either of the lines meeting at the most westerly station equals the departure of the line, and that of any other line is the algebraic sum of the double longitude of the preceding line plus the departure of that line plus its own departure. The algebraic sum of the products of the latitude of each line by its double longitude is then twice the required area.

Before proceeding to the determination of the area, the co-ordinates must be balanced. The calculation is best set out by adding columns to the table of co-ordinates, as in the following numerical example of the survey of Fig. 241. It is desirable, although not essential, that the first line tabulated should be that starting from the most westerly station, and rearrangement of the table may be required to effect this.

Line	Latitude in Links	Departure in Links	Double Longitude	Latitude × Double Longitude +	Latitude × Double Longitude −
AB	−298	+169	169		50,362
BC	−151	+362	700		105,700
CD	+630	+383	1,445	910,350	
DE	+301	−560	1,268	381,668	
EA	−482	−354	354		170,628
				1,292,018	326,690
				326,690	

Algebraic sum = twice area = 965,328 sq. links
Area = 482,664 sq. links,
= 4·827 acres.

In this example the positive products happen to exceed the negative products, but it is immaterial whether the algebraic sum is positive or negative The final division by two is apt to be overlooked, but the seriousness of this error contributes to its detection.

2 *By Departures and Total Latitudes* – The total latitudes of the various stations, as used in plotting the survey, may be employed for the calculation of the area according to the following

Rule *The area within the lines of a closed traverse survey is the algebraic sum of the products of the total latitude of each station by half the algebraic sum of the departures of the two lines meeting at the station*

Any station may be adopted as the reference point from which to calculate the total latitudes of the others In the survey ABCDE (Fig 242), A is used for this purpose, and the total latitudes of the

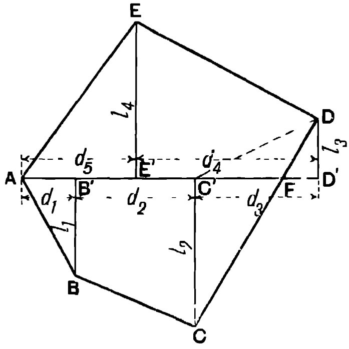


FIG 242

stations are shown as the lengths of the perpendiculars dropped from them on the east and west line through A The traverse is thus divided into triangles and trapezoids, and its area may be expressed as

$$\frac{d_1l_1}{2}+\frac{d_2(l_1+l_2)}{2}+\frac{d_4(l_3+l_4)}{2}+\frac{d_5l_4}{2}+\triangle C'CF-\triangle DFD',$$

but, joining C'D,

$$\triangle C'CF-\triangle DFD'=\triangle C'CD-\triangle DC'D',$$

$$=\frac{d_3l_2}{2}-\frac{d_3l_3}{2}$$

$$\therefore\ \text{Area}=\frac{d_1l_1}{2}+\frac{d_2(l_1+l_2)}{2}+\frac{d_3(l_2-l_3)}{2}+\frac{d_4(l_3+l_4)}{2}+\frac{d_5l_4}{2},$$

$$=\frac{l_1(d_1+d_2)}{2}+\frac{l_2(d_2+d_3)}{2}+\frac{l_3(-d_3+d_4)}{2}+\frac{l_4(d_4+d_5)}{2}.$$

On changing the signs throughout, it will be seen that, with the usual conventions as to signs, the above rule is verified, l_3d_3 being the only positive product, since l_3, being north, and d_3, east, are both positive.

As before, the division by two may be left till the finish if the tabulated products are the total latitude of each station by the algebraic sum of the departures of the lines meeting there

Working out the same numerical example as before, we have

Line	Latitude in Links	Departure in Links	Station	Total Latitude	Sum of Adjoining Departures	Products	
						+	−
AB	−298	+169	A				
BC	−151	+362	B	−298	+531		158,238
CD	+630	+383	C	−449	+745		334,505
DE	+301	−560	D	+181	−177		32,037
EA	−482	−354	E	+482	−914		440,548

Algebraic sum = twice area = 965,328 sq links
Area = 482,664 sq links,
= 4 827 acres

Since total latitudes are utilised for plotting the survey, this method involves less extra work than the first A satisfactory check is afforded by making the calculations by both methods.

Note—The area of the polygon is obtainable independently of the co-ordinates by dividing it into triangles and using the formula,

$$A = \tfrac{1}{2} ab \sin C,$$

where a and b are adjacent sides, and C is the angle they include In the case of figures of more than four sides, this method necessitates solution of the interior triangles, and proves very tedious

MEASUREMENT OF VOLUME

The reference made at the beginning of the chapter to the sources of error affecting the accuracy of calculated quantities applies with special force to the measurement of volume The calculations are based upon the results of levelling, and the field work is subject to the usual errors of observation, but an additional and more important source of inaccuracy consists in the deliberate omission to record many of the surface irregularities Although it would in general be a mere waste of time to measure all the minor undulations, the value of the field work is greatly increased by so setting out the lines of levels that the most faithful record of the surface features may be secured

The methods of calculation based on the field measurements are not of a precise nature, but involve assumptions as to the geometrical form of the solid which may still further impair the accuracy of the result. It is, however, desirable that the degree of refinement to which both field work and calculations are carried should mutually correspond.

General Methods of Measurement.—The methods of calculation may be classed according as the form of the solid is defined by (*a*) cross sections, (*b*) spot levels, (*c*) contour lines

Because of its general application, the measurement of earthwork is more particularly referred to below The unit employed for such quantities is the cubic yard When the methods are directed to the estimation of the capacity of a reservoir, the results are expressed in millions of gallons.

Measurement from Cross Sections

In this universally applicable method, the total volume is divided into a series of solids by the planes of cross sections The spacing of the sections should depend upon the character of the ground and the accuracy required in the measurement They are generally run at 66 or 100 ft centres, but sections should also be taken at points of change from excavation to embankment, if these are known, and at places where a marked change of slope occurs either longitudinally or transversely

Except when the calculations are made directly from the field notes, the sections are plotted, without vertical exaggeration , and, in the case of earthwork measurement, the new surface is represented on each, its level being obtained from the profile. On drawing the side slopes, the sectional form of the earth to be handled is represented on each section, and, considering two adjacent cross sections, these areas form the plane ends of a solid of length *L* equal to the distance between the sections (Fig 243) The cubic content of each of these solids is to be determined separately, the addition of those in cutting giving the total volume of excavation, and similarly for banking

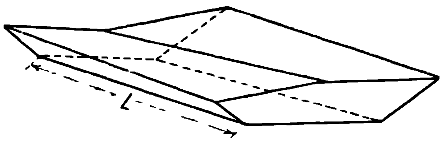


Fig 243

In water supply work, measurement of the cubic capacity of a proposed reservoir has to be made for various water levels to determine the surface level and area of the reservoir when impounding a given volume In this case, therefore, water levels are drawn across the sections, and the volume up to any of them is computed in the same way as for earthwork

Formulæ for Cross Sections.—In the case of the comparatively short cross sections defining the dimensions of the earthwork in road or railway construction, the form of the original ground surface may be determined by only a few field observations In the process of setting slope pegs (page 355), the particulars obtained are the side widths and side heights, as defined below, the depth of the earthwork on the centre line, with or without that at other points. In cross sections observed by clinometer, the usual data available are the centre depth of earthwork and the slope of the original ground on either side of the centre line

Formulæ for the dimensions of cross sections of cuttings and embankments for the simplest cases are given below, and should be verified by the student as exercises

Let $b =$ the constant formation, or "sub-grade," breadth,
s horizontal to 1 vertical $=$ the side slopes,
n horizontal to 1 vertical $=$ the transverse slope of the original ground,
$c =$ the depth of earthwork on the centre line,
d and $d_1 =$ the side widths, or "half" breadths, i e the horizontal distances from the centre line to the intersections of the side slopes with the original surface,
$D = d + d_1$,
h and $h_1 =$ the side heights, i e the vertical distances from formation level to the intersections of the side slopes with the original surface,
$= \frac{d - b/2}{s}$ and $\frac{d_1 - b/2}{s}$ respectively,
$w =$ the formation width of excavation in side-hill sections,
$A =$ the sectional area

Level Section (Fig 244)

$$d = d_1 = \frac{b}{2} + sc.$$

$$A = c(b + sc)$$

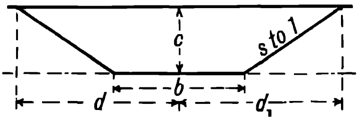


Fig 244

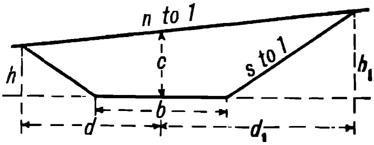


Fig. 245.

Two-Level Section (Fig 245).

$$d = \left[c + \frac{b}{2s}\right]\left[\frac{ns}{n+s}\right].$$

$$d_1 = \left[c + \frac{b}{2s}\right]\left[\frac{ns}{n-s}\right]$$

$$A = \frac{dd_1}{s} - \frac{b^2}{4s},$$

$$= shh_1 + \frac{b}{2}(h + h_1),$$

the formulæ given for the areas of three-level sections also being available.

Three-Level Section (Fig 246)

$$d=\left[c+\frac{b}{2s}\right]\left[\frac{ns}{n+s}\right].$$

$$d_1=\left[c+\frac{b}{2s}\right]\left[\frac{n_1 s}{n_1-s}\right].$$

In this case the expression for d or d_1 may apply to both side widths, according as the ground falls or rises from the centre to both sides

$$A=\frac{D}{2}\left[c+\frac{b}{2s}\right]-\frac{b^2}{4s},$$

$$=\frac{cD}{2}+\frac{b}{4}(h+h_1)$$

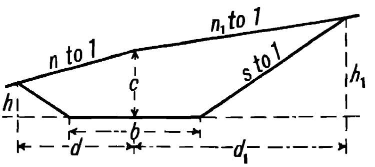


FIG 246.

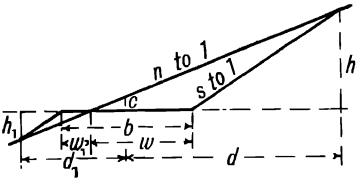


FIG 247

Side-hill Two-Level Section (Fig 247)

The centre line being in excavation, then for the excavation,

$$w=\frac{b}{2}+nc.$$

$$d=\left[c+\frac{b}{2s}\right]\left[\frac{ns}{n-s}\right],$$

$$=\frac{b}{2}+\frac{ws}{n-s}.$$

$$A=\frac{wh}{2}=\frac{w^2}{2(n-s)}.$$

For the embankment, $w_1=(b-w)$ must be substituted for w, and the first formula for side width becomes

$$d_1=\left[\frac{b}{2s}-c\right]\left[\frac{ns}{n-s}\right]$$

When the centre line is in embankment, $w=(\frac{b}{2}-nc)$, and the formulæ again apply.

The Prismoid.—To calculate the volumes of the solids between sections, it must be assumed that they have some geometrical form ;

and, since they most nearly take the form of prismoids, they are considered as such

The *Prismoid* may be defined as a solid having its end faces in parallel planes and consisting of any two polygons, not necessarily of the same number of sides, the longitudinal faces being plane surfaces extending between the end planes

The longitudinal or side faces can be regarded as generated by a straight line extending between the end polygons, and they take the form of triangles, parallelograms, or trapezia having their vertices at those of the end figures

A prismoid can be divided up into a series of prisms, wedges, and pyramids having a common length equal to the perpendicular distance between the parallel end faces The performance of such a division may require the temporary addition to the prismoid of one or more pyramids of the same length

Notes —(1) The prism forms the special case of the prismoid in which the end polygons are equal, the side faces being parallelograms If one of the ends degenerates to a line, and the other is a parallelogram, we have the wedge, the side faces being triangles and parallelograms If one end is a point, the other being any polygon, the solid becomes a pyramid, and the side faces are triangles

(2) The division of a prismoid into these solids of equal length is sometimes troublesome to follow on a sketch, and it will suffice to notice the simple case of Fig 248, in which ABCD and EFGH are the parallel end faces, a distance *L* apart The bottom longitudinal face BCGF is a rectangle, the side faces ABFE and CDHG are trapezia, while the upper longitudinal faces ADH and AHE are triangles By drawing JK on the end face EFGH to make JFGK ABCD, a prism is formed with these end faces To divide up the solid lying above the plane ADKJ, let HL be drawn parallel to KJ to meet FE produced at L HL is in the plane of ADH, and LEH forms the base of a pyramid having its apex at A The addition of this pyramid to the original prismoid makes the upper face one plane surface ADHL, and the solid having for its ends the area LJKH and the line AD can be split up into a wedge of base JKHM, obtained by drawing JM parallel to KH, and a pyramid JMLA Each of the constituent figures has the same length *L* as the prismoid The sum of their volumes, less that of the added pyramid, is the volume of the prismoid, and the algebraic sums of their areas at each end equal the corresponding end areas of the prismoid

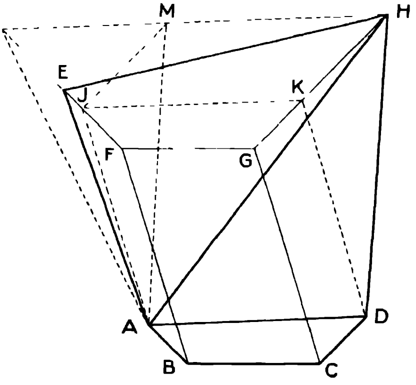


FIG 248

The circumstance that the prismoid can be divided up into prisms, wedges, and pyramids of the same length affords a simple method of deducing a formula for its volume.

Let L = the length of the prismoid, perpendicular to the parallel end planes,
A_1 and A_2 = the areas of those ends,
M = the mid-area, i e the cross sectional area in a plane parallel to the end planes and midway between them,
V = the volume of the prismoid,
a_1, a_2, m, v = the corresponding quantities for any prism, wedge, or pyramid, a_1 being regarded as the basal area in the latter two forms

Then, in the case of a prism, $a_1 = a_2 = m$,
,, ,, wedge, $a_2 = 0$ (a line), and $m = \frac{1}{2}a_1$,
,, ,, pyramid, $a_2 = 0$ (a point), and $m = \frac{1}{4}a_1$

For each prism, $v = La_1 = \dfrac{L}{6}(6a_1) = \dfrac{L}{6}(a_1 + a_2 + 4m)$

,, wedge, $v = \frac{1}{2}La_1 = \dfrac{L}{6}(3a_1) = \dfrac{L}{6}(a_1 + a_2 + 4m)$

,, pyramid, $v = \frac{1}{3}La_1 = \dfrac{L}{6}(2a_1) = \dfrac{L}{6}(a_1 + a_2 + 4m)$

The volume of each of the constituent parts of a prismoid can thus be expressed in the same terms, and the formula must therefore be applicable to any collection of those components With the notation applied to the prismoid, we therefore have the *Prismoidal Formula*,

$$V = \frac{L}{6}(A_1 + A_2 + 4M)$$

It must be distinctly understood that M is not the average of the end areas A_1 and A_2, except in the special case where the solid is composed of prisms and wedges only

Newton's Proof of the Prismoidal Formula.—A more straightforward proof was enunciated by Newton

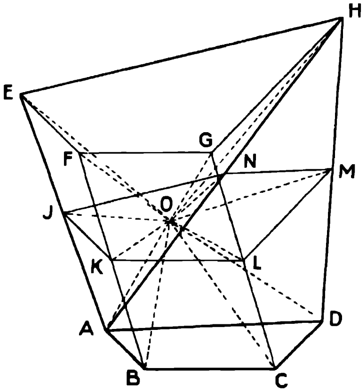


FIG. 249.

In Fig 249, let JKLMN represent the section midway between the end faces ABCD and EFGH and parallel to them Take any point O in the plane of the mid-section, and join O to the vertices of both end polygons The prismoid is thus divided into a number of pyramids, each having its apex at O, and the bases of these pyramids form the end and side faces of the prismoid

Denoting the end areas by A_1 and A_2, and the length of the prismoid by L, the volumes of the pyramids based on the end faces are, respectively, $\frac{1}{6}LA_1$ and $\frac{1}{6}LA_2$

To express the volume of the pyramids based on the longitudinal faces, consider, say, pyramid OABFE, and let the perpendicular distance of O from JK be h, then

$$\text{volume of pyramid OABFE} = \tfrac{1}{3}\text{ABFE} \times h$$
$$= \tfrac{1}{3}L \times \text{JK} \times h = \tfrac{1}{3}L \times 2\triangle\text{OJK}.$$

In the same manner, the volume of pyramid OAEH $= \frac{1}{3}L \times 2\triangle$OJN, and so on for the others, so that

total volume of the lateral pyramids $= \frac{1}{3}L \times 2$ area JKLMN $= \frac{2}{3}LM$

Summing, therefore, we have the volume of the prismoid,

$$V = \tfrac{1}{6}LA_1 + \tfrac{1}{6}LA_2 + \tfrac{2}{3}LM,$$
$$= \frac{L}{6}(A_1 + A_2 + 4M).$$

Comparison of Earth Solid and Prismoid.—The above formula is strictly correct for any prismoid, but it is desirable to compare the earth solid lying between adjacent cross sections with the prismoid as defined

(*a*) The ends of the prismoid are in parallel planes

This requirement is met in the earth solid provided the centre line is straight Since the cross sections are taken at right angles to the centre line, an error may be introduced on curves, owing to the non-parallelism of the ends, but this can be allowed for (page 298)

(*b*) The longitudinal faces of the prismoid are planes

In the earth solid, the new surfaces (the formation surface and side slopes) are frequently plane, and in the majority of cases where they are not, the departure from a plane is negligible in the distance between cross sections The original ground surface does not generally fulfil this condition, but will more nearly do so the greater the care taken in spacing the cross sections so that no marked change in the character of the ground occurs between them It may be shown that the prismoidal formula is strictly applicable to the case where the longitudinal faces of the solid, instead of being plane surfaces, are hyperbolic paraboloids The original ground surface may closely resemble these ruled surfaces if it exhibits gradual changes of transverse slope between cross sections

Calculation by Prismoidal Formula.—Although the solid to be measured may differ considerably from a prismoid, the prismoidal formula is generally accepted as correct for earthwork calculation It is, however, seldom used in a direct manner for ordinary measurements because of the difficulty in estimating the mid-areas These could, of course, be obtained by running mid-sections, but at the cost of doubling the field work Their values can be calculated comparatively simply when the sections are defined by two or three levels by taking the linear dimensions of the mid-area as the mean of the corresponding dimensions of the end areas, but with irregular sections the calculations are unwarrantably tedious

If the cross sections are at equal intervals, the difficulty may be overcome by treating every alternate sectional area as a mid-area

Thus, if $L =$ the distance between the sections,
$A_1, A_2 \ldots A_n =$ their areas,
the volume of the first prismoid of length $2L = \frac{2L}{6}(A_1+A_3+4A_2)$,
,, ,, second ,, ,, $\frac{2L}{6}(A_3+A_5+4A_4)$,
,, ,, last ,, ,, $= \frac{2L}{6}(A_{n-2}+A_n+4A_{n-1})$

Summing up, we have the total volume,

$$V = \frac{L}{3}\Big[A_1+4A_2+2A_3+4A_4+\ldots+2A_{n-2}+4A_{n-1}+A_n\Big],$$

which is Simpson's rule for volumes, an odd number of sections being required for its application

The solids of length $2L$ may, however, differ very considerably from true prismoids, and this method does not yield so good results as individual treatment of the solids between adjacent cross sections

The Method of End Areas.—The difficulty of applying the prismoidal formula to solids of which the mid-area has not been directly measured is avoided in practice by the use of the end areas rule, which states that the volume of a prismoid is given by

$$V = L\left(\frac{A_1+A_2}{2}\right).$$

This formula, although very convenient, is correct only if the mid-area is the mean of the end areas A_1 and A_2, *i e* if the prismoid is composed of prisms and wedges only Since the mid-area of a pyramid is half the average area of the ends, the volume of the general prismoid is overestimated by the formula, but, since the earth solid is not in general exactly represented by a prismoid, the method of end areas may be accepted as sufficiently accurate for most practical purposes, and is that almost exclusively used

The error made in estimating the volume of a prismoid by this method may be readily ascertained for the simple cases dealt with on pages 291 and 292 For the case in which the ground lines of the sections are level (Fig 244), the investigation is as follows

Let $L =$ the distance between adjacent sections,
$b =$ the constant formation breadth,
s hor to 1 vert $=$ the side slopes,
$c, c' =$ the centre heights of the sections,
$A, A' =$ their areas.

Since $A = c(b+sc)$, and $A' = c'(b'+sc')$,

by end areas, $$V = L\left[\frac{bc}{2}+\frac{bc'}{2}+\frac{sc^2}{2}+\frac{sc'^2}{2}\right].$$

Now the mid-area centre height $= \dfrac{(c+c')}{2}$,

$$\therefore M = \frac{(c+c')}{2}\left[b+\frac{s(c+c')}{2}\right],$$ so that, by prismoidal formula,

$$V = \frac{L}{6}\left[c(b+sc)+c'(b+sc')+2(c+c')\left(b+\frac{s(c+c')}{2}\right)\right],$$

$$= L\left[\frac{bc}{2}+\frac{bc'}{2}+\frac{sc^2}{3}+\frac{sc'^2}{3}+\frac{scc'}{3}\right]$$

On subtracting,

Volume by end areas − Volume by prismoidal formula $= \dfrac{Ls}{6}(c-c')^2$.

This quantity is always positive for level sections, and its form shows that a somewhat serious overestimation may result by averaging end areas in cases where adjacent sections differ considerably in area, as, for example, towards the ends of excavations or embankments.

Prismoidal Correction.—If in special cases it is required to obtain a closer approximation than is given by the above method, the precision of the measurement may be increased without undue labour by the application of a correction to the result given by the end area method. The difference between the volume as calculated and that which would be obtained by use of the prismoidal formula is termed the *Prismoidal Correction*, and, if it is known in any case, its application to the approximate result gives the volume as it would be obtained from the prismoidal formula with less trouble than is involved by the direct use of the latter. Since the end area method of calculation overestimates the volume of a prismoid, the correction for the prismoid is always subtractive, but in the case of the actual earth solid the correction, although usually subtractive, may fall to be added to the end area result.

The formula for the correction depends upon the number of levels defining the end areas, and, to deduce it once for all in any case, it is necessary to obtain the expression for the mid-area in order to formulate the volume by the prismoidal method. If the areas are of simple outline, this is accomplished by averaging corresponding linear dimensions, as above. Irregular sectional forms are treated by substituting for them equal areas of simple outline, from the dimensions of which those of a figure corresponding to the mid-area are obtained with more or less accuracy. For the substitution, level, two-level, or three-level equivalent sections are employed according to the degree of accuracy required: further refinement in equalising is of little practical importance.

Formulæ for the Prismoidal Correction.—With the previous notation (page 291), unaccented and accented symbols referring to adjacent sections, the following expressions are obtained in the above manner, and should be verified by the student

Level Sections (Fig. 244)

$$\text{P C} = \frac{Ls}{6}(c-c')^2.$$

Two-Level Sections (Fig 245)

$$\text{P C} = \frac{L}{6s}(d-d')(d_1-d_1')$$

Three-Level Sections (Fig 246)

$$\text{P C} = \frac{L}{12}(D-D')(c-c')$$

Side-hill Two-Level Sections (Fig 247)

The centre line being in excavation,

$$\text{P C for the excavation} = \frac{L}{12s}(w-w')(d-d')$$

$$\text{P C for the embankment} = \frac{L}{12s}(w_1-w_1')(d_1-d_1')$$

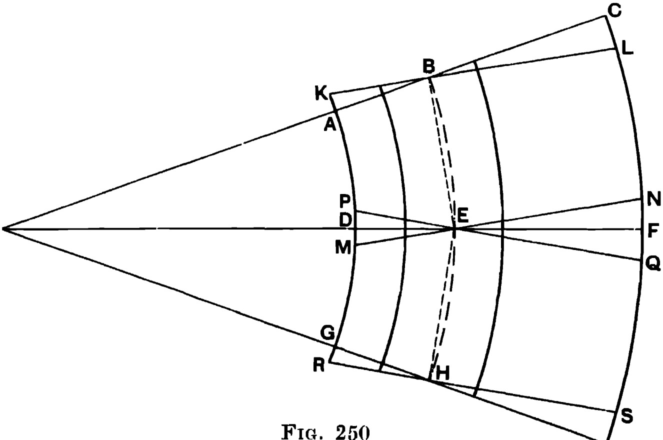


Fig. 250

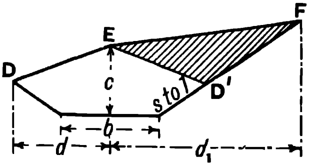


Fig 251.

Curvature Correction.—Cross sections on curves are run in radial lines, and consequently the earth solids between them do not have parallel end planes In computing the volumes of those solids, the common practice is to employ the usual methods, treating the ends as parallel to each other and normal to the chord, and, if circumstances warrant, to apply a correction for curvature

In Fig 250, B, E, and H are successive centre line pegs on a curve, and AC, DF, and GJ are lines of cross sections The solid from B to E is represented in plan by ADFC, but the volume calculated is that of the solid KMNL, the cross-sectional area normal to the chord being for all practical purposes equivalent to that normal to the tangent Evidently the calculated volume is too small by the volumes represented by CBL and FEN and too great by those represented by ABK and DEM When the cross-sectional area is symmetrical about the centre line, these wedge-shaped masses practically balance, and no curvature correction is required The more unsymmetrical the sectional area, the greater is the value of the correction, which falls to be applied positively or negatively to the calculated volume according as the greater half-breadth is on the convex or the concave side of the curve It is usual to compute the amount of correction per station, that for station E being (volume NEQ − volume MEP)

In Fig 251, representing cross section DF, if ED′ be drawn symmetrically with ED, the asymmetry of the sectional area is represented by ED′F, and the volume swept out by this area in turning through the angle NEQ also equals the required correction for station E With the previous notation, area ED′F is expressed by $\left(\frac{d_1-d}{2}\right)\left(c+\frac{b}{2s}\right)$ The solid swept out by this area, and represented in plan by ENQ, is, by Pappus' theorem, the product of the area by the length of the path of its centroid The horizontal distance from the vertical through E to the centroid of ED′F is $\frac{1}{3}(d_1+d)$, and the travel of the centroid is $\frac{L}{3R}(d_1+d)$, where L is the distance between the stations Otherwise, the solid ENQ may be treated as a truncated triangular prism The edges normal to the right section EF have lengths at E, D′, and F of 0, $\frac{Ld}{R}$, and $\frac{Ld_1}{R}$, respectively, so that the average length of the prism is $\frac{L}{3R}(d_1+d)$ By either method the required volume, or curvature correction for station E, equals

$$\frac{L}{6R}\left(d_1^2-d^2\right)\left(c+\frac{b}{2s}\right)$$

This expression obtains for stations between chords of equal length. The correction at a tangent point has half the above value, and that at a station between chords of different lengths is determined by proportion. The formula, deduced for a three-level section, is equally applicable to two-level ground, and may safely be used for irregular sections if the half breadths employed are those of an equivalent three-level section In the case of a two-level side-hill section (Fig 247), in which the width of excavation or embankment at formation level is w, the correction for the excavation or embankment is similarly found to have the value,

$$\frac{Lwh}{6R}(d+b-w),$$

$$=\frac{Lw}{6Rs}\left(d \quad \frac{b}{2}\right)\left(d+b-w\right)$$

Measurement from Spot Levels

This method of measurement is sometimes applied to large excavations The field work consists in dividing up the site of the work into a number of equal triangles, squares, or rectangles and observing the original surface level at each corner of those figures After completion of the earthwork, or during its progress, the lines dividing up the area are again set out, and the levelling is repeated on the new surface, the differences of level thus determined representing the depths of earthwork at a number of points of known position In estimating the volume of proposed earthwork, the levels of the new surface are obtained from the drawings The differences of level are regarded as the lengths of the sides of a number of vertical truncated prisms, i e prisms in which the basal planes are not parallel, the horizontal area of each of which is known The volume of each prism is obtained as the product of the area of the right section by the average length of the vertical edges.

Division of Ground.—The size of the unit areas into which the site should be divided will depend upon the degree of accuracy required in the measurement and upon the character of the ground. The aim should be to have the partial areas of such dimensions that the assumption that the ground surface within each is a plane is not greatly in error, and it is therefore desirable to reduce their size on irregular ground

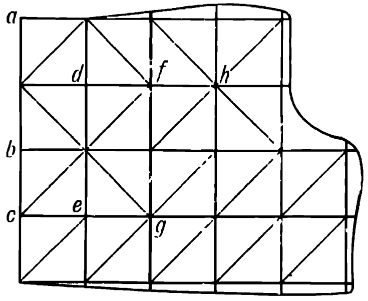


Fig 252

Good results are obtained by dividing up the site into a series of squares or rectangles and regarding each as divided by a diagonal into two triangles, the diagonal which more nearly lies along the surface of the ground being selected and noted (Fig 252) The surface within each rectangular area is then treated as consisting of two triangular planes, and the volume is computed from triangular prisms This system involves the same amount of levelling as the method of rectangles, but the freedom of choice in each rectangle as to the pair of planes which more nearly coincides with the ground surface necessarily tends to greater accuracy

It most frequently happens that the boundary of the work is irregular in plan, in which case there are a number of unequal partial areas adjoining it These figures approximate in shape to triangles or trapezoids, and may with sufficient accuracy be treated as such Their areas are obtained from measurements in the field or by scaling from the plan, and, since they are unequal, the volume of each prism must be separately computed

Method of Calculation.—In the calculation of the volume contained within the limits of the dividing lines, although each triangular prism may be computed separately, the fact that all have the same horizontal area suggests a method of reducing the arithmetical work It will be seen from Fig 252 that the corner *a* belongs to one triangle only, and the difference of level between the new and the original surface at *a* is therefore employed only once in the calculations That at *b* is used twice, that at *c* three times, and so on to that at *h*, which belongs to eight prisms, eight being the maximum number of times a particular height can be employed

If then A = the area of one triangle in square feet,

h_1 = the total sum, in feet, of all heights used once,

h_2 = ,, ,, ,, ,, twice, etc ,

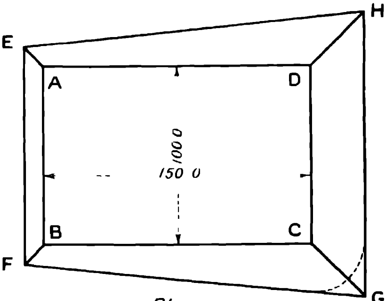
the total volume, in cubic yards, within the extreme dividing lines is given by

$$V = \frac{A}{27}\left[\frac{h_1+2h_2+3h_3+4h_4+5h_5+6h_6+7h_7+8h_8}{3}\right],$$

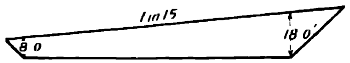
to which falls to be added that contained within the surrounding irregular strips

Note If certain parts of the ground are of a much rougher character than the rest, a few extra levels may be observed at those places, instead of reducing the size of the triangles throughout The additional levels are located by measurement of tie lines from the intersections of the reference lines The original prisms are thus split up into a number of smaller, and generally unequal, triangular prisms of known horizontal area, the volumes of which are computed separately

The division of a solid into a number of truncated prisms, not necessarily of constant area, affords a convenient method of obtaining the volume in cases like the following.



Plan



Longitudinal Section

FIG. 253

Example.—Calculate the volume of the excavation shown in Fig. 253, the side slopes being $1\frac{1}{2}$ horizontal to 1 vertical, and the original ground surface sloping at 1 in 15 in the direction of the centre line of the excavation.

The solid in question, although bounded by planes, is not a prismoid, since no two faces are parallel. The volume will be calculated as that of the vertical truncated prisms appearing in plan as ABCD, ABFE, DCGH, ADHE, and BCGF, of which the lengths of the vertical edges are known.

The horizontal breadths of the slopes on the left and right are computed as 10·9 ft. and 30 ft. respectively, so that we have

Prism ABCD

Area $= 150 \times 100 = 15{,}000$ sq. ft.
Average height $= \frac{1}{4}(8+8+18+18) = 13$ ft. — cub. ft.
Volume $= 15{,}000 \times 13 =$ 195,000

Prism ABFE

Area $= 110{\cdot}9 \times 10{\cdot}9 = 1{,}209$ sq. ft.
Average height $= \frac{1}{4}(0+0+8+8) = 4$ ft.
Volume $= 1{,}209 \times 4$ — 4,836

Prism DCGH

Area $= 130 \times 30 = 3{,}900$ sq. ft.
Average height $\frac{1}{4}(0+0+18+18) = 9$ ft.
Volume $= 3{,}900 \times 9$ — 35,100

Prisms ADHE and BCGF.

$$\text{Area} = 2\left[(150+40{\cdot}9)\times\frac{40{\cdot}9}{2}-\frac{10{\cdot}9^2}{2}-\frac{30^2}{2}\right] \quad 6{,}789 \text{ sq. ft.}$$

Average height $= \frac{1}{4}(0+0+8+18) = 6{\cdot}5$ ft.
Volume $= 6{,}789 \times 6{\cdot}5$ — 44,129

Total Volume = 279,065
= 10,336 cub. yd.

Notes.—(1) The solid may be divided in other ways, and the reader will find it instructive to verify the above result by a different treatment.

(2) It frequently happens, especially in embankments, that the side slope planes are not continued to intersections at the corners, but, as indicated by the dotted curve near C, the earth slope is rotated about the corner to generate a quarter cone there. Unless the original ground surface is horizontal, the bases of these partial cones are not normal to the axis, and the exact calculation of their volumes is unwarrantably troublesome. It is usually sufficient to estimate them as one-third the area in plan of the base times the axial height at the corner.

Measurement from Contour Lines

Rough estimates of volume may be made by treatment of the contour lines of the solid to be measured. In principle, this is no doubt the ideal method, and, if a highly accurate contoured plan were available with a contour interval sufficiently small that full particulars of the irregularities of the solid could be obtained, it would lead to a more precise result than the methods previously considered. Practically, however, this degree of accuracy is not realised because of the trouble of locating contours with an interval small enough to record the minor features of the ground. In dealing with contour lines, one must assume that the surface of the ground slopes uniformly from one contour to the next, and in most cases this assumption will be incorrect, the resulting error depending upon the vertical interval used as well as upon the character of the ground. To make an estimate of any practical value, the contours should not have a greater vertical interval than 5 ft. on ground of average character, but, if the surface is very irregular, an interval as small as 2 ft. would be required. The method therefore cannot compare with cross sectioning in point of convenience, and is not much used in practice except in the determination of the capacity of reservoirs or the measurement of subaqueous excavation, but even in these cases the method of cross sections is often preferred.

First Case.—The problem is a simple one in reservoir work and in earthwork when the made surface is level, for the new surface is then parallel to the equidistant level surfaces which define the contour lines.

To consider this case, let it be desired to determine the capacity of the reservoir shown in Fig. 254 when the water level is 165.0. The outline of the water surface is first obtained by interpolating the 165 contour. The volume is measured by regarding it as being divided up into a number of horizontal slices by the contour planes. The depths of these are known, and their end areas are obtained from the plan by any of the usual methods, the most expeditious and accurate being by planimeter. The area measured in each case is, of course, the whole area lying within a contour line and not that of the strip between two adjacent contour lines.

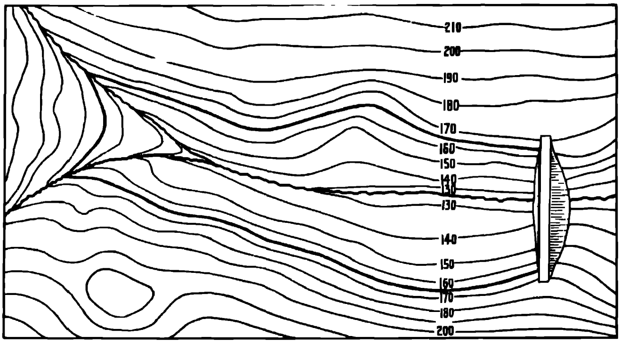


Fig. 254.

The nature of the data is precisely the same as in cross section work, the contour interval corresponding to the distance between cross sections, and the volume may be calculated either by the prismoidal formula or by the end area method. In using the prismoidal formula, every second area may be treated as a mid-area, or the mid-areas may be measured from contour lines interpolated midway between each original pair of contours. The latter method is recommended when the slopes are flat, so that there is a considerable difference between the areas within successive contour lines For most purposes, however, the method of end areas yields sufficiently good results

In connection with the location, design, and operation of reservoirs, it is not sufficient to know the storage capacity for one particular surface level The volumes in the reservoir for various water levels, such as the contour elevations, should be determined. If these computed volumes are plotted against the corresponding surface levels, the resulting curve is available for reading off the volume up to any water level or for finding the level corresponding to any given volume

Second Case.—The more general case in earthwork is that in which the ground is not brought to a level surface

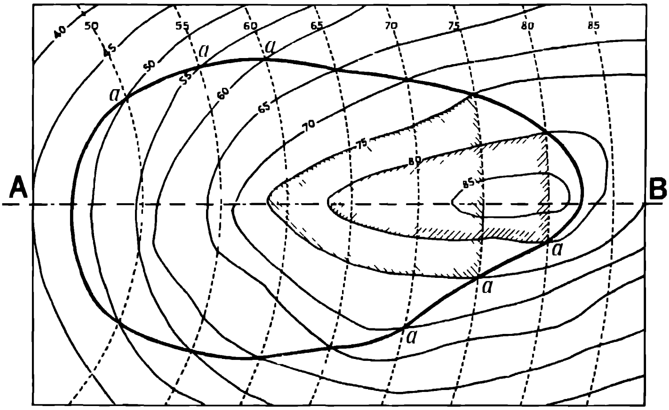
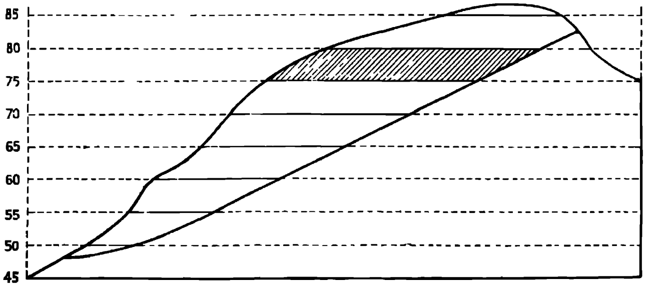


FIG. 255



Section AB

FIG. 256.

In Fig. 255, let the full lines denote the contours of the original surface, and the dotted lines those of the proposed new surface. By joining up the intersections *a* of original and new contours of equal value, the line in which the new surface cuts the original is obtained within this line excavation is necessary, the surrounding parts shown being in embankment. The methods of measurement may be considered with reference to the excavation, the same methods being applicable to embankments

First Method —It will be seen from the section (Fig 256) that the contour planes divide the solid to be measured into a series of horizontal layers The end areas of these can be obtained by planimeter from the plan, those for the strip between the 75 and 80 contours, which is hatched in section, being shown shaded in plan It frequently happens that an original contour line closes on itself without intersecting the equivalent contour of the new surface, as is the case with the 85 contour in the diagram, and the area enclosed by it is that to be measured If necessary, a sketch section will usually clear away any doubt as to the areas required. The required volume is then obtained by applying the method of end areas to each layer.

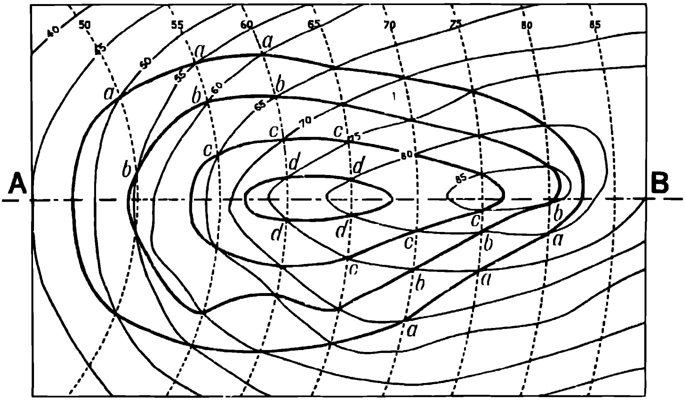
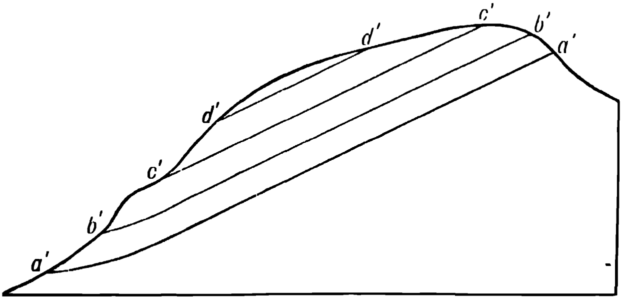


FIG 257



Section AB

FIG. 258.

Second Method.—Instead of dividing the solid by the contour surfaces, it may be divided into layers parallel to the new surface (Figs 257 and 258)

Just as *aaa* is the line along which the depth of excavation is zero, the depth will be exactly 5 ft along the line *bbb* got by joining up the intersections of the original contours with new ones of 5 ft lower elevation The total areas contained within *aaa* and *bbb* are the horizontal projections of the sloping surfaces which appear in section as the lines *a'a'* and *b'b'*, and the mean of those two areas times the *vertical* distance between them gives approximately the volume of the lowest strip By joining the intersections of the original contours with the new ones 10 ft lower, the line *ccc* of 10 ft excavation is obtained, *ddd* being similarly derived as the line along which the cut is 15 ft, and the areas within these lines are used in the same manner By inspection of the plan, the greatest depth of excavation appears to be about 17 ft, so that the highest strip has a vertical depth of 2 ft, the same method being applied to it as to the others, since, in this case, it more nearly resembles a wedge than a pyramid

This method usually involves less measurement of areas than the first, but has the disadvantage that the areas to be measured have to be specially traced out, and this may necessitate the interpolation of intermediate contours

THE MASS DIAGRAM

In the planning and execution of earthwork certain problems arise which are most simply studied by reference to what is termed a mass diagram.

Labour in earthwork may be analysed under four heads (*a*) Loosening, (*b*) Loading, (*c*) Hauling, (*d*) Depositing For a given class of plant, the unit cost of items (*a*), (*b*), and (*d*) depends almost entirely upon the character of the material, but that of (*c*) is a function both of the weight of the material and of the varying distance from the working face of the excavation to the tip end Haulage is the most variable item in the cost of earthwork, and, in cases where more than one scheme of distribution from excavation to embankment is possible, it is desirable to be able to compare different projects as regards economy of haulage. This may be accomplished with ample accuracy by means of the mass or haul diagram.

Definitions.—*Haul Distance* (*d*) is the distance at any time from the working face of an excavation to the tip end of the embankment formed from it.

Average Haul Distance (*D*) is the distance from the centre of gravity of a cutting to that of the tipped material

Haul is the sum of the products of each load by its haul distance $= \Sigma vd = VD$, where V is the total volume of an excavation.

Change of Volume—Since excavating involves the loosening and breaking up of the material, the volume available for the formation of an embankment is greater than that measured *in situ* in the unworked excavation On a rough average, this increase may be estimated at 20 per cent for earth When banked, however, the material commences to shrink, and after the lapse of a year or two will be found to occupy less space than it did originally in the excavation The amount of shrinkage depends on several factors, but for the present purpose may be taken as 10 per cent for all earths On the other hand, excavated rock is permanently swelled by about 50 per cent Change of volume does not influence ordinary measurements, which should always be based upon original volumes of excavation

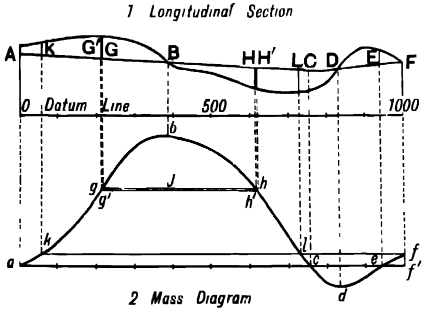
Construction of the Mass Diagram.—The mass diagram is a curve plotted on a distance base, the ordinate at any point of which represents the algebraic sum up to that point of the volumes of cuttings and banks from the start of the earthwork or from any arbitrary point In obtaining the algebraic sum, the usual convention is to consider cuttings plus and embankments minus. The mass diagram is therefore simply an integral or sum curve of the

Distance Ft	Volume Cub Yd		Total Volume. Cub Yd
	Cut +	Bank −	
0			0
	490		
100			+ 490
	927		
200			+1,417
	982		
300			+2,399
	279		
380			+2,678
		31	
400			+2,647
		226	
500			+2,421
		654	
600			+1,767
		1,160	
700			+ 607
		933	
800			− 326
		92	
831			− 418
	220		
900			− 198
	428		
1000			+ 230

volumes of the several cuttings and banks. When excavation and banking occur on the same section, as in side-hill work, their difference only is used in the summation, the sign being that of the greater volume.

The construction and properties of the diagram are best followed from a simple example. In Fig 259, let *1* represent a longitudinal section of, say, a railway siding The quantities required for plotting the mass curve are tabulated on page 307

In the example it is assumed that the nature of the material is unknown, and no allowance is made for change of volume In plotting, the curve should be placed directly above or below the longitudinal section, the same horizontal scale being used for each Positive total volumes are plotted above, and negative quantities below, the base line, and the ends of the ordinates are joined by a smooth curve, the resulting diagram being as in *2*, Fig 259.


1 Longitudinal Section

2 Mass Diagram

FIG 259

Characteristics of the Mass Diagram.—With the sign convention adopted :

(1) Upward slope of the curve in the direction of the algebraic summation indicates excavation Downward slope indicates embankment

(2) A maximum point occurs at the end of an excavation , a minimum point, at the end of an embankment

(3) The vertical distance between a maximum point and the next forward minimum point represents the whole volume of an embankment , that between a minimum and the next forward maximum point, the whole volume of a cutting

(4) The vertical distance between two points on the curve which have no maximum or minimum point between them represents the volume of earthwork between their chainages

(5) Between any two points in which the curve cuts the base line the volume of excavation equals that of embankment, since the algebraic sum of the quantities between such points is zero. The points *a* and *c*, for example, show, on being projected to A and C, that the earthwork is balanced between A and C, *i.e.* the material excavated from AB would suffice to form the embankment up to the point C. There is also balance from E to C.

(6) Any horizontal line intersecting the mass curve similarly serves to exhibit lengths over which cutting and banking are equalised. Thus, *gh* is a balancing line, the cut from G to

B just filling from B to H, the volume moved being represented by bj.

(7) When the loop of the mass curve cut off by a balancing line lies above that line, the excavated material must be hauled forward, *i e.* in the direction of summation of the volumes When the loop lies below, the direction of haul is backward

(8) The length of balancing line intercepted by a loop of the curve represents the maximum haul distance in that section. Thus, taking the base line as the balancing line, the greatest haul distance involved in disposing of excavation AB is ac = AC, so that no material should be hauled past C In general, the haul distance increases from zero at B to this maximum, and its value at any stage of the work is given by the length of a horizontal line intercepted within the loop Thus, gh is the haul distance when the face of the excavation is at G

(9) The area bounded by a loop of the mass curve and a balancing line measures the haul in that section To take the case of area $abca$, since haul = Σvd, consider a small volume or load, v, initially situated at GG′, and whose final position in embankment is HH′, obtained from the horizontals gh and $g'h'$ From (4) above, the vertical distance between gh and $g'h' = v$, and therefore area $ghh'g'$ = the haul of v The summation of all such small areas = the area of the loop = the haul involved in transferring cut AB to bank BC. Similarly, area cde = the haul from E to C. Regard must be paid to the scale If the horizontal scale is 1 in = x ft , and the vertical scale is 1 in = y cub yd , an area of n square inches represents a haul of nxy cub yd ft

(10) The haul over any length is a minimum when the balancing line is so situated that the sum of all areas cut off by it, without regard to sign, is a minimum.

Use of the Mass Diagram.—The exact interpretation of a mass diagram is entirely dependent upon the balancing line, each position of which exhibits a possible method of distributing the excavated material, and the selection of the most economical scheme is made by comparing those shown by various balancing lines Par (10) above is an important guide, but it most frequently happens that the condition of minimum haul necessitates the wasting or spoiling of material at one place and borrowing at another, the advisability or possibility of which depends upon circumstances. The haul involved in proposed wasting and borrowing is difficult of estimation, but it may be comparatively small if wasting is effected by widening an embankment, and borrowing by widening a cutting. The limit of profitable haul distance, beyond which it is economical to waste and borrow, is, of course, reached when the cost of excavating and hauling one cubic yard equals the cost of excavating and hauling to waste one cubic yard plus that of excavating and hauling one cubic yard from the borrow pit

Considering first the previous example of Fig 259, it will be seen that it is impossible to secure balance over the whole length, as there is an excess *ff'* of excavation Trying the base line as a balancing line, there is shown balance from A to E and wasting of the material between E and F But the haul (*abc*+*cde*), and the maximum haul distance, *ac*, can evidently be reduced by raising the balancing line, so that, if the haul required in wasting be assumed constant, the balancing line *klf* shows a preferable scheme, the material from A to K being wasted

Fig 260 illustrates a second example In this case, excavation and embankment are equal over the length shown, and borrowing and wasting would be unnecessary provided the excavated material were distributed by the method indicated by the base line as the balancing line But, on account of the large area below the base line, this arrangement involves considerable haul, with a maximum haul distance from *l* to *e* of about 1,350 ft, and it may be more economical to reduce the haul by wasting and borrowing A possible method is that shown by balancing lines *opqgr* and *ln*, in which there is borrowing from A to O and spoiling of the same volume from R to L The maximum haul distance is reduced to that from *r* to *g*, or about 880 ft Another possible arrangement is given by balancing lines *opq*, *stujv*, and *ln*, with a maximum haul distance *ju* of about 590 ft This scheme necessitates borrowing from A to O and Q to S and wasting from V to L, and is shown in longitudinal section By increasing the amount of borrowing and wasting, haul and maximum haul distance may be still further reduced In particular, if the haul distance is not to exceed 500 ft., the line *wx*, scaling that length, will be used as a balancing line. In selecting the most economical of the various methods of disposal, one must be guided by the relationship between the cost of haul and that of borrowing and wasting is so far as it can be estimated for the particular case

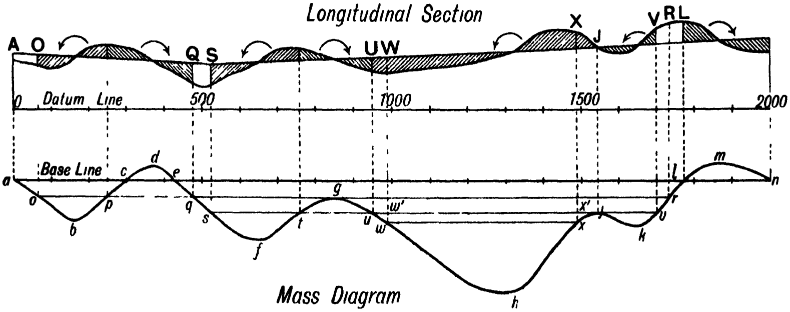


FIG 260.

Allowance for Change of Volume.—In comparing schemes for the disposal of proposed excavations, the available information regarding the nature of the material is not usually sufficiently reliable to warrant making refined allowances for the change of bulk caused by excavating, and the common practice in such cases is to neglect these and proceed as above If, however, sufficient data have been obtained, allowance should be made for the fact that rock increases considerably in bulk on being excavated. Soft earths also swell when loosened, but, chiefly owing to loss of material, they ultimately shrink to a smaller volume than they occupied in their original position in the excavation. The initial swell of earth is largely compensated for by the necessity for making embankments higher than they are intended to be to allow for subsidence, and the ultimate change of volume is in the direction of shrinkage

Allowance for change of volume is made, before summing the volumes, in either of two ways (*a*) by multiplying each computed volume of excavation by a factor which will convert it to the volume of embankment ultimately formed from the material ; (*b*) by multiplying each volume of embankment by a factor to give the volume of excavation from which it can be made. If we suppose that, on a rough average, 100 cub yd of solid rock will suffice for forming 150 cub. yd. of embankment, and that 100 cub yd of earth in an excavation will ultimately form 90 cub yd of embankment, the respective factors would be 1·5 and ·9 in the first method, and ·67 and 1·1 in the second Such average values are usually sufficient, since difficulties are encountered in estimating the best allowances for change of volume in a particular case. Shrinkage factors for soft earth depend upon the material, and are greater for low than for high embankments In addition, mixtures of different earths, and of earth and rock, may have to be dealt with, and, until the mass diagram is prepared and studied, it is not known from which cuttings the material for a given embankment will be derived

Overhaul.—The terms of contracts for earthwork may stipulate either that the price per cubic yard of excavation is inclusive of the cost of haul regardless of the haul distance involved or that the price includes the cost of haul within a specified distance only In the latter case, extra payment is made for the haulage of each cubic yard which has to be moved a distance exceeding this specified distance, termed the free haul distance The excess of haul distance above this amount is called overhaul distance, and the sum of the products of volumes by their respective overhaul distances is termed overhaul, and is the quantity for which extra payment is made. The unit of measurement of overhaul distance is commonly 100 ft , and that of overhaul is therefore cub yd × 100 ft.

The mass diagram affords a convenient aid to estimating overhaul. In the case of Fig 260, let the scheme with balancing lines *opq*, *stujv*, and *ln* be that adopted, and let the free haul distance be 500 ft. Overhaul is required on the section JU. The horizontal line *wx* of length 500 ft. having been drawn on the mass curve and projected to *w'x'* upon the balancing line, the area *w'whxx'* represents free haul. The overhaul required is given by the product of the volume between U and W or X and J multiplied by the excess beyond 500 ft. of the shift of its centre of gravity in being transferred from cutting to bank. The positions of the centroids may be computed or estimated, but, since the total haul on the section JU is represented by the area *uhj*, and the free haul by *w'whxx'*, the difference, or overhaul, is given by the sum of the two areas *uww'* and *x'xj*. These may be measured by planimeter, and are expressed in terms of cub. yds. × 100 ft.

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EXAMPLES

1. A piece of ground has three straight sides, AB, BC, and CD, and the fourth, AD, is irregular. The dimensions in feet are AB = 422, BC = 640, CD = 456, AD = 798, and AC = 842. Offsets outwards from AD to the irregular boundary have the values 0, 12, 4, 19, 0 at chainages of 0, 150, 330, 434, and 798 ft. from A. Calculate from these figures the area within the boundary expressing the result in acres, roods, and poles.

2. The cross section of a stream 30 ft. wide is measured by means of soundings taken 5 ft. apart. The depths recorded are

0, 1·5, 2·0, 3·5, 2·3, 1·0, 0.

The mean velocity is observed to be 3·4 ft. per second. Compute the discharge of the stream.

3. If areas are measured by planimeter as square inches of map surface, what are the factors for converting the results to acres in the case of the "6-inch" and "25-inch" Ordnance maps?

4. A survey was plotted to the 1/2,500 scale. A certain area on a photo-print reproduction was measured by planimeter, and was found to contain 9·36 square inches. The print shows a shrinkage of 1 per cent both up and down and along the sheet. Obtain the area measured, expressing the result in acres, roods, and poles.

5. In taking a cross section of a stream, a leveller first observed the staff held upon the 10-ft. mark on a gauge post, and obtained a reading of 0·32 ft. The staff was then held on points 20 ft. apart on the cross section, the readings, taken to the nearest tenth of a foot, being

0·3, 4·0, 6·8, 7·4, 9·0, 9·5, 10·2, 10·3, 8·0, 6·4, 4·2, and 0·5.

Obtain the area of flow when the surface of the water is at 5 ft. on the gauge, assuming that the bed of the stream slopes uniformly between the points observed.

6. Calculate the side widths and cross-sectional area of a railway embankment with a formation width of 30 ft., side slopes 1½ to 1, centre height 12 ft., and slope of original ground surface at right angles to the centre line of the embankment of 1 in 10.

7. The data of a closed traverse survey are as follows:

Line	Latitude in links	Departure in links
AB	− 300	+ 450
BC	+ 640	+ 110
CD	+ 100	− 380
DA	− 440	− 180

Calculate by any method the area contained within the survey lines, expressing the result in acres, roods, and poles. (R.T.C., 1913.)

8. The adjusted co-ordinates of a closed traverse are:

Line	Latitude in feet	Departure in feet
AB	526 N.	113 W.
BC	384 N.	316 E.
CD	227 S.	540 E.
DE	613 S.	104 W.
EF	364 S.	782 W.
FA	294 N.	143 E.

Calculate the area contained within the part ABCDEA of the figure, and express the result in acres, roods, and poles.

9. A corner of a piece of ground is bounded by lines AB, BC, and CD, of which the lengths and bearings are as follows:

Line	Length in feet	Bearing
AB	563	251° 42′
BC	212	193° 16′
CD	404	102° 23′

From a point X on AB it is required to lay off a straight line to a point Y on CD, such that XY will be parallel to BC, and XBCY will have an area of one acre. Compute the distances BX and CY.

10. A city building site, having an area of 5,749 sq. yd, is bounded by the following lines, of which the co-ordinates, expressed in feet, are

Line	Latitude	Departure
AB	+ 244 0	0
BC	- 32 8	+ 130 6
CD	- 17 4	+ 128 3
DE	- 271 5	- 144 7
EA	+ 77 7	- 114 2

It is desired to divide the ground by a straight line into two equal areas, each having the same frontage on the line AB F being the mid-point of AB, obtain the position of the dividing line FG by computing where G falls on DE.

11. A railway embankment is 30 ft wide with side slopes of $1\frac{1}{2}$ to 1. Assuming the ground to be level in a direction transverse to the centre line, calculate by the end area method the volume contained in a length of 6 chains, the centre heights at one chain intervals being, in feet,

$$2,\ 4,\ 4\tfrac{1}{2},\ 6,\ 5\tfrac{1}{2},\ 4,\ \tfrac{1}{2}.$$

12 Calculate the volume of the above embankment by the prismoidal method

13. Two irregular cross sections of a railway cutting 100 ft apart have areas of 450 sq ft and 194 sq ft respectively The formation breadth is 30 ft, and the side slopes are $1\frac{1}{2}$ to 1 By substituting equivalent level top sections, compute the mid-area and the volume of excavation between them.

14 A pier 300 ft long is to be built seawards with a top breadth of 40 ft and side batters of 1 in 8 The top level is to be 14 ft above low water of spring tides, and the foundation levels at intervals of 50 ft are 2, 4, 8, 10, 15, 22. and 27 ft below the same datum

Estimate by the use of the prismoidal formula the total volume of material required (R T C, 1921)

15 Calculate the value of the prismoidal correction for the first 300 ft of a railway cutting of which the formation breadth is 30 ft. and the side slopes are $1\frac{1}{2}$ horizontal to 1 vertical The original ground surface is level transversely to the centre line of the railway, and the centre depths of the earthwork are 0, 10, 14, and 20 ft at 100 ft. centres (T C D, 1927)

16 The slope of a certain piece of ground (which may be regarded as a plane surface) is 1 in 4 On the surface of this ground a line AB, 375 ft. long, is laid out at a gradient of 1 in 9

Find the slope of the ground in a direction at right angles to AB as seen in plan.

If AB be the centre line of the formation for a path 10 ft wide, horizontal transversely, and with side slopes of 2 horizontal to 1 vertical, calculate the volume of the earth to be moved in making the path There is to be neither cutting nor filling along the centre line (Univ of Lond, 1916)

17. A road having a formation breadth of 30 ft and side slopes of $1\frac{1}{2}$ to 1 is to be constructed on side-lying ground At adjacent sections, 100 ft apart, the depths of excavation at the centre line of the road are 2 ft and 1 ft respectively, and the original ground surface between the sections has a constant inclination of 4 to 1 in the direction at right angles to the centre line Calculate by the method of end areas the volume of excavation and the excess volume of excavation over embankment between the sections. (T C.D, 1931)

18. An embankment of 30 ft formation width with side slopes of 2 to 1 is to be formed on a curve of 10 chains radius. If the original ground surface slopes at 5 to 1 downwards towards the concave side, calculate the curvature correction per 100 ft, station when the centre height of the bank is 8 ft.

19. A rectangle ABCD, 100 ft by 80 ft, forms the plan of part of an excavation. AD and BC are the longer sides, and E is the point of intersection of the diagonals. From the following original surface levels and final excavated levels, calculate the volume of excavation within ABCD

Point	Original Level	Final Level
A	85 30	76 00
B	93 05	76 00
C	92 15	77 20
D	84 70	77 20
E	93 40	76 60

20 The areas within the contour lines at the site of a reservoir and along the face of the proposed dam are as follows

Contour	Area in sq feet.
500	7,054,000
495	6,427,000
490	5,083,000
485	4,612,000
480	2,958,000
475	1,573,000
470	836,000
465	115,000
460	3,000

Taking 460 as the bottom level of the reservoir, find

(*a*) the volume of water in the reservoir when the surface is at elevation 500, using the method of end areas

(*b*) the volume under the same conditions, as given by the prismoidal formula, taking every second area as a mid-area

(*c*) the surface level when the reservoir contains 300 million gallons

1 cub. ft = 6 24 gallons

CHAPTER VIII

SETTING OUT WORKS

THIS part of the duties of an engineer on location or construction involves the placing of pegs or marks to define the lines and levels of the work, so that construction may proceed with reference to them. Setting out is, in a sense, the reverse of surveying in that data are transferred from the drawings to the ground

In the case of works which may be completely defined by a series of straight lines, setting out is a simple operation, and requires little explanation (page 319) The greater part of this chapter is therefore devoted to a description of the setting out of curves

Location of Works.—Before proceeding to the detailed design of an engineering scheme, the relative merits of different sites must be carefully weighed, and the most economical location selected For this purpose, in unmapped countries the territory embracing all probable sites must be investigated by surveying In the case of large projects, time is saved by making successive surveys of increasing refinement, which are used for the gradual elimination of all locations except that finally adopted

In general, the first survey consists of a rapid examination of a wide area with the object of ascertaining which portions of it merit further investigation These are surveyed in greater detail, their relative advantages are studied, and finally the work is located in the best position on one of them The character and amount of the field work involved depend upon the nature of the proposed public works. In any large scheme a considerable amount of detailed investigation is required before the engineer can finally decide upon the site which, from considerations of first cost and running expenses, will prove the most successful.

Without entering into a discussion of the engineering factors which govern the location, the case of a railway may be taken to indicate the field operations undertaken by a locating party

Reconnaissance Survey.—The first, or reconnaissance, survey is extended over the area embracing all likely routes between the proposed terminals with the object of ascertaining the belt or belts of country which provide practicable routes. The field work has, therefore, the character of an exploratory survey, and is conducted either by rough traversing or by rapid triangulation, as described in Vol II, Chap VI. Traversing by prismatic compass is the most favoured method, and elevations are determined by barometric levelling with the aneroid (Vol II, Chap VI)

Although performed by rapid methods, the reconnaissance survey is important as forming the basis of the subsequent work. It is essential that no important topographical or geological features should be overlooked, and the engineer in charge should take full notes of all factors likely to influence the location A reconnaissance survey on the above lines is not required if topographical maps of the country are available, but, even so, it is usually desirable to supplement the information they afford by a rapid examination on the ground.

If no maps are available, the results of the survey are used to prepare a rough map showing the main topographical features which influence the location The possible routes are sketched down with due regard to the elevations of these controlling points

Preliminary Survey.—As a result of the reconnaissance, it may be discovered that a particular belt of country is better than any other as a route for the railway, and it remains to survey this strip in detail, so that the line may be located upon it in the most favourable position Otherwise, the reconnaissance may show that two or more belts of country appear equally suitable, and similar surveys are conducted over each to enable their relative merits to be studied and the line to be finally placed on one of them

The surveys required at this stage of the investigation are called preliminary surveys The method which has been most commonly used consists in laying out on the ground a trial line with suitable gradients in what appears on thorough examination of the ground to be the most economical position The line need consist only of a series of straights, no attempt being made to run in curves unless the angle of deflection between adjacent straights is considerable, in which case one or more short straights may be laid out in the approximate position of the curve The lines as laid out are surveyed by theodolite, plane table, or compass, as the conditions warrant, and chaining is performed continuously along them, marks being left at intervals of 100 ft or 200 ft An essential feature of the survey is the running of a longitudinal section along the traverse lines The profile constructed from it is used for the laying down of suitable gradients, and, although only approximate because of the neglect of curves, the result is sufficiently accurate for the purposes of the preliminary estimate.

To enable the relationship of the selected line to existing features to be exhibited in plan, a field party is detailed to "take topography." The positions and levels of any artificial features, such as roads and buildings, near the line are obtained, since they influence the question of possible deviations from the trial line. There is considerable variation in field practice regarding the extent to which topographical features are recorded. In some cases, no topography is taken other than particulars regarding the streams crossed: in others, a strip of country on either side of the projected line is completely contoured on the basis of a 5 ft. interval. If only one belt of country is being examined, a complete contour survey serves for the final location, but when two or more preliminary surveys are being run it is uneconomical to take much topography. Little or no record of the features of the adjacent ground is required in places where there is no doubt as to the best position of the line or where the ground is so flat that a deviation of the trial line will not appreciably affect the estimate of cost In doubtful situations, on the other hand, a few clinometric cross sections may be run out from the traverse On plotting these on the plan, and interpolating the contours, the most economical position of the line may be selected over the difficult part

Location Survey.—The lines projected by the preliminary surveys having been carefully studied and compared with regard to first cost and operating expenses, selection is made of the scheme to be adopted The remaining field work is directed in the first place to making a more detailed investigation of the ground along the preliminary line selected, with a view to improving the location with reference to the minor topographical features Finally, after the best centre line is obtained, it must be staked out

The final location may be performed entirely in the field, the engineer using the gradients shown on the preliminary profile as a guide in seeking for improvements in the alignment The detail work is then performed by a similar party to that employed on the preliminary survey, and operations are carried out on the same lines as before except that curves are selected and run in In the hands of a highly skilled locating engineer this method is sufficient, but the more usual and, in general, safer alternative is to make a contour survey of the chosen strip of country and to use the resulting map for the selection of the best centre line, thus making what is known as a paper location The traverse and levels of the preliminary survey furnish the horizontal and vertical control respectively for the contour survey. The contour interval is usually 5 ft, and the contours are located by clinometric cross sections, by hand or ordinary levelling, or by tacheometry. The width of country to be contoured depends upon the transverse slope of the ground, and is much smaller on steep than on flat ground, since in the latter case the gradient or the depth of earthwork may not be seriously affected by a considerable lateral movement of the centre line It frequently happens in the course of the survey that local deviations from the projected line appear desirable, and the survey is either widened or looped to include the topography of the alternative route. In making the paper location on the resulting map, the depths of earthwork along any trial line are readily obtainable by comparison of the proposed formation levels with the contour levels The process is one of successive trial until the line is obtained in which the gradients and curves are so arranged, with regard to the class of railway proposed, that the first cost plus capitalised working expenses is a minimum.

Location in Mapped Country.—The foregoing operations are greatly simplified when good detailed maps of the country are available. The preliminary trial lines may then be laid down directly on the maps and adjusted at doubtful points in the light of additional topographic data obtained at such places by cross sections or tacheometry These preliminary lines are transferred to the ground by means of scaled measurements, and the longitudinal sections are run

When the best scheme has been selected, lithographed drawings of the plan and profile are prepared for deposit in suing for legal power to proceed with the work These drawings must comply strictly with the statutory requirements as to scale and the amount of information to be exhibited

Final Location.—When authority has been obtained to proceed with the construction, the line as projected on the paper location is set out on the ground It is, however, still subject to such minor alterations as may be found desirable, but such deviations from the approved centre line must not be of greater extent than is allowed by the law of the country concerned. In this country, lateral deviations must not exceed 100 yards on either side of the centre line in country districts and 10 yards in towns and villages, and limits are also set as to sharpening curves and steepening gradients *

Setting Out in General.—Although in most cases the work is very simple in principle, difficulties are commonly encountered in practice, and indirect methods are frequently necessary, especially during the progress of construction As each case must be dealt with according to circumstances, it is difficult to formulate hard and fast rules, but one or two considerations are of general application.

The need for complete checking of the work is self-evident, as an undetected mistake might have serious consequences. When the work is not such as will enable the closing error to be ascertained, there is a greater sense of certainty if important pegs are located by two different methods based upon independent sets of calculations than if the same method is repeated.

Instruments should be tested at frequent intervals, and should be so used that errors of non-adjustment are reduced to a minimum. A steel band should be used for linear measurements

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* See "The Railways Clauses Act, 1845," "Standing Orders of the House of Lords," "Standing Orders of the House of Commons," and "Rules and Regulations Issued under the Ministry of Transport Act, 1919," or *The Preparation of Parliamentary Plans*, by R. T. Fuller.

In structural work, a setting out plan should be prepared showing those lines of the work which have to be set out As it may be impossible to locate these directly owing to obstructions, considerable use may have to be made of parallel lines, and it is therefore desirable at the outset to provide a convenient framework of reserve lines clear of the ground to be occupied by plant All intersections are marked by pegs, the exact point being indicated by the head of a tack driven into the peg, and full information regarding the linear and angular dimensions to these points should be entered on the plan

In abstracting the required dimensions from the drawings, the surveyor must be on his guard against possible errors in the figuring of the drawings Scaling should be resorted to as little as possible, and then only for minor features in the case of skew structures, the skew distances must be calculated Thus, for setting out a skew bridge the skew span and lengths of the abutments are computed from the given square span and width and the angle of skew Some of the dimensions necessary to fix the positions of the wing walls may have to be scaled In the process of setting out such a structure, it is first necessary to locate a centre line and drive a peg thereon to mark the centre point of the bridge On planting the theodolite over this peg, the angle of skew is set out, and points are established on the faces of the abutments Pegs are located on the lines of the abutment faces prolonged clear of the work, and their distances from the corners of the abutments are noted It is unnecessary in the first instance to provide pegs to mark the lines of foundations, back of wall, etc , as such lines can be obtained from that of the face The end of each wing wall is set out by means of a peg on the abutment face line and an offset distance, but one or more pegs should be driven on the face line of the wing wall clear of the excavation In the case of battered walls, only the foot of the wall is set out

CURVES

The ranging of curves is required in the location of various kinds of public works, but it forms so important a part of the setting out of the centre line of a railway that the following discussion refers more particularly to railway work. The methods, however, are similarly applied to other cases.

Except for special purposes (page 345) railway curves are circular, and may be classed as simple, compound, and reverse.

A *Simple Curve* consists of a single arc connecting two tangents (*a*, Fig 261)

A *Compound Curve* is composed of two arcs of different radii, curving in the same direction (*b*, Fig. 261). The centres of the two arcs are therefore situated on the same side of the curve, which itself lies completely on one side of the common tangent, i e the tangent to both arcs at their point of junction.

A *Reverse Curve* consists of two arcs, of the same or different radii, curving in opposite directions (*c*, Fig 261) In this case the centres are on opposite sides of the curve, and the curve crosses the common tangent

Note.—It is to be observed that if the arcs in *b* and *c*, instead of running tangentially into each other, have a straight, however short, introduced between them, they become simple curves

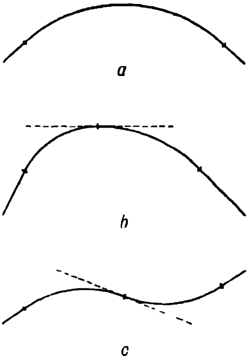

Fig 261

Designation of Curves.—A curve may be designated either in terms of its radius or with reference to the angle subtended at the centre by a chord of a particular length The former system is adopted in this country, and the unit may be either the foot or the chain, the latter being the more common The central angle, or degree, system is adopted throughout America The standard chord length is 100 ft , the angle which it subtends at the centre being called the degree of the curve Although perhaps not so convenient for plotting, the degree system possesses considerable advantages in setting out

Relationship between Radius and Degree.

Let R = the radius in feet,

and D = the degree of a curve

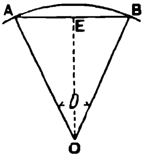

Fig. 262.

In Fig 262, let AB be a 100-ft chord, and let E, its mid-point, be joined to O, the centre of the arc, then, since OE is perpendicular to AB,

$$\sin \tfrac{1}{2}D = \frac{AE}{OA} = \frac{50}{R},$$

$$\text{and } R = \frac{50}{\sin \tfrac{1}{2}D}.$$

These relationships are precise, but, in calculating the radius corresponding to a given degree, or *vice versa*, advantage may often be taken of the circumstance that the sines of small angles are nearly proportional to the angles themselves, so that

$$\frac{R}{R_1} = \frac{\sin \tfrac{1}{2}D_1}{\sin \tfrac{1}{2}D} = \frac{D_1}{D}.$$

The radius of a one-degree curve is 5729·6, say 5730 ft , so that, approximately,

$$R = \frac{5730}{D}.$$

This formula is not applicable to curves of comparatively small radius, nor in special work requiring great accuracy, but is sufficiently near the truth for the large majority of railway curves. It underestimates the radius, thus, a 10° curve has actually a radius of 573·7 ft

Elements of a Curve.—Let AGHKB (Fig. 263) represent a simple circular curve of radius R, connecting the two straights EA and BF, A and B being the tangent points. Let the straights be produced to meet at C

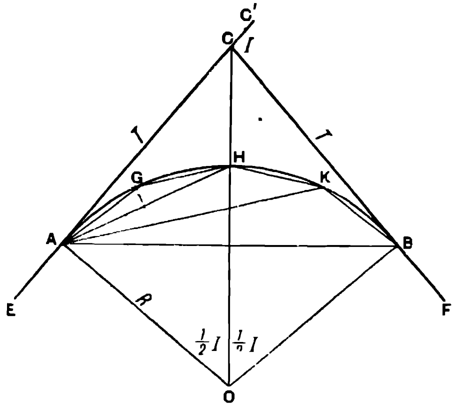

FIG 263

1 In the quadrilateral OACB, the angles OAC and OBC are right angles,

$$C'CB = AOB = I,$$

the *Intersection Angle*, or *Angle of Deflection*, this quantity representing the amount of deviation given by the curve.

2 The equal distances AC and CB are called the *Tangent Lengths*, T Join CO

In the triangle ACO, $\frac{AC}{AO} = \tan AOC = \tan \frac{1}{2}I$,

$$\therefore \quad T = R \tan \tfrac{1}{2}I.$$

3 Let AG, GH, HK, and KB be equal chords Join G, H, K, and B to A.

The angles AOG, GOH, etc , subtended at the centre by these chords or arcs, are equal, and are each twice the angles GAH, etc , subtended at the circumference The angle CAG between the tangent and the first chord is also half the central angle subtended by that chord, and therefore equals the angles GAH, etc

The angles CAG, CAH, etc , contained between the tangent and the rays from A to the various points on the curve, are called the *Deflection Angles* of those points The most common method of locating points on a curve is based on the use of these angles, the values of which have therefore to be calculated. Since the deflection angle to any point is the sum of the circumferential angles for the chords or arcs lying between the origin of the curve and that point, it is sufficient to examine the calculation of these circumferential angles

Formulæ for Circumferential Angles (Fig 264)

Let C = chord length in feet,
A = corresponding arc length in feet,
R = radius of curve in feet,
D = degree of curve,
δ = circumferential angle

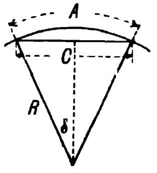

FIG. 264.

In terms of C and R.

$$\text{Sin } \delta = \frac{\frac{1}{2}C}{R},$$

$$\text{or } \delta = \sin^{-1}\frac{\frac{1}{2}C}{R}.$$

In terms of A and R

$$\delta, \text{ in radians}, = \frac{\frac{1}{2}A}{R},$$

$$\text{and, in degrees}, = \frac{180 \times \frac{1}{2}A}{\pi R} = \frac{28{\cdot}65\,A}{R},$$

$$\text{or } \frac{1718{\cdot}9\,A}{R} \text{ minutes.}$$

In terms of C and D

$$\text{Sin } \delta = \frac{\frac{1}{2}C}{R},$$

$$\text{but } R = \frac{50}{\sin \frac{1}{2}D},$$

$$\sin \delta = \frac{\frac{1}{2}C \sin \frac{1}{2}D}{50}.$$

With the approximation that the sines of small angles are proportional to the angles themselves,

$$\delta = \frac{CD}{200}$$

If C is 100 ft., δ is *exactly* $\frac{D}{2}$

In terms of A and D

$$\delta = \frac{28{\cdot}65A}{R},$$

$$= \frac{28{\cdot}65A \sin \frac{1}{2}D}{50},$$

$$= {\cdot}573A \sin \frac{1}{2}D$$

Since the triangle ACB (Fig. 263) is isosceles, the total deflection angle CAB between the tangent and the chord joining the tangent points = $\frac{1}{2}I$.

4. The length L of the curve AB = the central angle AOB in radians × R.

$$= \frac{\pi IR}{180} = {\cdot}01745\,IR, \text{ where } I \text{ is in degrees.}$$

If n = the number, integral or fractional, of arcs of length A forming the curve, then $n = \frac{\frac{1}{2}I}{\delta}$,

$$\text{and } L = nA = \frac{\frac{1}{2}IA}{\delta}.$$

These relations are precise, but if the ratio $\frac{R}{A}$ is great, A and C become nearly equal, and

$$L = nC = \frac{\frac{1}{2}IC}{\delta} \text{ approximately.}$$

◆

SETTING OUT SIMPLE CURVES BY THEODOLITE

Location of Tangents.—In pegging out a line composed of straights and curves, the straights, or tangents, must be set out before the curves connecting them can be located It is assumed that the surveyor is provided with a copy of the working plan, upon which the line is shown in relation to the controlling traverse of the preliminary survey and to existing features By scaling distances from the traverse lines or from buildings, fences, etc , several points on each tangent can be obtained on the ground by tape measurements Such points being temporarily marked, the tangents may then be set out by theodolite, so that, by trial and error, they run as nearly as possible through the marks

Peg Interval.—It is necessary for purposes of reference, and for conveniently obtaining distances along the line, that the pegs should be at equal intervals from the start of the railway to the end There must be no break in the regularity of their spacing in passing from a tangent to a curve or *vice versa* The setting out must therefore start at the beginning of the railway and be continued forwards The interval between the pegs may be either 66 ft or 100 ft The former has the advantage that every tenth peg marks a furlong, and in this country is that generally adopted. In America the 100 ft interval is universal, while in countries in which the metric system is used a distance of 20 metres is most commonly employed On sharp curves additional pegs may be driven at 33 ft or 50 ft centres

Location of Tangent Points.—For a given pair of straights, there is only one point at which a curve of given radius or degree may leave the first straight tangentially in order to sweep tangentially into the second The points of commencement and termination of the curve must therefore be determined with greater precision than would be possible by merely scaling their positions from the plan

1. Having located the two tangents and defined them by ranging poles, peg out the first tangent EA (Fig 265) up to about the estimated position of A, the theodolite being placed on EA and aligned on one of the poles. By means of the instrument, produce the straight to align two poles *a* and *b* a few feet apart, one being placed on each side of C, the position of which is estimated by the chainman from the line of the poles on BF.

2 Transfer the instrument to some convenient point on the second straight, and produce the latter to meet a string stretched between *a* and *b* The point of intersection C of the two tangents thus obtained is marked by a peg

3 Set up the theodolite over C, and measure the angle ECF By subtracting the result from 180°, the value of the intersection angle I is obtained Calculate the tangent lengths from $T = R \tan \frac{1}{2}I$

4 From C measure back the lengths CA and CB = T, the tangent points A and B being aligned from the instrument at C Mark A and B in a distinctive manner, either by painted pegs or by three ordinary pegs, the centre one of which defines the point

5. Transfer the instrument to A, and set it over the tangent point peg Measure the angle CAB, which should equal $\frac{1}{2}I$ This provides a convenient check on the equality of the tangent lengths, which may, however, both be in error by the same amount through a mistake in the measurement of I or in the calculation of T

6 The chaining of the first straight may now be completed, the chainage of the point A being noted

Location of Points on the Curve by Deflection Angles.—In setting out pegs on the curve, the most generally adopted method involves the use of one theodolite only Consideration will be given later to the manner of employing two instruments (page 334)

The interval between the chainage pegs on the curve should strictly be measured as the arc intercepted between them, and consequently, in employing chords to locate the pegs, the length of chord between two adjacent pegs would apparently have to be calculated In the great majority of cases in practice, however, the difference in length between the chord and the arc is quite negligible, and it is sufficient to use the peg interval as the chord length In describing the field work, it will be supposed that this is done

It will seldom happen that the tangent point A falls exactly at a peg interval from the last peg station on the first straight In consequence, since the chainage must be continuous, the chord AG to the first point G on the curve will be shorter than the regular length C Such a chord is termed a *Sub-chord* There will also in general be a sub-chord at the end of the curve Let their lengths be denoted by c and c'

1 From the chainage of A obtain the length of the first sub-chord Thus, if E (Fig 265) is the last chainage peg on the straight, $c = C - \text{EA}$

2 Calculate the circumferential angle δ for a chord C by any of the above formulæ, whether involving A or C, since these are being regarded as equal The circumferential angle CAG for the sub-chord may be obtained in the same manner, but with sufficient accuracy may be taken as $\frac{c}{C}\delta$

3. Draw up a table of the deflection angles to the various points

$$\text{1st deflection angle } d_1 = \frac{c}{C}\,\delta,$$

$$\text{2nd} \quad ,, \quad ,, \quad d_2 = \frac{c}{C}\,\delta + \delta,$$

$$\text{3rd} \quad ,, \quad ,, \quad d_3 = \frac{c}{C}\,\delta + 2\delta,$$

and so on, up to a maximum of $\frac{1}{2}I$

Instead of multiplying δ by the factors 2, 3, etc , it is preferable to obtain each deflection angle from the previous one by the addition of δ In this way a mistake made at one point will be continued, and may be detected in the following manner Calculate, from the value of the angle I, the length L of the curve Then

Chainage of A$+L$ = Chainage of B

The amount by which the chainage to B exceeds an exact number of peg intervals is the length of the end sub-chord c', and the calculation of the deflection angles is checked if the circumferential angle corresponding to c' equals ($\frac{1}{2}I$—the deflection angle to the end of the last whole chord)

The tabulated values of d_1, d_2, d_3, etc are the angles to be set on the circle of the theodolite in locating the curve, provided the direction of curvature from the first straight to the second is towards the right, as in Fig 265, since this direction is the same as that of the graduation of the circle of the instrument If, however, the curvature is towards the left, and the circle is not graduated both ways, each of the above angles must be subtracted from 360°, and the resulting values tabulated

4 With the instrument at A, set the vernier to zero, and sight on C Keeping the horizontal circle clamped, set off the first deflection angle d_1 on the vernier, so that the line of sight is now directed along AG The point G having to be a distance c from A, the rear chainman holds the chain with the reading c at the peg A, while the leader, holding the end of the chain against a pole, is directed by the surveyor into the line of sight When the chain is taut, and the pole is at the same time in the line of sight, the position of G is obtained, and a peg is driven at the point

5 Set the vernier to read the second deflection angle d_2, so that the line of sight is along AH If the chords are 66 ft or 100 ft long, the rear end of the chain is held at G, and the forward chainman swings about G until his end of the chain is in the line AH, when H is marked This procedure is repeated up to L, the end of the last whole chord, the position of each peg being determined by a ray from A and a measurement from the previously driven peg

6. Measure the length LB of the terminal sub-chord If the result agrees with the calculated length c', the chaining is checked To test the accuracy of the whole work, set the vernier to read $\frac{1}{2}I$,

If there has been no accidental change in the orientation of the instrument, the line of sight should again pass through B. Now relocate B by this ray and by measuring c' from L If the new position of B does not coincide with the tangent peg, the distance between the two points is the actual error of tangency, the allowable amount of which will depend entirely upon the circumstances of the case If this is exceeded, the whole work would require to be checked *ab initio* In situations where a few inches are of little importance, it is usually permissible to adjust the last few pegs to secure tangency in city or tunnel work the required degree of precision is, of course, greater

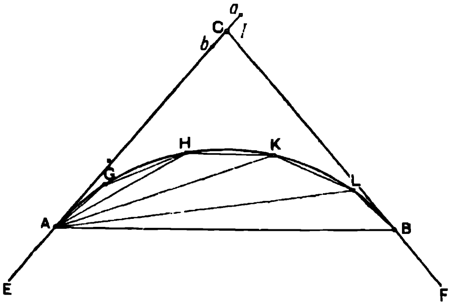

FIG 265.

7 The first chainage peg on the second straight will be the amount $(C - c')$ distant from B, and the chaining of this line may now be proceeded with on the transfer of the instrument to B or to another convenient point on BF

Notes —(1) Assuming the error in the length of the chain is negligible, the most common source of error in the above work is in setting off the wrong value for a deflection angle, the effect of this mistake being continued along the remainder of the curve The surveyor should therefore always compare each reading, as set, with the tabulated value of the angle Quite a satisfactory method of detecting mistakes on other than rough ground is for the chainmen, and the surveyor himself where possible, to observe at frequent intervals whether the pegs which have been driven appear to lie on a smooth curve In this way it is possible to discover a comparatively small local error

(2) Much time may be saved on a long curve if at each point to be fixed the leading chainman puts himself roughly in position before being guided by the instrument man He should do so by referring to the position of the previous chord produced After pegging point H, for example, and pulling forward the chain, he should hold it approximately in the line GH, and then swing his handle through the constant deflection distance to reach K The surveyor should tell him this distance, which $= \frac{C^2}{R}$ provided the chord produced is not a sub-chord (page 337)

Alternative Method of Setting Off Deflection Angles.—If there were no sub-chord at the commencement of the curve, the values of the deflection angles would be δ, 2δ, 3δ, etc , and a table of these could be prepared beforehand and used repeatedly in setting out curves of the particular radius to which it applies In cases where there is an initial sub-chord, the manipulation of the instrument at A may, however, be arranged so that advantage can be taken of such a table.

The chainage of A and the length and deflection angle of the sub-chord having been obtained in the usual manner, set off this angle *behind* the zero of the circle, so that the reading is (360°—the computed angle) for a right-hand curve. With the vernier clamped to this reading, sight on C. Turn the vernier to read zero: the line of sight is now along AG (Fig. 265), and the point G may be obtained. By then successively setting off δ, 2δ, 3δ, etc., the tabulated angles, the remaining pegs are located. In setting out a left-hand curve on this system, the tabulated angles can be used without subtraction from 360° provided the vernier reading for the tangent backsight is ($n\delta$+the deflection angle for the initial sub-chord), where n is the number of whole chords in the curve.

Example of Calculations.—Suppose that in ranging a right-hand curve of 32 chains radius by chords of one chain, with an instrument capable of reading to 30″, the intersection angle I is found to be 14° 38′. Let it be required to calculate the quantities necessary for the setting out.

$$\begin{aligned} T &= R \tan \tfrac{1}{2}I, \\ &= 32 \times \tan 7^\circ 19', \\ &= 4{\cdot}11 \text{ chains} \end{aligned}$$

On measuring back this distance from C, let the chainage of A be found to be 265·73 chains (commonly written 265+73). The initial sub-chord c has therefore a length of 27 links.

The circumferential angle for a chain chord, in minutes,

$$= \frac{1718{\cdot}9\,A}{R}, \text{ and with the approximation that } A = 1 \text{ chain,}$$

$$= \frac{1718{\cdot}9}{R \text{ in chains}} = 53'{\cdot}71,$$

$$\text{and for the sub-chord} = \frac{27}{100} \times 53'{\cdot}71 = 14'{\cdot}50$$

Table of deflection angles to nearest 30″

$$\begin{array}{llr} d_1 & & = \ 14'\ 30'' \\ d_2 = & 14'{\cdot}50 + 53'{\cdot}71 = 68'{\cdot}21 & = 1^\circ\ \ 8'\ \ 0'' \\ d_3 = & 68'{\cdot}21 + 53'{\cdot}71 = 121'{\cdot}92 & = 2^\circ\ \ 2'\ \ 0'' \\ d_4 = & 121'{\cdot}92 + 53'{\cdot}71 = 175'{\cdot}63 & = 2^\circ\ 55'\ 30'' \\ d_5 = & 175'{\cdot}63 + 53'{\cdot}71 = 229'{\cdot}34 & = 3^\circ\ 49'\ 30'' \\ d_6 = & 229'{\cdot}34 + 53'{\cdot}71 = 283'{\cdot}05 & = 4^\circ\ 43'\ \ 0'' \\ d_7 = & 283'{\cdot}05 + 53'{\cdot}71 = 336'{\cdot}76 & = 5^\circ\ 37'\ \ 0'' \\ d_8 = & 336'{\cdot}76 + 53'{\cdot}71 = 390'{\cdot}47 & = 6^\circ\ 30'\ 30'' \\ \tfrac{1}{2}I & & = 7^\circ\ 19'\ \ 0'' \end{array}$$

To check the calculation of the deflection angles, we have,

$$\begin{aligned} L = \frac{\pi I R}{180^\circ} &= \frac{3{\cdot}1416 \times 14^\circ 38' \times 32}{180^\circ}, \\ &= 8{\cdot}17 \text{ chains} \\ \text{Chainage to } B &= 265{\cdot}73 + 8{\cdot}17 \\ &= 273{\cdot}90 \text{ chains.} \end{aligned}$$

Length of final sub-chord $c' = 90$ links.

The circumferential angle subtended by 90 links $= \frac{90}{100} \times 53'{\cdot}71 = 48'{\cdot}34$,

but $(\tfrac{1}{2}I - d_8) = 7^\circ 19' 0'' - 6^\circ 30' 30'' = 48' 30''$, The calculations are therefore checked, the final difference, due to the assumption that C and A are equal, being negligible.

Note—If, instead of preparing a list of deflection angles as above, the engineer uses one of the published tables of deflection angles, he calculates the circumferential angles for the sub-chords, *viz* 14′ 30″ and 48′ 30″, as before. He sets the vernier to read 359° 45′ 30″ for the backsight AC, and moving the vernier to 360° directs his line of sight towards the first peg on the curve Thereafter, he sets off the seven successive tabulated angles, 53′ 30″, 1° 47′ 30″, etc, up to 6° 16′ 0″ The reading for the sight on B should be 6° 16′ 0″ + 48′ 30″ = 7° 4′ 30″

If the curve were left-handed, the vernier setting for the sight AC would be 6° 16′ 0″ + 14′ 30″ – 6° 30′ 30″ The successive settings are then the tabulated angles, 6° 16′ 0″, 5° 22′ 30″, 53′ 30″, and 360°, which is the setting for the peg before the tangent point B For the sight on B, the vernier should read 360° - 48′ 30″ = 359° 11′ 30″

Accuracy of Fix of Pegs on Curve.—Since the position of each peg on the curve other than the first is determined as the point of intersection of the instrument line of sight by an arc centered at the previously driven peg and of radius equal to the chord length, the probable accuracy of the location of the peg will depend upon the character of this intersection There will not as a rule be any doubt as to which of the two possible intersections is that required, but, if the angle between the chord and the line of sight is considerable, these two intersections approach each other, and the fix becomes unsatisfactory, in that carelessness in chaining produces a relatively great displacement in the position of the point sought

In Fig 266, L is the true position of the point to be located, and K is the last driven peg Let e be the error of measurement of the length KL = C, so that the length of the chord as measured is $KL_1 = (C - e)$, and L_1 is the point actually set out If the angle KLA be denoted by α, the displacement LL_1 in the direction of the line of sight is approximately given by $e \sec \alpha$ The instrument being over the first tangent point, $\alpha = (d_L - \delta)$, and, to secure a satisfactory fix, it is desirable that d_L should not exceed about 30° If the curve is such that I is greater than 60°, so that some of the deflection angles may exceed 30°, the instrument should be shifted forward according to the method of the following Case 1

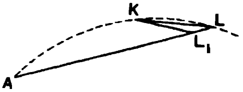

FIG 266

Difficulties in Ranging Simple Curves

Case 1.—When the Complete Curve cannot be Set Out from the Starting Point.—It has hitherto been assumed that the curve is sufficiently short and the ground so flat and free from obstructions that all the required pegs on the curve can be set out from the one position of the instrument at A. It is, however, very commonly impossible to do so.

In Fig. 267, the pegs G, H, and J have been located by the deflection angles d_1, d_2, and d_3 from A, but let it be supposed that, on setting off d_4, the line of sight AK is found to be obstructed

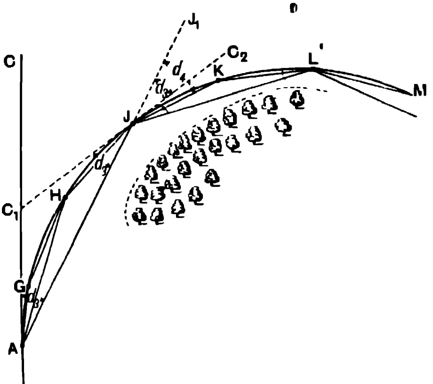

FIG 267

1 Transfer the instrument from A, and centre it over J

2 Set and clamp the vernier to the angle it read at A when sighting C, *i e* either zero or (360°—the deflection angle for the first sub-chord) according to the method used Sight back on A

3 Transit the telescope Set the vernier to d_4, the tabulated deflection angle for the point K, and the line of sight is now directed along JK, for, if C_1JC_2 represents the tangent at J,

$$C_1JA = J_1JC_2 = d_3,$$
$$J_1JK = d_3 + \delta = d_4$$

4 Continue the setting out from J in the usual manner

It may not be possible to complete the curve from the station J, in which case the further procedure is as follows

1 Set up over L, the last point located from J

2 Sight back on the last point occupied by the instrument or on any peg on the curve, with the vernier reset to read the deflection angle for that peg Thus, if J is sighted, the vernier must first have been brought to read d_3

3 Transit the telescope Set the vernier to d_6, the tabulated angle for the next peg M The line of sight is now along LM, and the setting out may be continued

Notes —(1) Points which are to serve as instrument stations, or to which backsights are to be taken, should be marked with particular care, as otherwise it will be found troublesome to check in at the end of the curve.

(2) The above method is arranged to possess the advantage that the checked tabular values, d_1, d_2, etc , are employed throughout On completing the work by sighting B from the last instrument station, the vernier should therefore read $\frac{1}{2}I$ An alternative, but less convenient, method consists in setting off the angles δ, 2δ, etc , anew from the tangent at each instrument station.

Case 2.—When an Obstacle Intervenes on the Curve.—If an obstruction, whether it can be seen over or not, is such that it cannot be chained across, it will be necessary to omit the location of the line across it until the obstacle is removed during construction To obtain the positions of the pegs beyond the obstacle the usual procedure must be modified as follows.

1. Having located in the ordinary way the points G and H (Fig 268) up to the obstruction, find, by setting off the successive tabulated deflection angles, a clear line of sight to a point on the curve. Let AL be this line, the deflection angle being d_5 It is supposed that the point K, although clear of the obstacle, cannot be seen from A

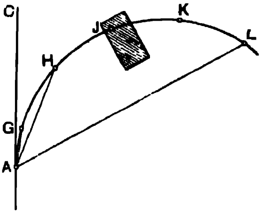

FIG 268

2 Calculate the length of the long chord AL from the formula,

$$\frac{\frac{1}{2}\text{ chord}}{\text{radius}} = \sin \tfrac{1}{2} \text{ central angle subtended by chord,}$$

i e $\text{AL} = 2R \sin d_5$

3 Measure out this distance from A, aligning the chaining from the instrument at A, and peg the point L

4 Continue the curve from L in the usual manner.

Notes —(1) If necessary, pegs such as K can be located by offsets from the long chord by the methods of page 338, or, if the instrument is transferred to L, they may be set out by deflection angles from L Otherwise they may be left over until the obstruction has been removed

(2) It may happen that no clear line AL can be obtained on account of obstacles of the type of Case 1 In such circumstances, having calculated the length of the curve, and so determined the length of the final sub-chord, the curve may be set out from the end B in the reverse direction up to K

Case 3.—When the Point of Intersection of the Tangents is Inaccessible.—This difficulty is of frequent occurrence Since the intersection point C is employed both in the measurement of I and as the starting point from which the lengths T are measured back, the field work must be arranged to supply the twofold deficiency when C is inaccessible

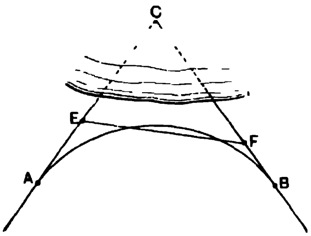

FIG 269

1 Select any convenient intervisible points E and F (Fig 269) on the straights

2 Measure angles AEF and EFB, then $I = 360° - \text{AEF} - \text{EFB}$ Calculate the tangent lengths from $T = R \tan \frac{1}{2} I$.

3 Measure EF, and solve triangle CEF for CE and CF

4 To locate A and B, measure from E the distance $\text{EA} = (T \sim \text{CE})$, and from F the distance $\text{FB} = (T \sim \text{CF})$

5. Complete the pegging out of the curve in the usual way.

Note —It frequently happens that no clear line EF can conveniently be obtained, in which case it is necessary to run a traverse between E and F, the required angles AEF and EFB and the distance EF being obtained by calculation

Case 4.—When the First Tangent Point is Inaccessible.—The field work must first be directed to the determination of the chainage of the inaccessible tangent point, since the length of the initial sub-chord and the positions of the pegs on the curve cannot otherwise be known.

1 On chaining back the tangent length from C (Fig. 270) and finding that A is inaccessible, note the measurement from C to a point F clear of the obstacle, then AF = (T—CF)

2 By any method of measuring past an obstacle, *e g.* by solution of a triangle such as FDE, determine the distance from F to some convenient point E on the straight and at the other side of the obstacle

3 Obtain the chainage to E, then the chainage of A = the chainage of E + EF — AF

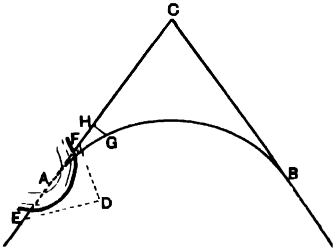

Fig 270

4 Compute the length of the curve, and so obtain the chainage of B

5 Set off the curve in the reverse direction from B The result may be checked by measuring the length of the offset GH from the last peg located to the tangent, its required length being $\frac{AH^2}{2R}$ approximately

Note —If it is found inconvenient to set out from B, the method of Case 6 may be employed

Case 5.—When the Second Tangent Point is Inaccessible.—In this case the continuation of the chainage along the second straight forms the difficulty Let A (Fig 270) be the second tangent point

1 On chaining the second tangent length, establish a point F at a known distance from C

2 Obtain, as in the last case, the distances FA = (T—CF), and FE to a convenient point E on the straight

3 The chainage of A having been computed from that of the starting point and the length of the curve, the chainage of E = the chainage of A + FE — FA

4 From the point E locate the first accessible chainage peg on the second straight.

Case 6.—When both Tangent Points are Inaccessible.—1 Having obtained the length AF and the chainage of A by the method of Case 4, compute, from FD $= R - \sqrt{R^2 - AF^2}$, the value of the perpendicular offset from F to the curve (Fig 271).

2 With the theodolite at F, set out the point D very carefully. D will be employed as an instrument station from which the curve may be continued.

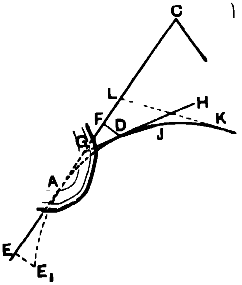

Fig 271.

3. Calculate the length of arc AD from the circumstance that it subtends at the centre an angle $\sin^{-1}\frac{AF}{R}$, then the chainage to D = the chainage of A + arc AD.

4 Draw up a table of deflection angles referred to the tangent DH.

5 Set the instrument over D. To lay the line of sight along DJ towards the first peg J, distant c from D, a backsight may be taken on F, and the vernier turned through $FDJ = 90^\circ + \sin^{-1}\frac{AF}{R} + \frac{c}{C}\delta$ But, unless DF has been made reasonably long, it is much better to secure a long backsight by regarding the curve as extended beyond A to E_1, the point E_1 being established by an offset EE_1 On sighting E_1 from D, and turning through angle $E_1DJ = 180^\circ - \frac{1}{2}\left(\sin^{-1}\frac{AE}{R} + \sin^{-1}\frac{AF}{R}\right) - \frac{c}{C}\delta$, the line of sight will be directed towards J

6 Continue the location until the obstacle at B is reached. Check the work by measuring an offset from the curve to the second tangent, and use the method of Case 5 in order to continue the chainage

Notes —(1) The whole of the field work involved in this method requires considerable care in order that the closure may be effected The instrument stations should be carefully aligned and marked by a tack on the peg Checks on the progress of the work on a long curve may be made at intermediate instrument stations, such as K, by noting where the tangent KL cuts the first tangent The distance CL, for instance, should be found to equal $\left[T - R\tan\frac{1}{2}\left(\frac{180^\circ \times \text{arc AK}}{\pi R}\right)\right]$.

(2) Having determined the chainage of A, the following alternative method of setting out the curve is preferable when the intersection point C is readily accessible Set up the theodolite at C, and bisect angle ACB Compute the apex distance, or distance from C to the mid-point of the curve, and, by measuring out this distance along the bisector, locate M, the mid-point of the curve The chainage of M = the chainage of A + $\frac{1}{2}L$. Transfer the instrument to M, backsight on C, and set out the curve in both directions from M

(3) A third method consists in locating an instrument station on the curve by measuring out from a point, such as E, on the straight Assuming that D is a point of known chainage to which a measurement can be made from E, the distance ED and the angle CED are obtained by solution of triangle EAD since, the chainage of A having been ascertained, EA, AD, and angle EAD can be evaluated Alternatively, triangle EGD may be solved The point D having been pegged, the setting out can be continued from D after backsighting on E.

The foregoing difficulties may occur simultaneously, particularly in the course of setting out in cities. Thus, in Fig 272, which represents the centre line of part of an underground railway, the setting out is based upon the points D and E on one tangent and F and G on the other. Very careful traversing must be executed between DE and FG to determine the intersection angle and the positions of A and B To obtain any point on the curve, such as the centre of a shaft H, the co-ordinates of the point are computed from, say, E as origin and DE as meridian. A traverse is conducted from E towards H, the length of the last line being computed so that it ends on the required point The bearing of the tangent at H is also calculated, and is set out from the traverse to form a base for the location of the curve underground.

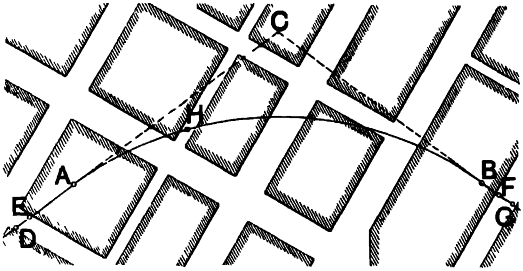

FIG 272

Horse Shoe Curves.—If, as in Fig 273, the angle I subtended at the centre of a curve exceeds 180°, the point of intersection C of the tangents EA and BF lies on the same side of the curve as the centre O The angles at C are as marked, and evidently from triangle AOC

$$T = R \tan (180° - \tfrac{1}{2}I)$$

The deflection angles may be laid off from AD, the production of the first straight, and they are tabulated in the usual way up to the maximum angle, DAB = $\frac{1}{2}I$

In country necessitating the introduction of such curves it will usually be impracticable to obtain the value of I at the intersection point C, and the method of running a series of traverse courses or, if possible, a single transversal between points on the two straights is generally required. If EF be the traverse closing line or the transversal, then $I = \text{AEF} + \text{EFB}$

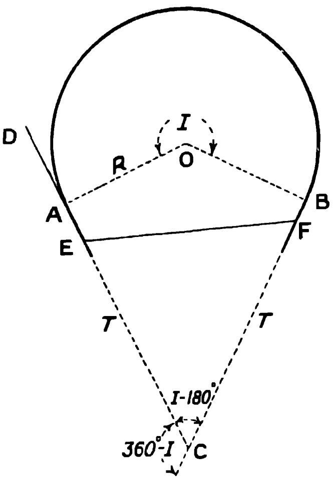

FIG 273

Setting out Simple Curves by Two Theodolites.—The positions of pegs on a curve may be determined, without the necessity of chaining the chord lengths, by locating them as the points of intersection of the lines of sight from two theodolites placed one at each end of the curve Since two instrument men are required, the method is not in general use, but might be warranted in cases where the curve lies on ground of such character that there would be difficulty in performing the chaining with the required degree of accuracy

1 Having determined the positions of A and B, the chainage of A, and the length c of the initial sub-chord, calculate the length of the curve and from it the chainage of B and the length c' of the terminal sub-chord

2 Draw up tables of deflection angles from A and B respectively Thus, for the curve of Fig 274, we should have

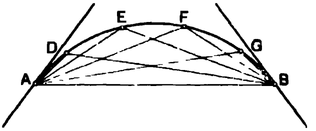

Fig 274

Point	Angle from A	Angle from B
A	—	$\frac{c'}{C}\delta+3\delta+\frac{c}{C}\delta=\frac{1}{2}I$
D	$\frac{c}{C}\delta$	$\frac{c'}{C}\delta+3\delta$
E	$\frac{c}{C}\delta+\delta$	$\frac{c'}{C}\delta+2\delta$
F	$\frac{c}{C}\delta+2\delta$	$\frac{c'}{C}\delta+\delta$
G	$\frac{c}{C}\delta+3\delta$	$\frac{c'}{C}\delta$
B	$\frac{c}{C}\delta+3\delta+\frac{c'}{C}\delta=\frac{1}{2}I$	—

The fact that the sum of the deflection angles to each point on the curve equals $\frac{1}{2}I$ provides a check in addition to the usual one

3 With the instruments at A and B respectively, set off these angles consecutively, the chainman being guided to each point until the pole held by him is simultaneously in the two lines of sight

Note —Should it be found necessary to shift one or both instruments to sub-stations on the curve, the tabulated angles will be employed throughout if the orientation of the instrument at each setting is performed as in Case 1, page 329.

SETTING OUT SIMPLE CURVES BY CHAIN AND TAPE ONLY

The location of a simple curve may be performed without the use of an angular instrument in cases where :

(*a*) A high degree of accuracy is not demanded, as in roads, avenues, etc

(*b*) The curve is short, as in certain railway and tramway curves, corners of buildings, curved wing walls, etc.

Location of Tangent Points.—In short curves of small radius, the positions of the tangent points can generally be taken from a large scale plan with sufficient accuracy to permit of the fitting in of the curve between them on the ground Otherwise, the following method may be used

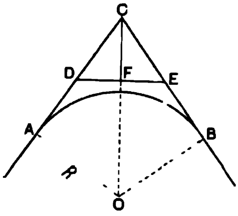

Fig 275

1 Produce the two straights by eye to meet at C (Fig 275)

2 Select any pair of intervisible points D and E, one on each tangent and equidistant from C, making CD and CE as long as is convenient

3 Measure DE, mark its mid-point F, and measure CF

4 Since triangles CDF and COA are similar,

$$\frac{CA}{OA} = \frac{CF}{DF},$$

so that $T = \frac{CF}{DF} \cdot R$

Notes—(1) If CD and CE cannot conveniently be made equal, select any points D and E in suitable positions on the respective tangents Measure the three sides of triangle CDE, and solve for angle DCE = 180° − I. Calculate T from $T = R \tan \frac{1}{2}I$, and measure out the differences DA and EB

(2) If C is inaccessible, angles ADE and DEB may be evaluated by solution of triangles such as A′DF and FEB′ from the measured lengths of their sides, A′ and B′ being convenient points on the tangents Thereafter the procedure is as in Case 3, page 331

Location of Points on the Curve by Deflection Distances.—This is the most useful method for a long curve

Case (a) When there is no initial sub-chord, or the chainage is not required to be continuous

1 With the tangent point A as centre (Fig 276), swing the chord length AD = C on the chain until the perpendicular offset ED from the tangent $= \frac{C^2}{2R}$ D is the position of the first peg on the curve

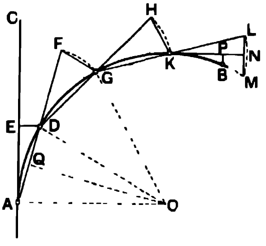

Fig 276

2. By means of poles at A and D produce AD its own length to F. Mark F with a pole or arrow, and, with centre D, swing the chord length to the position DG, so that FG $= \frac{C^2}{R}$. Peg the point G.

3. Produce DG to H, and repeat

4. To check the work at the second tangent point B, KB being found to be a sub-chord of length c', set out the points L and M as for a whole chord. Bisect LM at N, then KN is the tangent

to the curve at K From KN set off the perpendicular offset $PB = \frac{c'^2}{2R}$, and the error of tangency will be determined

Notes —(1) The above expressions for the offsets from the tangents and from the produced chords are precise If Q is the mid-point of AD, and A, Q, D, and G are joined to the centre O, the triangles AED and OQA are similar

$$\frac{ED}{C} = \frac{\frac{1}{2}C}{R},$$

$$\text{or } ED = \frac{C^2}{2R}$$

Again triangles DFG and ODG are similar, since both are isosceles and FDG = 180° - 2GDO = DOG,

$$\frac{FG}{C} = \frac{C}{R},$$

$$\text{or } FG = \frac{C^2}{R}.$$

(2) Unless the radius is very small, it is in general sufficiently accurate and more convenient to make AE = C, so that the offset $ED = \frac{C^2}{2R}$ is not perpendicular to AE

In similar circumstances, LN may be set out perpendicular to KL and equal to $\frac{C^2}{2R}$ without the necessity of locating M

Case (b) When there is an initial sub-chord, c.

A slight modification is necessary in this case on account of the circumstance that the second peg G could not be located in the usual manner from the production of the short chord AD (Fig. 277)

1 Obtain the position of D by swinging the length c about A until $ED = \frac{c^2}{2R}$

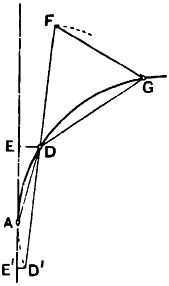

FIG 277

2 Consider the curve extended as shown dotted, and locate the whole chord DD′ by turning it about D until the perpendicular offset E′D′ has the length given by the approximate formula $\frac{(C-c)^2}{2R}$

3 Produce D′D a whole chord length to F, and swing through $FG = \frac{C^2}{R}$ as before

Notes —(1) If the radius is more than, say, three times the chord length, it is usually quite allowable to locate D′ by measuring back AE′ = $(C - c)$ and erecting the offset from E′

(2) If the sub-chord AD is not too short for reasonably accurate production, an alternative method of locating G is to produce AD to F′, making DF′ a whole chord length C With centre D, the chord length is swung to DG, the distance F′G being approximately $\frac{C(C+c)}{2R}$

Location of Points on the Curve by Tangent and Chord Offsets.—These methods are useful for short curves. In such cases it is usually unnecessary that the pegs should be equally spaced, and the field work requires no explanation Formulæ from which to calculate the lengths of the offsets are as follows:

(a) Offset o at a distance l on a tangent from the tangent point (Fig 278)

From triangle EDO,

$$R^2 = (R - o)^2 + l^2,$$

$$\text{whence } o = R - \sqrt{R^2 - l^2},$$

$$= \frac{l^2}{2R} \text{ approximately}$$

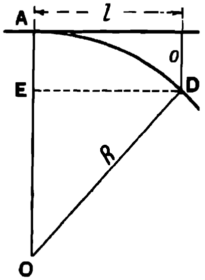

FIG 278

(b) Offset o_1 from the mid-point of a chord of length C (Fig 279)

In the same manner,

$$o_1 = R - \sqrt{R^2 - \frac{C^2}{4}},$$

$$= \frac{C^2}{8R} \text{ approximately}$$

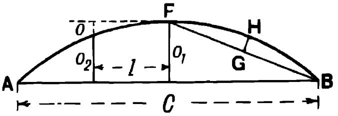

FIG 279

(c) Offset o_2 from the chord AB at a distance l from its mid-point

Since a tangent to the curve at its mid-point F is parallel to AB, o_2 is less than o_1 by the offset o above formulated, so that

$$o_2 = o_1 - R + \sqrt{R^2 - l^2},$$

$$= o_1 - \frac{l^2}{2R} \text{ approximately}$$

(d) Having obtained F by the offset o_1, an alternative method of locating other points on the curve consists in bisecting chord FB and erecting an offset $GH = \frac{FB^2}{8R}$ from its mid-point By dealing similarly with the chords FH and HB, etc, any number of points may be determined.

Note—The approximate expressions above should not be used unless l or C is small in relation to the radius.

COMPOUND CURVES

Elements of a Compound Curve.—In Fig. 280, ADB represents a compound curve, the two branches of which join at D and have centres O_1 and O_2 and radii R_1 and R_2 respectively, R_2 denoting the greater whether it comes first or second in the direction of chainage. The end straights, on being produced, meet at C, giving an intersection angle I The tangent lengths AC and BC are necessarily unequal, and will be distinguished as T_1 and T_2, of which T_2 is the greater and is adjacent to the arc of greater radius. The common tangent at D meets these end tangents at C_1 and C_2, and makes with them the angles I_1 and I_2 equal to the respective central angles subtended by the arcs For the two simple curves, we have

$$C_1A = C_1D = R_1 \tan \tfrac{1}{2}I_1 = t_1,$$
$$C_2B = C_2D = R_2 \tan \tfrac{1}{2}I_2 = t_2$$

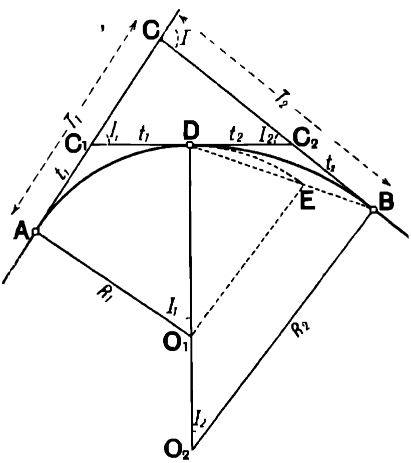

Fig 280

The following properties of the compound curve are evident from the figure

(1) The centres O_1 and O_2 are in the same straight line with D, since the radii O_1D and O_2D are both perpendicular to the common tangent

(2) $I = (I_1 + I_2)$, since I is an exterior angle of the triangle CC_1C_2

(3) $CC_1 = C_1C_2 \dfrac{\sin I_2}{\sin I}$,

$$T_1 = t_1 + (t_1 + t_2)\frac{\sin I_2}{\sin I}, \quad \ldots\ldots\ldots \quad (1)$$

and similarly $T_2 = t_2 + (t_1 + t_2)\dfrac{\sin I_1}{\sin I}$ . ...(2)

(4) T_1 must be greater than $R_1 \tan \frac{1}{2}I$, or R_1 must be smaller than $T_1 \cot \frac{1}{2}I$. for, if $T_1 = R_1 \tan \frac{1}{2}I$, the curve would be simple, and if T_1 were less than $R_1 \tan \frac{1}{2}I$, tangency at the second straight could be secured only by making the radius of the second branch less than R_1, which has been premised to be the smaller radius. Similarly, T_2 must be smaller than $R_2 \tan \frac{1}{2}I$, and R_2 greater than $T_2 \cot \frac{1}{2}I$.

(5) If the arc AD is extended to E, so that the tangent at E is parallel to CB, then D, E, and B are in the same straight line. For O_1E and O_2B are parallel, and if DE and DB be joined, the triangles DO_1E and DO_2B are similar, since both are isosceles and their angles at O_1 and O_2 are equal Therefore angles O_1DE and O_2DB are equal, and DEB must be one straight line Similarly, the corresponding point on the arc BD produced is in the same straight line as D and A

Relationships between the Parts of a Compound Curve.—The seven quantities involved in a compound curve of two branches, *viz* I, I_1, I_2, R_1, R_2, T_1, and T_2, are interrelated in a manner deducible from the above expressions (1) and (2) as follows

$$T_1 = t_1 + (t_1 + t_2)\frac{\sin I_2}{\sin I}, \quad \ldots \quad (1)$$

$$= R_1 \tan \tfrac{1}{2}I_1 + (R_1 \tan \tfrac{1}{2}I_1 + R_2 \tan \tfrac{1}{2}I_2)\frac{\sin I_2}{\sin I},$$

$$\text{but } \tan \tfrac{1}{2}I_{1 \text{ or } 2} = \frac{1 - \cos I_{1 \text{ or } 2}}{\sin I_{1 \text{ or } 2}},$$

$$\therefore\ T_1 = R_1\frac{1-\cos I_1}{\sin I_1} + \left(R_1\frac{1-\cos I_1}{\sin I_1} + R_2\frac{1-\cos I_2}{\sin I_2}\right)\frac{\sin I_2}{\sin I},$$

$$\text{or } T_1 \sin I = R_1\left(\frac{1-\cos I_1}{\sin I_1}\right)(\sin I + \sin I_2) + R_2(1 - \cos I_2)$$

Substituting $\sin(I - I_1)$ for $\sin I_2$, this reduces to

$$T_1 \sin I = R_1(\sin I \sin I_1 + \cos I \cos I_1 - \cos I) + R_2(1 - \cos I_2),$$

$$= R_1(\cos(I - I_1) - \cos I) + R_2(1 - \cos I_2),$$

$$= R_1(\cos I_2 - \cos I) + R_2(1 - \cos I_2),$$

which may preferably be expressed as

$$= R_1\{(1 - \cos I) - (1 - \cos I_2)\} + R_2(1 - \cos I_2),$$

$$\text{or } T_1 \sin I = R_1 \operatorname{versin} I + (R_2 - R_1) \operatorname{versin} I_2 \quad . \ . \quad . \ (3)$$

$$\text{Similarly, } T_2 \sin I = R_2 \operatorname{versin} I - (R_2 - R_1) \operatorname{versin} I_1 \quad . \quad . (4)$$

Expressions (3) and (4) in conjunction with the relationship $I = (I_1 + I_2)$ provide three simultaneous equations, the solution of which will determine three of the seven parts of the curve if the remaining four form the data. In practice, it will almost always be possible to locate the end straights first, so that I will be known If, in addition, both radii are given, there remains one quantity to be fixed, and this may be either a tangent length, T_1 or T_2, or one of the angles I_1 or I_2 If, as is generally the case, the curve is already plotted, it will usually be preferable to scale a tangent length, but the length of one of the arcs may be fixed instead, in which case I_1 or I_2 is known By utilising the above three simultaneous equations, the manner of setting out the curve is made independent of the choice of data, but for definiteness in describing the field work it will be assumed that R_1, R_2, and T_1 are known, and that I is measured in the field.

Setting Out a Compound Curve.—*Data* R_1, R_2, T_1, and I.

1. Locate the end straights, and produce them to their intersection C (Fig. 280) Measure I
2. Calculate T_2, I_1, and I_2
3. Chain T_1 and T_2 from C, and peg A and B. Determine the chainage of A.

4 Calculate the lengths of the arcs AD and DB and thence the chainages of D and B. Prepare a table of deflection angles for the arc AD : the deflection angle to D should work out as $\frac{1}{2}I_1$. Similarly, tabulate deflection angles up to $\frac{1}{2}I_2$ for the arc DB referred to the tangent at D

5 Set out the arc AD from A.

6 Transfer the instrument to D, and set the vernier to $\frac{1}{2}I_1$ behind zero Sight A, transit, and set the vernier to the first tabulated deflection angle for arc DB The line of sight is now directed towards the first peg on DB Continue setting out to B.

Notes —(1) Having included the value of T_1 in the data, A must be located by measurement from C as above and not merely by its chainage as scaled from the plan If A is fixed by the latter method, its distance from C must be measured to give the value of T_1 to be used in the calculations

(2) A check on the positions of A and B is obtained, if B is visible from A, by observation of the angle CAB, the value of which can be calculated by solution of the triangle CAB, but it frequently happens that the sight AB cannot be obtained If B is visible from D, the check may consist in establishing the direction of the common tangent at D from a backsight on A, or other point on the arc AD, and observing whether $C_2DB = \frac{1}{2}I_2$ or, in other words, verifying that $ADB = (180° - \frac{1}{2}I)$

Example of Calculations for a Compound Curve.—Let $R_1 = 20$ chn , $R_2 = 40$ chn , $T_1 = 17{\cdot}54$ chn , and the measured value of $I = 63° \; 29'$, the arc of smaller radius coming first in the direction of chainage

I_2 is first obtained from

$$T_1 \sin I = R_1 \operatorname{versin} I + (R_2 - R_1) \operatorname{versin} I_2, \quad . \; . \; . \; . \; . \quad (3)$$

whence $I_2 = 39° \; 45'$,

and $I_1 = (I - I_2) = 23° \; 44'$

T_2 can now be derived from

$$T_2 \sin I = R_2 \operatorname{versin} I - (R_2 - R_1) \operatorname{versin} I_1, \qquad (4)$$

whence $T_2 = 22{\cdot}85$ chn

On measuring back T_1, let the chainage of A be found to be 252·46 chn

$$\text{Arc AD} = \frac{\pi I_1 R_1}{180°} = 8{\cdot}28 \text{ chn},$$

chainage of D = 252·46 + 8·28 = 260·74 chn

$$\text{Arc BD} = \frac{\pi I_2 R_2}{180°} = 27{\cdot}75 \text{ chn},$$

chainage of B = 260·74 + 27·75 = 288·49 chn

The tabulation of the deflection angles may then proceed as for simple curves.

Difficulties in Ranging Compound Curves.—The procedure in the case of obstacles or inaccessibility of C, A, or B is as for simple curves When D is inaccessible, the second arc is best set out from B, but, if B is also inaccessible, the location may be made by means of observations from A or any other instrument station on the first arc.

Thus, in Fig 281, let it be required to locate H, one of the chainage pegs on the second branch, from the station G on the first arc. In triangle GDH, GD and DH can be obtained from the known lengths of arcs GD and DH, and $GDH = (180° - C_1DG - C_2DH)$.

But $C_1DG = (\frac{1}{2}I_1 - d_G)$, and C_2DH = the tabulated deflection angle of H from D, so that the triangle can be solved for GH, DGH, and DHG Now AGH = $(180° - \frac{1}{2}I_1 - \text{DGH})$ GH may therefore be set out and H pegged On transferring the instrument to H, the work may be checked by observing whether GHB $= (180° - \frac{1}{2}I_2 - \text{DHG})$ in the case where this method is employed with B accessible. To enable the tabulated deflection angles to be utilised in setting out arc HB, the backsight on G should be made with the vernier reading (360°−GHD) On turning the vernier to read d_K, the deflection angle for the next peg K, and transiting, the line of sight will be directed for the pegging of K

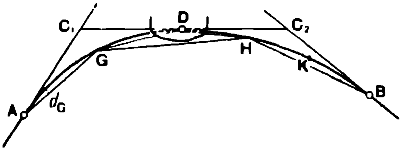

FIG 281

Alternative Method of Setting Out a Compound Curve.—An alternative method of locating a compound curve of which the radii are known is designed to avoid the necessity of extending the end straights to their intersection C, and is very useful where C is distant or inaccessible or when the measurement of T_1 and T_2 would be troublesome It involves the setting out of the common tangent by means of scaled dimensions from the plan in the same manner as for the end straights The points of intersection, C_1, C_2 (Fig 282), of the common and end tangents are pegged, and I_1 and I_2 are measured. In order that a compound curve of given radii may fit the three tangents, the necessary condition is that the length of the common tangent should $= (t_1 + t_2) = (R_1 \tan \frac{1}{2}I_1 + R_2 \tan \frac{1}{2}I_2)$

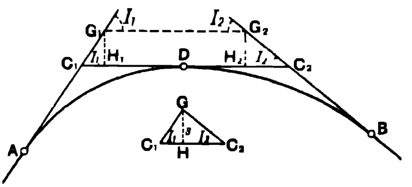

FIG 282

The process of setting out is as follows

1 Locate the end straights and the common tangent Measure I_1 and I_2 and the length L of the common tangent as laid out

2 Calculate t_1 and t_2 If $L = (t_1 + t_2)$, locate D at t_1 from C_1, and set out the arcs AD and DB

3 If, however, L differs appreciably from $(t_1 + t_2)$, I_1 and I_2 being assumed to have been correctly measured, the discrepancy is due to the line not occupying the position required for common tangency The change required to make $L = (t_1 + t_2)$ is most conveniently effected by a movement of the line parallel to itself, so that the values of I_1 and I_2 are unaltered The direction of shift will be outward, or towards C, if $L > (t_1 + t_2)$, and inward if $L < (t_1 + t_2)$, and the amount of shift can be calculated as follows.

In Fig. 282, let G_1G_2 be the line set out in place of C_1C_2, its length L being found $< (t_1 + t_2)$. The perpendiculars G_1H_1 and

G_2H_2 represent the shift s. Let the triangles $G_1C_1H_1$ and $G_2C_2H_2$ be supposed moved together until G_1H_1 and G_2H_2 are in coincidence. In the resulting triangle GC_1C_2,

$$C_1H = s \cot I_1,$$
$$\text{and } C_2H = s \cot I_2,$$
$$\therefore C_1H + C_2H = s(\cot I_1 + \cot I_2),$$
$$= (t_1+t_2) - L,$$

or, for any case, $$s = \frac{(t_1+t_2) \backsim L}{\cot I_1 + \cot I_2}$$

Note.—It is usually quite justifiable to make the shift a parallel one, as the direction of the common tangent may be set out as accurately as those of the end straights. Alteration in the direction of the common tangent has the effect of changing the lengths of the component arcs and the position of the whole curve.

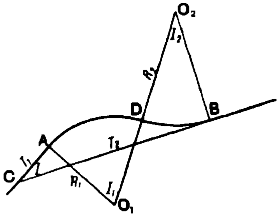

Fig 283

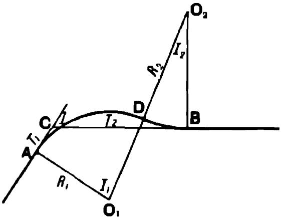

Fig 284

REVERSE CURVES

Since sudden reversal of curvature is objectionable at high speeds, the use of reverse curves should, where possible, be avoided on main lines, but they are frequently required in sidings, etc., where the traffic is slow.

The geometrical principles involved are similar to those for compound curves, but the radii of the component arcs of a reverse curve may be equal, and the point of intersection C of the end straights may fall on either side of the common tangent and on either side of the tangent points A and B. As before, R_2 will denote the greater radius, but T_2, the adjacent tangent length, does not necessarily exceed T_1, as in the compound curve. The intersection point C may coincide with a tangent point, so that one of the tangent lengths vanishes. Figs. 283 to 287 illustrate various conditions.

(*a*) $I_1 > I_2$. C on O_1 side of common tangent and before A (Fig. 283).

(*b*) $I_1 > I_2$. C on O_1 side of common tangent and beyond A (Fig. 284).

(*c*) $I_1 = I_2$. C at infinity (Fig. 285).

(*d*) $I_1 < I_2$. C on O_2 side of common tangent and before B (Fig. 286).

(*e*) $I_1 < I_2$. C on O_2 side of common tangent and beyond B (Fig. 287).

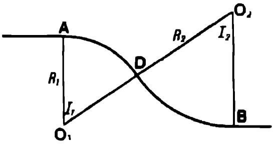

Fig 285

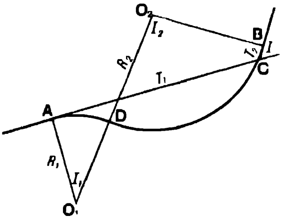

Fig 286

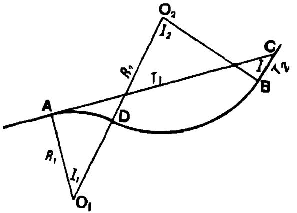

Fig. 287.

As before, the angle I represents the angular deviation produced by the curve, but, in the case of reverse curves,

$$I = I_1 \sim I_2$$

The remaining relationships between the seven parts, corresponding to (3) and (4) (page 340), and derived in the same manner, are

$$T_1 \sin I = (R_1 + R_2) \operatorname{versin} I_2 - R_1 \operatorname{versin} I,$$
$$T_2 \sin I = (R_1 + R_2) \operatorname{versin} I_1 - R_2 \operatorname{versin} I$$

These formulæ are applied in setting out in the same manner as the similar formulæ for compound curves.

Case of Parallel Tangents.—In case *c* (Fig. 285) I is zero, and T_1 and T_2 are infinitely great, so that the general formulæ do not serve to enable the curve to be set out. This case is of considerable practical importance, and, since the data almost invariably include the distance between the parallel tangents, formulæ for its solution may be derived as follows.

Let P = the perpendicular distance between the parallel tangents.

In Fig. 288, let EDF be drawn through D parallel to the tangents and meeting O_1A and O_2B in E and F respectively.

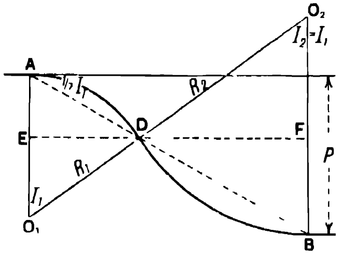

FIG. 288

Then $P = AE + FB$,

$$= R_1 \operatorname{versin} I_1 + R_2 \operatorname{versin} I_2,$$
$$= (R_1 + R_2) \operatorname{versin} I_1,$$

$$\text{or } \operatorname{versin} I_1 = \frac{P}{(R_1 + R_2)}$$

If the radii are known, this formula suffices for locating D and enabling the two simple arcs to be set out. Alternatively, if the radius and central angle of one arc are given, the required radius of the other can be found.

In practice it frequently happens that a reverse curve between parallel tangents is comparatively short, and use may be made of the chord AB in setting out.

It will readily be seen from Fig. 288 that D is always in the line AB. Now $AD = 2R_1 \sin \frac{1}{2} I_1$, and $DB = 2R_2 \sin \frac{1}{2} I_1$,

$$\therefore AB = 2 \sin \tfrac{1}{2} I_1 (R_1 + R_2) = 2 \frac{P}{AB} (R_1 + R_2),$$

$$\text{or } AB = \sqrt{2P(R_1 + R_2)}$$

TRANSITION CURVES

A *Transition* or *Easement* curve is a non-circular arc introduced between a straight and a circular curve, or between the two branches of a compound or a reverse curve The function of a transition curve in railway alignment is two-fold

1 To eliminate the sudden change of curvature in passing from a straight to a curve or from one arc to another of different radius or direction of curvature

2 To provide a medium for the gradual introduction, tailing out, or change of the required superelevation of the outer rail

Change of Curvature.—In passing from a straight into a circular curve, the sudden change of curvature from zero to a finite quantity at the tangent point causes a lurching of the rolling stock due to the abrupt calling into play of centrifugal force. The effect is to throw the train suddenly against the outer rail, and is evidenced by increased wear of that rail in the region of the tangent point The resulting oscillation not only depends upon the speed, but also increases with increase of change of curvature, and it is therefore more severe at the point of common tangency on a reverse curve and less marked between the two arcs of a compound curve than at the ends of a simple curve of similar radius By the introduction of a transition curve at those places, the change of curvature may be made continuous and as gradual as desired

Superelevation.—In railway curves, in order to equalise the wear of the two rails and to eliminate the possibility of the train mounting the outer rail by the action of centrifugal force, that rail is elevated above the inner one by such an amount that the resultant of the weight of the train and the centrifugal force acting upon it is equally distributed to both rails This is accomplished if the superelevation or cant is such that

$$\frac{\text{cant}}{\text{distance between centres of rails}} = \frac{\text{centrifugal force}}{\text{weight of train}}$$

The centrifugal force acting upon a body of weight W, travelling with a velocity of v ft. per sec round a curve of radius R ft , is given by $\frac{Wv^2}{gR}$, where g is 32·2 ft per sec ^{2}, the acceleration due to gravity. The required amount of cant may therefore be expressed as

$$\text{cant in inches} = \text{distance between centres of rails in inches} \times \frac{v^2}{gR}.$$

A more convenient form of this relation is

$$\text{cant in inches} = \frac{\cdot 8 \times \text{distance between centres of rails in ft} \times (\text{speed in miles per hr.})^2}{\text{radius in ft.}}$$

With the standard 4 ft. 8½ in gauge the centres of the rails are about 4 ft 11 in. apart, and, on substituting this value, the formula becomes :

$$\text{cant in inches} = \frac{3{\cdot}9 \times (\text{speed in miles per hour})^2}{\text{radius in feet}},$$

$$\text{or} \quad = \frac{{\cdot}06 \times (\text{speed in miles per hour})^2}{\text{radius in chains}},$$

$$\text{or} \quad = 00068\ (\text{speed in miles per hour})^2 \times \text{degree of curve}$$

In fixing the cant for a particular curve, it is impossible to suit all classes of traffic, and practice varies as to the method of assigning a speed which will determine the value to be adopted. Cant is sometimes made to accord with the probable speed of the fastest trains, regard being paid to circumstances of locality, since speed is reduced on rising gradients and near stations and junctions. Experience shows, however, that too much cant is as objectionable as too little, and it is better to base the cant on some speed less than the maximum, say the average speed of passenger trains. The speed to be allowed for is sometimes fixed from both the maximum and minimum speeds on the curve Thus, the rule on certain railways is to provide cant suitable for a speed

$$V = \sqrt{\tfrac{1}{2}(V^2\,\text{max} + V^2\,\text{min})}$$

If the cant, as calculated, exceeds a certain limiting value, the speed of fast trains should be reduced on the curve The most commonly adopted maximum value is 6 in , a greater cant than this being very rare

At the junction of a straight and a circular arc, since the necessary superelevation for the curve cannot be introduced abruptly, three courses are open in the absence of a transition curve ·

1 To commence tilting the track on the straight, so that when the tangent point is reached the full superelevation is provided.

2 To introduce the cant along the curve, the rails on the straight being level transversely up to the tangent point

3. To combine the above methods, so that the cant is introduced partly on the straight and partly on the curve.

These methods are objectionable, since the requirement that superelevation should be proportional to curvature is not fulfilled. The effect of a transverse inclination on the straight is to cause lurching of the rolling stock against the low rail, with excessive wear of that rail With insufficient cant on the curve, the effect is transferred to the elevated rail. The first and third methods are not uncommon, and if the latter is accompanied by an arbitrary "sweetening" of the junction when laying the rails, its faults are to some extent removed. The objections to the above methods are, however, entirely obviated by the introduction, between the straight and the circular curve, of a transition arc of such varying curvature as will enable the cant to be introduced at any desired rate, the curvature at any point corresponding to the cant required at that point

Requirements of a Transition Curve connecting a Straight and a Circular Arc.—These are as follows

1 The transition should be tangential to the straight

2 Its curvature should be zero at the origin on the straight

3 The curvature should increase along the transition at the same rate as the cant increases

4 Its length should be such that at the junction with the circle the full cant has been attained

5 It should join the circular arc tangentially.

6 Its curvature at this point should be the same as that of the circle.

The conditions as to the tangency and curvature at the origin and the end of the transition are definite, but the rate of change of curvature and the length are dependent on the rate at which the cant is introduced It is the universal practice to increase the cant at a uniform rate, and, in consequence, the curvature of the transition at any point is proportional to the distance of that point from the origin

Length of Transition Curves.—The length of transition curve required to enable the cant to be applied at a suitable rate may be determined in three ways

Let L = length of transition in ft ,
V = speed on which cant is based, in miles per hour,
v = ,, ,, ,, ,, in ft per sec ,
C = cant in inches for standard gauge,
R = radius of circular curve in ft

First Method —The length may be that required to permit the superelevation to be introduced according to an arbitrary gradient. This is a common system, and the rate of canting varies in practice from 1 in in 25 ft to about 1 in in 100 ft , so that L runs from 300 to about 1,200 times the cant

On this principle, if the gradient is 1 in. in s ft.,

$$L = \frac{3{\cdot}9 V^2 s}{R}.$$

Second Method —The length may be such that trains are canted at an arbitrary time rate. This rate may range from 1 in. to over 2 in. per second.

If r = rate of canting in inches per second,

the time taken in travelling over the transition $= \frac{L}{v} = \frac{C}{r} = \frac{3{\cdot}9V^2}{Rr}$ sec.,

$$\text{so that } L = \frac{5{\cdot}72V^3}{Rr}$$

Third Method.—The length may be such as to enable the rate of change of radial acceleration on the transition curve to be limited to the extent that no sensation of discomfort may be experienced by passengers. Mr. Shortt* states that in his experience a rate of change of acceleration of 1 ft. per sec.² per sec. is the maximum that will pass unnoticed.

Let a = allowable rate of change of acceleration in ft. per sec.³,
v, V = *max.* speed in ft. per sec. and miles per hour, respectively.

The time taken to travel over the transition $= \frac{L}{v}$ sec.

Acceleration attained in that time $= \frac{aL}{v}$ ft. per sec.²,

$= \frac{v^2}{R}$, the radial acceleration on the circular arc,

$$\text{whence } L = \frac{v^3}{aR},$$

or, for a = 1 ft. per sec.³,

$$L = \frac{3{\cdot}16V^3}{R}.$$

Either of the methods making $L \propto V^3$ is preferable to the first.

The Ideal Transition Curve.—Let

R = the radius of the circular curve (Fig. 289),
r = the radius of the transition at any point P, l ft. along it from the origin T,
c = the cant at P,
ϕ = the angle between the straight and the tangent to the transition at P,
ϕ_1 = the value of ϕ at S, the point of junction with the circle,
L = the total length of the transition.

Since cant will increase uniformly along the transition,

$$\frac{c}{l} = \text{constant},$$

but cant is inversely proportional to radius,

$$\therefore\ rl = \text{constant} = RL$$

---

* "A Practical Method for the Improvement of Existing Railway-Curves," by W. H. Shortt, *Min. Proc. Inst. C.E.*, Vol. CLXXVI.

Now $\frac{1}{r} = \frac{d\phi}{dl}$, the curvature,

$$\frac{l}{RL} = \frac{d\phi}{dl},$$

or $d\phi = \frac{ldl}{RL}$,

and integrating,

$$\phi = \frac{l^2}{2RL} + k$$

When $l = 0$, $\phi = 0$,

$\therefore k = 0$,

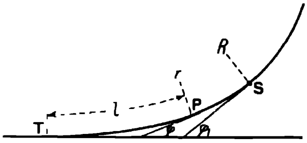

FIG 289

so that $\phi = \frac{l^2}{2RL}$, *the intrinsic equation of the ideal transition spiral.*

For the purpose of setting out this curve by offsets from the tangent at the origin T, the Cartesian co-ordinates, referred to the tangent as the x-axis, would require to be calculated, and may be obtained as follows

$$\frac{dy}{dl} = \sin\phi = \left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - . \qquad\right),$$

but, from the intrinsic equation, $dl = \frac{RLd\phi}{l}$, and, eliminating l,

$$dl = \frac{RLd\phi}{\sqrt{2RL\phi}} = \frac{\sqrt{2RL}\cdot d\phi}{2\sqrt{\phi}};$$

$$dy = \frac{\sqrt{2RL}}{2}\left(\phi^{\frac{1}{2}} - \frac{\phi^{\frac{5}{2}}}{3!} + \frac{\phi^{\frac{9}{2}}}{5!} - . \qquad\right)d\phi,$$

by integrating which, $y = \sqrt{2RL}\left(\frac{\phi^{\frac{3}{2}}}{3} - \frac{\phi^{\frac{7}{2}}}{7\cdot3!} + \frac{\phi^{\frac{11}{2}}}{11\cdot5!} - \qquad\right)$

the constant of integration being zero, since $y = 0$ when $\phi = 0$.

By substitution of $\frac{l^2}{2RL}$ for ϕ, the value of the offset at a distance l along the curve from the origin is given by

$$y = \frac{l^3}{3(2RL)} - \frac{l^7}{7\ 3!(2RL)^3} + \frac{l^{11}}{11\ 5!(2RL)^5} -$$

Similarly, $\frac{dx}{dl} = \cos\phi = \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \qquad\right)$

$$\text{and } x = \sqrt{2RL}\left(\phi^{\frac{1}{2}} - \frac{\phi^{\frac{5}{2}}}{5\ 2!} + \frac{}{9\cdot4!} - \qquad\right)$$

$$\text{or } = l - \frac{l^5}{5\cdot2!(2RL)^2} + \frac{l^9}{9\ 4!(2RL)^4} -$$

The curve therefore does not lend itself to simple expression in Cartesian co-ordinates. The series, however, rapidly converge, since in the practical case ϕ never exceeds a small fraction of a radian. The x co-ordinate is of minor importance, since the offsets y could be employed in conjunction with lengths l along the curve, the setting out being then performed by chords and offsets.

In place of setting out the curve by offsets, use may be made of the method of deflection angles and chords from the starting point T Let d be the deflection angle from the tangent at T to a point l ft. along the curve, then

$$\tan d = \frac{y}{x} = \frac{\dfrac{\phi^{\frac{3}{2}}}{3} - \dfrac{\phi^{\frac{7}{2}}}{7\cdot 3!} + \dfrac{\phi^{\frac{11}{2}}}{11\cdot 5!} -}{\phi^{\frac{1}{2}} - \dfrac{\phi^{\frac{5}{2}}}{5\cdot 2!} + \dfrac{\phi^{\frac{9}{2}}}{9\cdot 4!} -}$$

$$= \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{26\phi^5}{155{,}925} +$$

This resembles the expansion,

$\tan\dfrac{\phi}{3} = \dfrac{\phi}{3} + \dfrac{\phi^3}{81} + \dfrac{2\phi^5}{3{,}645} +$, sufficiently closely for the required values of ϕ to justify the approximation,

$$d = \frac{\phi}{3},$$

or, on substituting $l^2/2RL$ for ϕ,

$$d = \frac{l^2}{6RL} \text{radians}$$

Modifications of the Ideal Transition Curve.—With a view to simplifying the calculation of the quantities required in setting out a transition, various forms of curve, differing within the length required only slightly from the ideal curve, have been proposed, and are in use.

The Cubic Spiral.—In the above expressions for the offset y, the error involved in rejecting the terms after the first is entirely negligible in practice, so that we have

$$y = \frac{l^3}{6RL}, \text{ the equation of the cubic spiral}$$

The approximation made is equivalent to assuming

$$\sin\phi = \phi, \text{ or } \frac{dy}{dl} = \phi,$$

$$\text{but } \phi = \frac{l^2}{2RL},$$

$$\therefore dy = \frac{l^2 dl}{2RL},$$

$$\text{and } y = \frac{l^3}{6RL}, \text{ the constant of integration being zero.}$$

The cubic spiral is well adapted for location by chords and offsets from the tangent. If, however, the method of deflection angles is preferred, the approximation,

$$d = \frac{\phi}{3} = \frac{l^2}{6RL} \text{ radians, is quite valid.}$$

The Cubic Parabola.—If it is desired to make use of both Cartesian co-ordinates, then, in addition to the above approximation for y, the approximation that $x = l$ may be similarly obtained by neglecting the terms in its series after the first The curve is thus modified to $y = \frac{x^3}{6RL}$, the cubic parabola, which is sometimes known as Froude's transition curve

The cubic parabola is inferior to the cubic spiral, as not only are approximations for both x and y introduced, but the error involved in the assumption that $x = l$, or $\cos \phi = 1$, is greater than that made in the approximation for y, since the x series is the less rapidly convergent of the two The curve has, however, been extensively used owing to the ease with which it may be set out by rectangular co-ordinates It is not so well adapted for setting out by angles and chords, since the expression for d,

$$d = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{x^2}{6RL},$$

does not contain l The length of the chord between two points x_1y_1 and x_2y_2 is, however,

$$[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{\frac{1}{2}},$$

or, practically, $(x_2 - x_1) + \frac{(y_2 - y_1)^2}{2(x_2 - x_1)}$.

In the large majority of practical cases the differences between the three curves discussed are very small, and it is generally allowable to regard convenient properties of one as applicable to any of them

Relationship between Transition and Main Curves.—The effect of introducing transition arcs at the ends of a simple curve being to lessen the curvature there, it would appear necessary to increase the curvature of the main arc in order to enable the whole curve to fit the tangents. It is, however, much preferable to preserve the radius proposed for the circular arc and accommodate the transitions by shifting the main curve to a position farther from the intersection point of the tangents. Alternatively, the circular curve may be located in the position it would assume without transitions if the two tangents are shifted outwards from the curve by an amount which will permit of the introduction of the transitions. The location will be considered with reference to the case of shifting the main curve, this being the usual practical case.

In Fig. 290, TC represents the original tangent, and AC′ the shift tangent, the perpendicular distance between them being the shift s The transition curve of length L joins the circular arc, placed between the shift tangents, at S, at which point the offset SU $= Y$. SV is the tangent to both the transition and the circular arc at S, and the angle ϕ_1 which it makes with the tangent TC is evidently equal to the angle AOS Let SW be drawn perpendicular to OA, then

$$s = Y - \text{WA},$$
$$= Y - R \text{ versin } \phi_1,$$
$$= \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R},$$

with sufficient accuracy for any of the curves discussed above.

$$\text{Now } \phi_1 = \frac{L}{2R} = \frac{\text{AS}}{R},$$
$$\text{AS} = \frac{L}{2}$$

But in practical cases the divergence between AS and MS is sufficiently small that we may put MS $=$ AS, so that M is the midpoint of the transition arc

$$\text{Again, the offset MX} = \frac{(\frac{1}{2}L)^3}{6RL} = \frac{L^2}{48R},$$

i e the transition curve bisects the shift

Setting Out.—The setting out of a transition curve, say the cubic spiral, may be performed (*a*) by deflection angles, (*b*) by tangent offsets

By Deflection Angles —1 Locate the tangent point T by measuring out the tangent length CT, which may with sufficient accuracy be taken $= (R+s) \tan \frac{1}{2} I + \frac{1}{2}L$ Alternatively, the position of T may be found by first locating X from the measurement CX $= (R + s) \tan \frac{1}{2} I$, setting out the perpendicular offset XM $= \frac{1}{2}s$, and swinging the distance MT $= \frac{1}{2}L$

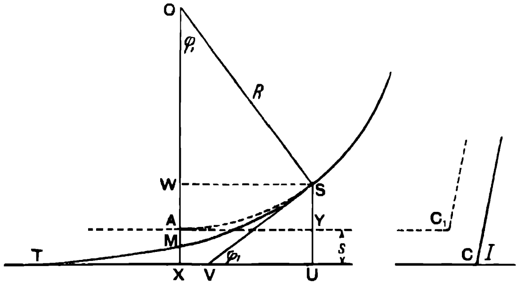

FIG 290

2. Obtain the chainage of T and from it the length of the first chord in the transition. It is desirable to make the standard chord

length C on the transition ½ or ¼ of the usual peg interval, and the deflection angles are obtained from

$$d = \frac{l^2}{6RL} \text{ radians,}$$

$$= \frac{n^2C^2}{6RL},$$ where n = the number, integral or fractional, of chord lengths C from the origin,

$$= \frac{573n^2C^2}{RL} \text{ minutes}$$

Tabulate the deflection angles up to that of S, which $= \frac{1}{3}\phi_1 = \frac{L}{6R}$ radians $= \frac{573L}{R}$ minutes

3 Obtain the chainage of S by adding L to that of T, and tabulate the deflection angles for the circular curve referred to the tangent at S The maximum deflection angle is $(\frac{1}{2}I - \phi_1)$, and the length of the circular arc $= \frac{\pi(I - 2\phi_1)R}{180^\circ}$.

4 Set the theodolite over T, sight along TC, and proceed to lay off the successive deflection angles and peg the curve up to S The position of S may be conveniently checked by measuring the offset $\text{SU} = \frac{L^2}{6R}$

5 Transfer the theodolite to S Since TSV $= \frac{2}{3}\phi_1$, the direction of SV may be obtained from a backsight on T For a right-hand curve this angle should be set off behind zero After sighting T and transiting the telescope, the vernier can be set to the first deflection angle for the circle

6 Set out the circular arc in the usual way up to the end S_1, and obtain a check by measuring the offset to the second tangent at that point

Notes —(1) Continuity of the chainage is sometimes disregarded on the transition curve, no initial sub-chord being used The chainage of S and the position of the first peg on the circular arc are, however, obtained as above.

(2) In place of setting out the circular curve from S, the theodolite may be set up at A and the deflection angles laid out with reference to the shift tangent This method has usually the advantage of affording a longer backsight The necessary changes in the computations will be easily deduced

By Tangent Offsets.—In this method, T is located and its chainage obtained as before Offsets are computed from $y = \frac{l^3}{6RL} = \frac{(nC)^3}{6RL}$ for the initial sub-chord and the succeeding uniform chords, each peg being located by swinging the chord length from the preceding peg until the required offset is obtained. The chainage of S is determined as before, and the setting out of the circular arc proceeds by deflection angles from S or A

Note.—In the case of the cubic parabola, convenient abscissæ are measured out along TC

Transitions between Branches of Compound and Reverse Curves.—The length L of transition arc required between the branches of a compound curve is that necessary to deal with the difference of cant or with the difference of radial acceleration, according to the principle employed for determining length The formulæ of pages 347 to 351 are therefore made applicable by putting $\frac{R_1R_2}{(R_2-R_1)}$ for R In this case the shift at the junction between the branches is the radial length of gap between the two shifted arcs. Its value is obtained by means of the same substitution for R, giving $s=\frac{L^2(R_2-R_1)}{24R_1R_2}$. As before, the transition bisects, and is bisected by, the shift. The values of deflection angles are obtained by similarly modifying the previous formulæ, but in this case the length of the transition curve is frequently so short that it is commonly sufficient to peg the mid-point and ends of the transition and, if necessary, locate one additional peg on each half by offsets

In the case of a transition between the two branches of a reverse curve, the length L is the sum of the lengths l_1 and l_2 necessary for the individual radii R_1 and R_2 The gap between the two shifted arcs at the original point of reverse is $\frac{L^2(R_1+R_2)}{24R_1R_2}$ This original inflection point bisects the transition, but, unless the radii are equal, it does not bisect the gap, the individual shifts of the circular arcs from the common tangent being given by $\frac{l_1{}^2+3l_2{}^2}{24R_1}$ for the curve of radius R_1, and by $\frac{3l_1{}^2+l_2{}^2}{24R_2}$ for that of radius R_2 *

The most convenient method of setting out the transition is to locate the points at which the transition joins the shifted curves by measuring out $\frac{1}{2}L$ on either side of the original point of reverse Points on the transition are then obtained by laying off deflection angles or by cubic offsets from the shifted arcs. The point of inflection of the transition coincides with that of the original curve only when the radii are equal.

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* For derivation of these formulæ see Perrott and Badger, *The Practice of Railway Surveying and Permanent Way Work*

MISCELLANEOUS OPERATIONS IN SETTING OUT

Setting Slope Stakes.—The operation of slope staking consists in locating and pegging points on the lines in which proposed earth slopes intersect the original ground surface

In the case whether cross sections have been plotted and the new work is shown thereon, all that is necessary is to scale the horizontal distances representing the side widths or " half-breadths " (page 291) and to set out these measurements on the ground Slope stakes can, however, be located without the use of plotted cross sections by a trial and error process, which is best followed from an example

In Fig 291, let the formation level at a certain point on an embankment, as derived from the profile, be 123·60, the top width being 30 ft , and the side slopes $1\frac{1}{2}$ horizontal to 1 vertical. Let the instrument height of a level commanding the ground be, say, 121·3 The difference of 2·3 ft between instrument height and formation level represents a quantity, called the grade staff reading, which in this case falls to be added to readings of the staff when held on ground points in order to give the depths below formation of those points If, for example, the centre height of the earthwork is not known, by reason of a local deviation or otherwise, and the staff is held on the ground at the line peg C, giving a staff reading of, say, 5·6 ft , the centre height of the embankment will be 7 9 ft

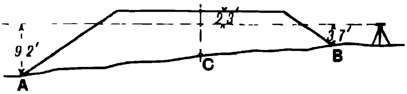

FIG 291

To locate the slope peg A, the staffman must estimate where the foot of the slope will come, and he holds the staff there, measuring also its distance from the centre peg C Let us suppose the staff reading to be 8·3 ft This represents a depth from formation level of $8{\cdot}3+2{\cdot}3=10{\cdot}6$ ft , and, if the point selected is really on the toe of the slope, the corresponding side width ought to be $1\frac{1}{2}\times 10{\cdot}6+15 =30{\cdot}9$ ft. If the measured distance happens to have this value, the staffman has estimated the position correctly, but it will usually require a second or third trial before the correct point is found Thus, if the staff reading of 8 3 ft was obtained at a distance of 27 ft from C, the surveyor, on finding that the computed side width of 30·9 ft does not correspond with the measurement, would direct the staff holder to proceed farther from the centre. The next results might be 9·2 ft on the staff and 32 ft on the tape, which would be practically correct. In the same manner, the results of the final trial in setting the right-hand slope stake B might be 3·7 ft on the staff at 24 ft from C.

This method is very convenient for taking and recording cross sections, and is invariably adopted by American railway engineers.

The results in the case of the three level section of our example are conveniently written,

$$\frac{-11{\cdot}5}{32{\cdot}0} \qquad 7{\cdot}9 \qquad \frac{-6{\cdot}0}{24{\cdot}0},$$

the upper figures in the case of the side points representing the vertical distances between formation and ground levels, reckoned positive for cuttings and negative for embankments, and the lower figures their distances from the centre Additional points between the centre peg and the slope stakes may be recorded by observing the staff reading and distance from the centre and applying the grade staff reading positively or negatively to the former From the side widths and centre and side heights, the area may be computed by the formulæ on pages 291 and 292

Setting Out Vertical Curves.—At the junction of two railway gradients it is desirable to introduce a vertical curve to round off the angle and give a gradual change from the one gradient to the other For convenience in setting out, the parabola is generally employed for this purpose

The length of curve required to afford a suitable rate of change should evidently depend upon the algebraic difference between the gradients, but the most suitable length for any particular case is largely a matter of opinion The practice recommended by the American Railway Engineering and Maintenance of Way Association, and which will serve as a good guide, is that for first-class railways the gradient should not change at a greater rate than 0·1 ft per 100 ft station on summits and 0·05 ft per station in sags, while for second-class lines the rate of change should not be more than twice as much According to this rule, the curve joining a rising gradient of 1 in 100, or 1%, with a falling gradient of 0·5% at a summit on a first-class road would require to have a length of $\frac{1+0{\cdot}5}{0{\cdot}1} \times 100$ ft $=1{,}500$ ft The length of curve need not, however, exceed that of the longest train likely to pass over it

Vertical curves are also required in road work In this case they are of special importance at summits, and should be of such length as will afford drivers a suitable limit of vision

Whatever length is adopted, half of it is placed on either side of the apex at which the straight gradients meet. To enable the curve to be set out, its elevation at each chainage peg is obtained from the corresponding elevation of the tangent gradients, given on the working profile, by computing and applying the vertical distance between the tangent and the curve

In Fig. 292, let two gradients meet at C, and let it be required to join them with a parabolic vertical curve n chains long. Pegs A and B, situated $\frac{1}{2}n$ chains from C, mark the beginning and end of the curve ADB. By taking the average of the known elevations of A and B, obtain that of E, the mid-point of AB, which is situated on the vertical through C The elevation of C being given on the profile, the dimension CE is now known, and, since the parabola bisects CE at D, the elevation of D is obtained. Now, from the property of the parabola that offsets from a tangent are proportional to the squares of distances along the tangent, the elevations of points on the curve may be computed. Thus, let $\frac{1}{2}n = 5$ chains, the chainage of C being integral as is usual, and let it be supposed that CD, $= \frac{1}{2}$ CE, has been found to be 0·70 ft , then the offsets in order from A, as shown dotted, are respectively $\frac{1}{25}$, $\frac{4}{25}$, $\frac{9}{25}$, and $\frac{16}{25}$ of 0·70 ft , the corresponding offsets from CB having the same values.

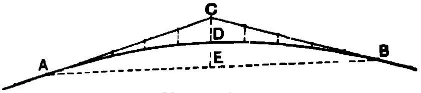

Fig 292.

Setting Out Tunnels.—The setting out of a tunnel is an operation demanding a high order of precision throughout The usual type of engineer's theodolite is not always sufficient, particularly with regard to the quality of the telescope and general stability, and a 6-in. or larger instrument is sometimes necessary

The simplest case occurs when the two ends of the tunnel are intervisible or visible from an intermediate point, so that its direction can be laid out on the ground without difficulty and continued beyond the entrances Permanent ranging marks are constructed clear of the work, and from these the line may be projected into the tunnel at each end If the tunnel is also to be driven from shafts, these must be carefully aligned on the surface, and, when the shaft is sunk, the alignment at the bottom is commonly obtained by suspending two plumb lines from a frame above the shaft Each consists of copper, brass, or steel wire, and carries a rather heavy weight immersed in a pail of water for steadiness The plumb lines should be capable of fine adjustment into line, and the upper end of each should therefore be mounted in a manner permitting lateral movement by means of a screw The distance between the wires forms a short base which is prolonged underground by setting the theodolite in line with them, numerous face right and face left observations being taken to reduce instrumental and observational errors to a minimum A lamp, screened by having a sheet of tracing cloth pasted over the glass, is placed behind the far wire when the instrument is being aligned, and a similar lamp is used behind any mark being set out When the shaft is not on the centre line of the tunnel, the plumb lines are usually placed so that the line through them is roughly normal to the centre line This line is prolonged on the surface to meet the centre line, and the intersection angle is measured, as well as the chainage of the intersection point and its distance from the plumb lines. By setting out the same dimensions underground, the corresponding point on the centre line and the direction of the tunnel are obtained, Similar care is required in the levelling. The relative levels of formation at the two ends of the tunnel must be carefully determined by running two or more lines of levels between them, the precautions against error following those adopted in precise levelling (Vol. II, Chap. V). The formation levels at the various shafts are computed, and are established underground by means of vertical steel tape measurements from level marks at the surface

The more usual case occurs when the ends of the tunnel are not intervisible, and their relative positions must then be carefully determined. This may sometimes be accomplished by traversing, but in most cases it is necessary to connect the ends by triangulation extended from a carefully measured base line (Vol II, Chap. III) The length of the tunnel is ascertained, and intermediate points are located, by co-ordinate calculation from the triangulation, and the centre line, whether straight or curved, may then be set out on the surface Instrument stations are established, and by observations from them the plumbing wires at the shafts are aligned as before In the case of curves, the centre line is best located underground by offsets from a series of chords

The procedure in setting out tunnels naturally varies according to local conditions Short of actual experience, an adequate knowledge of the methods employed can be derived only by a careful study of the descriptions given in the publications of engineering societies and in the technical press

Setting Out Viaduct Piers on a Curve.—In setting out a river viaduct, the location of points on the piers and abutments has generally to be performed several times at various stages of the work The first operation during construction may consist in the location of staging surrounding each pier, and, according to the class of foundation adopted, cofferdams, cylinders, or caissons will subsequently have to be aligned On completion of the foundations, the outlines of the piers and abutments are set out upon them, and it may at this stage be practicable to employ instrument stations on the piers. The methods will be sufficiently indicated by describing the location of the centre lines of the piers of a curved viaduct by observations from the river banks only.

Two cases may occur . (1) The tangent points may fall on the banks behind the abutments, so that the whole viaduct is on the curve , (2) The tangent points may be situated in the river, so that only a part of the viaduct is on the curve.

Case 1.—Tangent Points Accessible.—The preliminary data will include · (*a*) the radius of the curve ; (*b*) the chainage to the face of the first abutment ; (*c*) the angle of skew of that abutment relatively to the tangent to the curve at the point at which the latter cuts the face ; (*d*) the angles between adjacent piers or piers and abutment faces, if they are not parallel , (*e*) the lengths of the square or the skew spans , (*f*) the dimensions of the piers and abutments. The situation fixed for the first abutment governs the position of the other lines to be located, and calculation may be made of the further items: (*g*) the chainage to the centre of each pier and to the second abutment face; (*h*) the angles of skew made by the centre lines of the piers and by the face of the second abutment with the curve These quantities may be derived as follows

In Fig 293, NPQ and STU represent the parallel centre lines of two piers or an abutment face and a pier situated on a curve of radius R

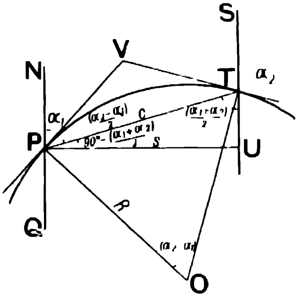

FIG 293.

Let α_1 and α_2 = the angles of skew made by these lines with the tangents at P and T respectively,

s = the square span PU,

c = the chord length PT = the skew span

Given R, s, α_1, and the chainage of P, it is required to calculate α_2 and the chainage of T

The angle subtended at the centre O = the change in direction of the curve from P to T = $(\alpha_2-\alpha_1)$,

$$\therefore\ c = 2R \sin\frac{(\alpha_2-\alpha_1)}{2}$$

But PTU $= \frac{(\alpha_1+\alpha_2)}{2}$,

$$\therefore\ c = s \operatorname{cosec}\frac{(\alpha_1+\alpha_2)}{2}$$

Equating these values of c, we have

$$2R \sin\frac{(\alpha_2-\alpha_1)}{2} \sin\frac{(\alpha_1+\alpha_2)}{2} = s,$$

i e $R(\cos\alpha_1 - \cos\alpha_2) = s$,
whence α_2

Chainage of T = chainage of P + arc PT,

$$= \text{chainage of P} + \frac{\pi(\alpha_2-\alpha_1)R}{180°}.$$

If NPQ is not parallel to STU, there is no square span, and c is substituted for s in the data To α_2, as calculated on the assumption of parallelism, it is then necessary to apply the known angle between NPQ and STU.

Setting Out—1 C being inaccessible, locate the tangent points A and B (Fig. 294) by the method of Case 3, page 331, the length of a transversal EF being obtained by measuring out a base line such as ED, observing two or all angles of the triangle EDF, and solving for EF.

2 A and B, being found accessible, are to be used as instrument stations from which the required points can be located by simultaneous observations In setting out a point on the centre line, such as G, the deflection angles CAG and CBG are readily computed from the lengths of the arcs AG and GB obtained as above

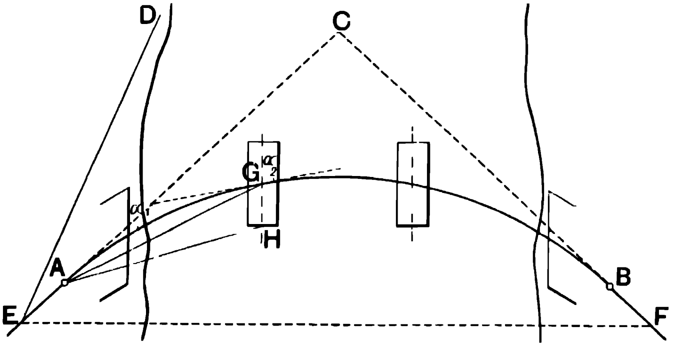

FIG 294.

3 The same principle applies to the setting out of points such as H, the angles CAH and CBH being calculated Thus, CAH = CAG + GAH, of which CAG is known, while triangle AGH can be solved for GAH, since GH will be known, AG is the chord subtending a known arc, and AGH = (α_2—CAG) Any point can be located in this manner provided its position is specified relatively to some point of known chainage on the centre line

Note —If A and B, although on shore, are unsuitable for observing from, it will be necessary to establish more convenient instrument stations of known chainage If these are on the curve, the above methods apply, each deflection angle being calculated and measured from the tangent through the instrument station If the stations have to be selected on the straights, the methods of the following case must be used

Case 2.—Tangent Points in River.—In this case the only difference in the data will be that the angle of skew of the first abutment is measured from the first straight. In Fig 295, NPQ and STU represent the parallel centre lines of two piers or an abutment face and a pier, such that the tangent point A falls between them With the previous notation, being given R, s, α_1, and the chainage of P and A, let it be required to calculate α_2 and the chainage of T

Through A draw N_1AQ_1 parallel to NPQ; then $N_1AV = \alpha_1$, and the methods of the previous case apply to the lines N_1AQ_1 and STU, the perpendicular

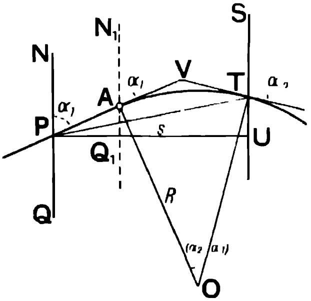

FIG. 295.

distance between which is $Q_1U = (PU - PQ_1) = (s - \text{PA} \sin \alpha_1)$. Corresponding to the previous results, we therefore have

$$R\,(\cos \alpha_1 - \cos \alpha_2) = (s - \text{PA} \sin \alpha_1),$$

$$\text{and chainage of T} = \text{chainage of P} + \text{PA} + \frac{\pi(\alpha_2 - \alpha_1)R}{180^\circ}.$$

Setting Out.—1 Having solved triangle CEF as before, and found that A and B are inaccessible (Fig 296), select two instrument stations, such as E and F, from which to set out simultaneous deflection angles The chainage of E being known, that of F is readily found since EA, arc AB, and BF are known

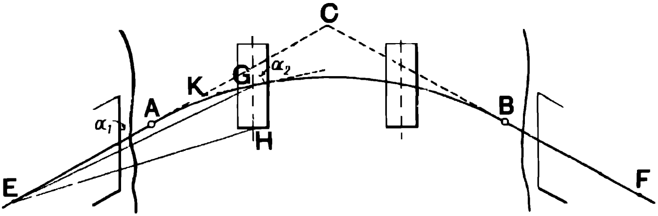

FIG 296

2 Calculate the deflection angles for a centre line point such as G, these angles are CEG and CFG KG being the tangent to the curve at G, CEG is obtained by solution of triangle EKG, since EKG = $(180^\circ - \iota)$, where ι is the central angle subtended by arc AG, KG $= R \tan \frac{1}{2}\iota$, and EK $= (\text{EA} + R \tan \frac{1}{2}\iota)$ Similarly for CFG.

3 The value of angle CEH required in locating H is obtained by solution of triangle EGH, in which EG is obtained from triangle EKG, GH is known, and EGH $= (\alpha_2 - \text{KGE})$ Similarly for CFH

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EXAMPLES

1. The chainages at the beginning and end of a 2° curve are 320 + 34 and 339 + 75 respectively. Find the intersection or central angle.

2. If in the above case the intersection angle is increased by 5° by turning the second tangent about the vertex, find the new chainages of the tangent points of the 2° curve.

3. To ascertain the radius of an existing curve, two 100 ft. chords, AB and BC, are measured out, and the versed sine of chord AC is found to be 4 ft. 9 in. Find the radius.

4. Show that the tangent offset at the end of the Nth chord on a $D°$ curve is approximately given by $\frac{7}{8}N^2D$ ft.

5. A certain railway curve is to have a radius of 20 chains, the tangents intersect at chainage 320 + 30, and the angle of deflection is 37° 50′, find the following quantities: (*a*) tangent distances, (*b*) apex distance, or distance of middle point of the curve from the intersection, (*c*) length of curve, (*d*) chainage of the beginning, end, and apex of the curve. (Inst. C. E., 1907.)

6. Some particulars of part of a railway survey are given in the annexed table:—

Point or Apex	Apex Angle	Radius or degree of Curve	Distance from previous Apex
A_0			
A_1	163° 18′	R = 1200 ft.	4872.4
A_2	165° 36′	4° Curve	6319.5
A_3			3824.7

The line starts at A_0, and all distances are in feet. The distances are from A_0 to A_1, and so on.

You are required to find the whole length, along the centre line of the railway, from A_0 to A_3, and the chainage of each tangent-point. (Univ. of Lond., 1916.)

7. From a tangent AB, having a bearing of 133° 42′, a left-hand curve of 3,000 ft. radius is to be set out in 100 ft. chords to join a tangent BC, the bearing of which is 126° 18′. The chainage of the point of commencement of the curve is 19,638 ft. Give a list of the bearings to the pegs from the initial tangent point, expressing them to the nearest ½ min.

8 The bearing of the first tangent to a railway curve of 40 chains radius is 72° 19′ What length of curve is required if the second tangent is to have a bearing of 88° 55′? If the chainage of the first tangent point is 236 24 chains, calculate the first three deflection angles to the nearest 20″, for chords of one chain.

9 A curve of 30 chains radius has been pegged out to connect two railway tangents having the intersection angle $I - 15°\ 26'$, and the chainage of the initial tangent point has been found to be 384 17 chains On further examination of the ground it is decided to alter the radius to 45 chains Calculate the chainage of the new initial and final tangent points, and the distance between the new and original curves at their mid-points (T C D, 1928)

10 The lengths and bearings of three successive courses in the final location survey for a railway are

Line	Length in chains	Bearing
PQ	63 22	122° 39′
QR	39 87	149° 57′
RS	46 25	131° 21′

It is proposed to connect *PQ* and *QR* by a curve of 80 chains radius, and *QR* and *RS* by one of 60 chains radius If the chainage of the tangent point on *PQ* is found to be 642 35, find that of the other tangent points

11 The positions of eight columns are to be set out on a railway station platform, the columns to be 40 ft apart centre to centre The centre points of columns 1 to 5 are to lie upon a circular arc of 1,905 ft radius, and the tangent to this arc through column No 1 has been established The remaining columns will lie on a straight line tangential to the arc at column No 5 Obtain to the nearest 10″ the deflection angles required in locating the columns on the curve from a theodolite placed at the centre point of the first column

12 Two railway tangents have been located on the ground, and the difference between their bearings has been found to be 21° 48′ They are to be connected by a circular arc, and it is desired that the tangent lengths should not exceed 500 feet in length Calculate the degree of curve, to the nearest quarter-degree, which will be most suitable, and find the corresponding tangent lengths and the length of the curve

13 Part of the traverse survey on a railway location is as follows .

Line	Bearing	Length in feet
PQ	93° 14′	1,967
QR	75° 44′	1,608
RS	101° 8′	2,710

Tangents are laid along these lines in the course of the final location, and it is decided to connect *PQ* and *QR* by a $1\frac{1}{2}$ degree curve What is the maximum radius which can be used for the curve joining *QR* and *RS* so that there may be not less than 100 ft of straight on *QR* between the end of one curve and the beginning of the other? (T C D, 1931)

14. From a line AB, 820 ft long, perpendiculars AD and BC are laid out of lengths 122 ft and 264 ft. respectively. A pond is to be constructed in a public park, and will have straight sides lying along AB and DC, the ends being formed of circular arcs to which AB, DC, and the end perpendiculars are tangential Calculate the radii of the two ends and the perimeter of the pond

15. A 10-chain curve AB runs through a number of obstacles, for the clearing of which points C and D are to be established on the curve by running out chords AC and AD at 10° and 15° to the first tangent. Calculate the lengths of the chords.

16. In laying out a circular railway curve it is found that the tangents intersect at chainage 257 + 34 and that the deflection angle is 27° Find the radius of the curve which will pass through a point 40 ft from the intersection and equally distant from the tangents, and write down the chainage at the beginning of the curve (Inst C E , 1906)

17 The internal angle ABC between two tangents is 120°, and they are to be connected by a circular curve Owing to the presence of buildings it is found necessary that the curve should pass through a point D, the length of the perpendicular DE on to the tangent AB being 24 ft , the distance BE being 500 ft Find the radius and tangent distance of a suitable curve If the chainage at the intersection point is 150 chains 20 links, find the chainage at the end of the curve (R T C , 1913)

18 In the course of a railway survey the following bearings were observed AB, 77° 18′ ; BC, 92° 54′ , and CD, 105° 12′ BC is 12 08 chains long AB and CD are to be connected by a circular arc, right-handed from AB, which is required to pass through a point E on BC situated at 4 90 chains from B Determine the required radius of curve

19 A curve *AB*, of 60 chains radius, is to be set out by chain chords to join two tangents *AC* and *CB*, the intersection angle of which is 19° 46′ The chainage of the initial tangent point *A* is found to be 521 28 chains The part of the curve adjacent to *A* passes through a dense strip of wood, and will not be pegged out in the first instance, but a line *AD* can be laid off from the tangent point, making angle *CAD* = 3° 54′. Calculate (*a*) the length of *AD* so that *D* may be on the curve, (*b*) the chainage of *D*, (*c*) the length of curve to be set out from *D* to *B* (T C D , 1928)

20. A light railway is to be carried round the shoulder of a hill, and its centre line is to be tangent to each of the three lines *AB*, *BC*, and *CD* as follows

Line	Bearing	Length
AB	North 30° East	—
BC	East	600 ft
CD	South	—

Calculate the radius of the curve and the lengths required for setting out the tangent points (Univ. of Lond , 1913)

21. A curve of 60 chains radius has been pegged out to connect two railway tangents, the difference between the bearings of which is 18° 0′. It is found necessary to alter the alignment by introducing a straight length of 6 chains midway between the existing tangents and connecting it to them by two arcs of equal radius If the original tangent points are to remain in the same position, compute the radius of the required curves and the maximum distance between the original and amended centre lines.

22. *AC* and *BC* are two railway straights which are to be connected by a 2° curve The intersection point *C* is found to be inaccessible, and a line *AB* is run out between convenient points on the straights. The length of *AB* is found to be 900 feet, and the observed values of the angles *BAC* and *ABC* are 9° 20′ and 8° 30′ respectively Calculate the length of the curve and the positions of its tangent points relatively to *A* and *B* respectively.

23. The intersection point *C* of two tangents *AC* and *CB* of a railway which is being set out is found to be inaccessible, and a traverse is run from a point *D* in *AC* to a point *H* in *CB* with the following results ·

Line	Bearing	Length in feet	Latitude	Departure.
DA	344° 38′			
DE	123° 12′	186	- 101 8	+ 155 6
EF	159° 30′	427	- 400 0	+ 149 5
FG	100° 48′	144	- 27·0	+ 141 4
GH	186° 0′	251	249 6	- 26 2
HB	143° 34′			

If the tangents are to be joined by a curve of 2,640 feet radius, calculate the distance of the initial tangent point from *D* (T C D , 1929)

24 A 4° curve is to connect two straights, the bearings of which are 20° and 80° respectively. The intersection point is inaccessible, and from a point A of chainage 13,350 ft on the first straight a traverse is run to D on the second straight as follows

Line	Bearing	Length
AB	60°	468
BC	110°	360
CD	30°	950

Find the chainage of the first tangent point and the distance from D to the second (R T C., 1920)

25. C is the inaccessible intersection point of two railway straights AC and CB which are to be joined by a circular curve. Points D and E are selected in AC and CB respectively, and the following bearings are observed AD, 173° 42′, DE, 187° 24′ EB, 198° 18′ The distance DE is measured and found to be 960 feet If on examination of the ground it is found desirable to locate the initial tangent point of the curve at D, obtain the necessary radius in feet and the distance from E to the second tangent point

26 A right-hand curve of 60 chains radius, of which A and B are the tangent points, has been pegged out to connect the tangents AC and CB, the intersection angle of which is 19° 18′ It is desired to alter the alignment of the tangent CB to DE, D being on CB and 1 57 chains from B towards C, and angle CDE on the side of CB remote from A being 177° 16′. If the original radius of the curve is preserved, find the distances of the new tangent points from A and D respectively.

27 From a given point A it is required to set out a tangent AD to an existing 40-chain curve BDC A straight line ABC is run to intersect the curve, making AB = 10 20 chains and BC = 4 32 chains Calculate the angle BAD and the length of arc BD.

28. ABC is a railway line of which AB is straight and BC is a 20-chain curve towards the left 2 chains to the right of AB and parallel to it is a straight railway DE Find at what distance from B a 40-chain curve must leave BC to join DE

29 AC and CB are two railway tangents giving an intersection angle of 12° 26′. They are joined by a curve AB of 60 chains radius, the chainage of the initial tangent point A being 346 27. From a point D in AC situated at 3 04 chains from A it is desired to set out a tangent DE to the curve. Calculate (*a*) the angle CDE, (*b*) the chainage of E, (*c*) the distance from B at which DE produced meets BC.

30 It is proposed to lay down a right-hand curve of 792 ft radius at a dock, and the initial and terminal tangent points, A and B, will occupy the positions given by the following total co-ordinates :

	Total Latitude in feet.	Total Departure in feet
A	+ 3,479	- 1,096
B	+ 2,963	- 1,208

Calculate the shortest distance between the curve centre line and a point C the total co-ordinates of which are + 3,204 ft. and − 1,129 ft. in latitude and departure respectively.

31. The initial tangent point of a 40-chain railway curve is situated in water, but its chainage is found to be 1,936.44 chains. It is desired to set out the peg at 1,941 chains by sighting from station 1,934 on the straight. Calculate, to the nearest 20″, the angle to be set off from the tangent and the distance to be measured out to locate the peg.

32. In setting out a compound curve of radii 20 and 40 chains, the intersection angle of the arc of smaller radius is found to be 90°, and that of the other 60°. The common tangent is then chained and found to have a length of 43 chains. Find by how much and in what direction it will be necessary to shift this tangent parallel to itself, so that the curve may be accurately set out. Explain how this subsequent setting out is performed. (R.T.C., 1913.)

33. A compound curve is to consist of an arc of 30 chains followed by one of 40 chains radius, and is to connect two straights which yield an intersection angle $I = 84°\ 32'$. At the intersection point the chainage, if continued along the first tangent, would be 77.64, and the starting point of the curve is selected at chainage 45.14. Calculate the chainage at the point of junction of the two branches and at the end of the curve.

Describe briefly the steps involved in setting out the curve. (R.T.C., 1915.)

34. A railway siding is to be curved through a right angle. In order to avoid buildings the curve is to be compound, the radii of the two branches being 8 chains and 12 chains. The distance from the intersection point of the end straights to the tangent point at which the arc of 8 chains radius leaves the straight is to be 10.08 chains. Obtain the second tangent length, or distance from the intersection point to the other end of the curve, and the length of the whole curve. (T.C.D., 1927.)

35. A compound railway curve ABC is to have the radius of the arc AB 60 chains and that of BC 40 chains. The intersection point V of the end straights is located, and the intersection angle is observed to be 35° 6′. If the arc AB is to have a length of 20 chains, calculate the tangent distances VA and VC. (T.C.D., 1930.)

36. A reverse curve having the two branches of common radius is to connect two parallel straights 30 ft. apart. If the distance between the tangent point on one straight and that on the other is 240 ft., find the necessary radius.

37. The first branch of a reverse curve has a radius of 8 chains. Find the second radius so that the curve may connect parallel tangents 84 links apart, the distance between tangent points to be 5 chains. Calculate also the lengths of the branches.

38. Two parallel railway straights, 27 ft. apart centre to centre, are to be connected by a reverse curve the radius of each branch of which will be 10 chains. You are given the position of one of the tangent points. Describe in detail how you would set out the curve either by offsets from the line joining the tangent points or by deflection angles to points ½ chain apart on the curve. Give all the necessary calculations. (T.C.D., 1926.)

39. A rule which is sometimes used for determining the required superelevation of the outer rail on railway curves on standard gauge is that the superelevation should equal the offset from the centre of a chord of length equal to the distance travelled in one second. Compare the superelevation given by this rule with the theoretical value derived from mechanical principles.

40 Calculate (*a*) the super-elevation required on the outer rail of a curve of 30 chains radius for a speed of 45 miles per hour, (*b*) the length of transition curve necessary so that the rate of change of radial acceleration may be 1 ft per sec ^{3}, (*c*) the amount of shift required for this transition

41 A railway curve of 50 chains radius and about half a mile long is to be set out to connect two tangents The maximum speed on this part of the railway will be 60 miles per hour, and transition curves are to be introduced at each end of the curve Find a suitable length for the transition curves, and calculate the necessary " shift " of the circular arc

Describe how you would set out the first transition and locate the initial pegs on the circular curve (T C D , 1926)

42 Explain the utility of introducing a vertical curve at the junction of two railway gradients

The levels of a number of successive points 100 ft apart, as shown on the longitudinal section, are .

236 42, 235·62, 234 82, 234·02, 235·13, 236 24, 237 35

Calculate the levels of the pegs which should be set out to define a vertical curve 600 ft long The curve may be either a parabolic or a circular arc. (T C D , 1927)

CHAPTER IX

TACHEOMETRICAL SURVEYING

TACHEOMETRY or, as it is otherwise termed, Tachymetry or Telemetry, is a branch of angular surveying in which both the horizontal and vertical positions of points are determined from the instrumental observations, the labour of chaining being entirely eliminated The usual instrument employed is called a Tacheometer, and is nothing more than a transit theodolite adapted for distance measuring by the provision of special fittings

Scope.—The fundamental object of tacheometry is the preparation of a contoured plan For this purpose it possesses the merit that the field work can be executed with considerable rapidity, more especially in rough country where ordinary levelling is tedious and chaining is both slow and inaccurate This advantage has led to its wide adoption by engineers in location surveys for lines of communication, reservoirs, etc Tacheometry is, however, sometimes applied in small surveys in which elevations are not determined.

Systems.—The underlying principle common to different systems of tacheometry is that the horizontal distance between an instrument station, A, and a point, B, as well as the elevation of B relatively to the instrument, can be deduced from (1) the angle at A subtended by a known short distance at B, and (2) the vertical angle from A to B The various tacheometric methods available employ the principle in different ways, and differ from each other in methods of observation and reduction, but may be classified under two heads .

(*a*) The Stadia System, in which the necessary observations to the point are secured with one pointing of the telescope.

(*b*) The Tangential System, in which two pointings are required

The general conduct of the field work does not differ materially in the two methods, and their principles and the instruments used will first be considered.

THE STADIA SYSTEM

This is the more extensively used system of tacheometry, particularly for detailed work such as is required in location surveys. The principle appears to have been discovered in 1770 by James Watt, who constructed an instrument, and used it in Scotland in 1771.

Principle.—Let it be supposed that the theodolite employed differs from the ordinary transit only in having the diaphragm fitted with two additional horizontal hairs, called stadia* hairs, one on each side of the centre hair. The only other article of equipment required is a graduated staff or stadia rod, which may be an ordinary levelling staff, and which is held on the points to be located with respect to the instrument station

On viewing the staff through the telescope, the stadia hairs are seen to intercept or subtend a certain length, which will be greater the farther off the staff is held, and from the observed value of the intercept the distance to the staff station is deducible This constitutes the *Fixed Hair Method* The procedure may be modified in that, instead of the intercept being the variable, a constant length of staff between two targets may be used, provided the stadia hairs are adjustable so that they can be set to subtend it. Provision is made in this case for the measurement of the variable interval between the hairs, from which quantity the required horizontal distance is computed. This system is distinguished as the *Movable Hair Method* The former is much the more widely adopted method of stadia surveying, but the latter can be usefully applied to surveys involving sights of greater length than is consistent with accurate reading of a graduated staff.

Fixed Hair Method. Distance and Elevation Formulæ.—Reduction formulæ for the horizontal distance and elevation of a staff station with respect to the instrument will be deduced first for the case where the staff can be read with the telescope horizontal. In the general case, the telescope is inclined, and the data include the vertical angle

(1) *Horizontal Sights*—In Fig 297, O is the optical centre of the object glass of an external focussing telescope, a, b, c represent the three horizontal hairs, and A, B, C, the points on the staff which appear cut by the hairs, so that ab is the length of the image of AB

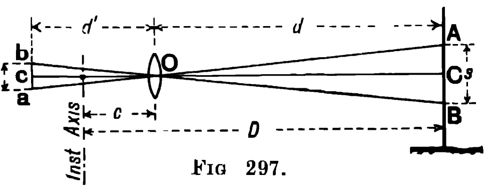

FIG 297.

Let $f =$ focal length of object glass,
$i =$ stadia hair interval, ab,
$s =$ staff intercept, AB,
$d =$ horizontal distance from O to the staff,
$d' =$,, ,, ,, to the plane of the hairs,
$c =$,, ,, ,, to the vertical axis of the instrument,
$D =$ horizontal distance from the axis to the staff.

---

* The term *stadia* should strictly be confined to the staff or stadia rod, but is loosely used in referring to the hairs, the entire instrument, and the method generally.

The rays AOa and BOb passing through O are straight lines, so that triangles AOB and aOb are similar, whence

$$\frac{d}{d'}=\frac{s}{i}.$$

But d and d' are conjugate focal distances (page 11),

$$\frac{1}{f}=\frac{1}{d'}+\frac{1}{d},$$

or $d=\frac{fd}{d'}+f$, on multiplying throughout by fd

Substituting $\frac{s}{i}$ for $\frac{d}{d'}$, we have

$d=\frac{fs}{i}+f$, and, on adding c to each side,

$$D=\frac{fs}{i}+(f+c)$$

The intercept s is observed as the difference of the stadia hair readings, and, to evaluate D, the quantities, $\frac{f}{i}$ and $(f+c)$, must be known for the particular instrument used The elevation of the staff station is obtained in this case exactly as in ordinary levelling by observation of the reading C of the centre hair

Note —The reader may deduce the distance formula by similar treatment with the other easily drawn rays, *viz* those passing through either the interior or exterior principal focus of the object glass

(2) *Inclined Sights* —In this case the staff may be held either vertically or normal to the line of sight, the former method being generally preferred.

(a) *Staff Vertical* (Fig 298) —Let θ be the angle of elevation or depression of the line of sight from the horizontal The inclined distance D from the trunnion axis G to the point C on the staff could be obtained directly from the previous formula, were it not that the observed intercept AB is not, as before, normal to OC. Let an intercept A′B′ be drawn through C perpendicular to OC Since ACA′ $=\theta$, and AA′C is practically 90°, A′B′ may be expressed as $s\cos\theta$ with negligible error, whence

$$D=\frac{fs}{i}\cos\theta+(f+c),$$

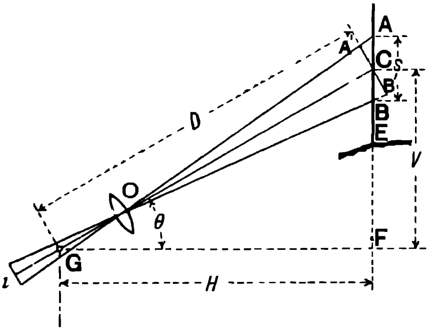

FIG 298

and, since the required horizontal distance $H = D\cos\theta$,

$$H = \frac{fs}{i}\cos^2\theta + (f+c)\cos\theta$$

Let the difference of level FC between the telescope axis and the point C on the staff be denoted by V. The value of V can be obtained from $H\tan\theta$ or $D\sin\theta$, so that

$$V = \frac{fs}{i}\cos\theta\sin\theta + (f+c)\sin\theta,$$

$$\text{or} = \frac{fs}{i}\tfrac{1}{2}\sin 2\theta + (f+c)\sin\theta$$

Denoting CE, the reading of the centre hair, by h, the difference of level between G and E for an angle of elevation is given by

$$\text{FE} = V - h,$$

and, if the elevation of the trunnion axis above datum is expressed by H I, Reduced level of E = H I $+V-h$

In the case of a depressed sight, FE $= V+h$, so that

Reduced level of E = H I $-V-h$

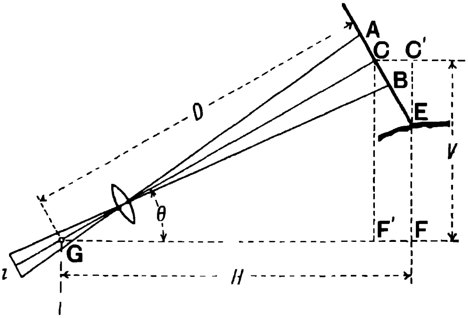

FIG. 299.

(*b*) *Staff Normal* (Fig. 299)

In this case, $D = \frac{fs}{i} + (f+c)$,

but $H = D\cos\theta + \text{CC}' = D\cos\theta + h\sin\theta$,

$$H = \frac{fs}{i}\cos\theta + (f+c)\cos\theta + h\sin\theta,$$

the ($h\sin\theta$) term being subtractive when θ is a depression, since the staff then leans away from the instrument.

The vertical component $V = \text{F}'\text{C} = D\sin\theta$, whence

$$V = \frac{fs}{i}\sin\theta + (f+c)\sin\theta.$$

For an angle of elevation, FE $= V - h\cos\theta$, and

Reduced level of E = H.I. $+V-h\cos\theta$

For an angle of depression, FE $= V + h\cos\theta$ and

Reduced level of E = H.I $-V-h\cos\theta$.

Movable Hair Method. Distance and Elevation Formulæ.—The above formulæ apply in this case, but s is now the constant distance between the staff targets, and the variable i is measured The hairs are set to intercept s by means of screws with micrometer heads, and their distance apart is read in terms of the pitch of the screws

Let p = pitch of the screws,
n = number of pitches between the stadia hairs, or the number of turns of the micrometer heads necessary to carry both stadia hairs from the centre hair until they subtend s

Then the expression, $D = \frac{fs}{i} + (f+c)$, becomes

$$D = \frac{fs}{np} + (f+c),$$

which involves the two instrumental constants $\frac{f}{p}$ and $(f+c)$ The remaining formulæ are derived from the fundamental expression as before

The Tacheometer.—Although an ordinary transit fitted with stadia hairs or points can be employed for tacheometry, accuracy and speed are promoted if the instrument is specially adapted for the work The telescope formerly provided in small theodolites by some makers is not sufficiently powerful for tacheometry The magnification should be at least 20 diameters, and, to obtain a sufficiently bright image, the effective aperture of the objective should not be less than $1\frac{3}{8}$ in For the same reason, it is inadvisable to employ an erecting eyepiece on account of the resulting loss of light It is desirable that the objective should produce a flat and undistorted image, since the stadia hairs are not in the optical axis Two pairs of stadia hairs are sometimes fitted The interval between the extreme hairs corresponds to a value of $f/i = 50$ and is twice that of the inner pair, for which $f/i = 100$ To facilitate reading the extreme hairs, the eyepiece may be arranged to move up or down by means of a rack and pinion The widely spaced hairs are intended for the taking of short sights, but are seldom used

Since the measurement of vertical angles constitutes an important item of the observations, the instrument should have a sensitive spirit level mounted upon the vernier arm of the vertical circle. The vertical circle is commonly graduated in quadrants with the zeros horizontally opposite each other, but some surveyors prefer a whole circle graduation, as thereby there is no need to distinguish readings as elevations or depressions in booking, and mistakes are avoided (page 62).

The instrument required for observations on the movable hair system differs from the simple tacheometer only in having a special diaphragm. In the commonest form, the fixed horizontal and vertical hairs are stretched across the frame, and each movable hair is mounted upon a slide, which can be raised or lowered by a micrometer screw To prevent fouling, the hairs lie in slightly different vertical planes. The screws are provided with milled heads and drum scales The drums are divided into 100 parts, and are read against a fixed index to 0·1 of a division either by estimation or by vernier Readings are therefore made to 0·001 of the pitch of the screws A comb scale with teeth of the same pitch as the screw is provided for the purpose of exhibiting the number of complete pitches intercepted between the axial hair and either stadia hair.

A transit theodolite provided with either fixed or movable stadia hairs and having a telescope of the quality specified above may be classed as a tacheometer, but the term is sometimes limited to instruments in which the telescope is fitted with an anallatic lens.

The Anallatic Lens.—The formula, $D=\frac{fs}{i}+(f+c)$, shows the staff intercept to be proportional to $D-(f+c)$, the distance between the staff station and the exterior principal focus of the objective. The latter point therefore forms the apex of a constant visual angle between the sides of which the quantity s is intercepted If this apex were situated on the vertical axis of the instrument, the term $(f+c)$ would vanish, and D would be proportional to s. This was accomplished by Porro by the introduction in the telescope of an additional convex lens, called an anallatic lens, placed between the eyepiece and object glass, and at a fixed distance from the latter

The anallatic lens is generally provided in external focussing tacheometer telescopes by English makers While its use simplifies the reduction of observations, it is open to the objection that it increases the absorption of light in the telescope with consequent reduction in brilliancy of the image. It is not fitted in internal focussing telescopes (see page 377)

Theory of the Anallatic Lens.—In Fig. 300, O represents the optical centre of the object glass, and L that of the anallatic lens, the rays drawn being two of those passing through the principal focus, F_1, of the latter The actually formed image of the staff intercept AB is represented by ba, and b′a′ is that which would be produced if no anallatic lens were interposed. The effect of the introduction of this lens is best followed by reference to these two images.

Let D, d, f, and c = the same quantities as before (page 371),

d' = distance from optical centre of objective to b′a′,

d_1 = distance from optical centre to actual image ba,

e = distance between optical centres of object glass and anallatic lens,

f_1 = focal length of anallatic lens,
i = length b′a′,
I = length ba, the actual stadia hair interval

The equations governing the size and position of $b'a'$ are, as before,

$$\frac{1}{f} = \frac{1}{d'} + \frac{1}{d}, \qquad \ldots\ldots \qquad (1)$$

$$\text{and } \frac{d}{d'} = \frac{s}{i} \qquad \ldots\ldots \qquad (2)$$

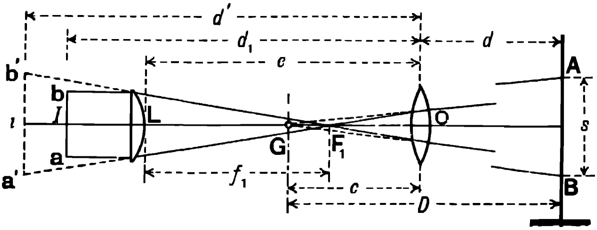

FIG 300

Due to refraction through the anallatic lens, the rays which would converge to form b′a′ actually form the image ba, so that b′a′ and ba are conjugate, and their distances $(d'-e)$ and (d_1-e) from L are connected by

$$\frac{1}{f_1} = \frac{1}{(d_1-e)} - \frac{1}{(d'-e)}, \qquad \ldots\ldots \qquad (3)$$

the negative sign being required since b′a′ and ba are on the same side of L (page 11)

The lengths of b′a′ and ba are proportional to their distances from L, so that

$$\frac{i}{I} = \frac{(d'-e)}{(d_1-e)} \qquad \ldots\ldots \qquad (4)$$

An expression for D can now be obtained by eliminating d', d_1, and i from these equations

From (3), $(d_1-e) = \dfrac{f_1(d'-e)}{(f_1+d'-e)}$, whence, from (4),

$$i = \frac{I(f_1+d'-e)}{f_1},$$

but, from (1), $d' = \dfrac{fd}{(d-f)}$,

$$\therefore i = \frac{I\left(f_1 + \dfrac{fd}{(d-f)} - e\right)}{f_1}.$$

A second expression for i follows from (1) and (2), *viz.*

$$i = \frac{fs}{(d-f)}.$$

Equating these two expressions for i, and solving for d, we have

$$\frac{fs}{(d-f)} = \frac{I\left(f_1 + \frac{fd}{(d-f)} - e\right)}{f_1}$$

$$\text{whence } d = \frac{ff_1 s}{I(f+f_1-e)} - \frac{f(e-f_1)}{(f+f_1-e)},$$

$$\text{and} \quad D = \frac{ff_1 s}{I(f+f_1-e)} - \frac{f(e-f_1)}{(f+f_1-e)} + c \quad . \; . \quad . \quad (5)$$

Now the condition that D should be proportional to s requires that the second and third terms should vanish, so that

$$\frac{f(e-f_1)}{(f+f_1-e)} = c,$$

which is secured by placing L so that

$$e = f_1 + \frac{fc}{(f+c)} \, . \quad . \; . \; . \; . \; ..... \; ...(6)$$

Under these conditions, the apex of the tacheometric angle is situated at G, the centre of the trunnion axis The focal length f_1 must be such that e is less than f, and the values of f_1 and I must be so arranged that the multiplier $\frac{ff_1}{I(f+f_1-e)}$ is a suitable round number, say 100, so that

$$D = 100s$$

The formulæ for inclined sights on either the fixed or the movable hair system are similarly modified by omitting the term involving the second constant

The anallatic lens is usually provided with means for adjusting its position in the telescope, so that, if the constant is found to differ from 100, the distance e may be adjusted until the desired value is obtained

The Internal Focussing Telescope in Tacheometry.—If the instrument is fitted with an internal focussing telescope, the distance formula, $\frac{fs}{i} + (f+c)$, is not applicable, nor can the internal lens be regarded as the exact equivalent of an anallatic lens The system consisting of the object glass and the internal lens is not of constant focal length, since the interval between these lenses is varied in focussing, whereas an anallatic lens is placed at a constant distance from the object glass. Investigations of the distance formula for the internal focussing telescope are given by Louis and Caunt* and by Henrici.†

The variation of focal length of the objective system is very small for sights of more than, say, 100 ft, and practically it is sufficient to adopt the same basic reduction formula, $D=Cs+C_1$, as in the case of the externally focussed telescope The multiplier C is a constant, usually 100, but C_1; although practically constant for long sights, is not so far all distances, and may be regarded as a correction term which neutralises the effect of the varying focal length Its value is usually only a few inches, and it is often disregarded.

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* *Tacheometer Tables* London, 1919,

† " The Use of Telescopes with Internal Focussing for Stadia Surveying " *Trans Optical Society*, Vol XXII

It is impossible to proportion a focussing lens to serve as an anallatic lens, but several designs have been evolved with the object of reducing to negligible limits the variations in the magnitude and position of the tacheometric angle In that patented by Messrs Zeiss in 1910, the internal lens, when focussed for very distant objects, is placed about midway between the plane of the stadia hairs and the image projected by the object glass of the point in which the apex of the tacheometric angle then lies When focussed for very near objects, the internal lens, if negative, is moved half-way at most towards the reticule This scheme places the apex of the tacheometric angle in a practically constant position at the instrument axis Alternatively, if the internal lens is placed about midway between the object glass and the plane of the hairs when the telescope is focussed for very distant objects, the apex is situated in the object glass The additive term is then the constant c In both arrangements the tacheometric angle is practically constant for all sights over, say, 20 ft in length

Messrs J S Wilson and E W Taylor, of Messrs Cooke, Troughton and Simms, patented in 1925 a design in which the focal length of the internal lens is equal in magnitude, but opposite in sign, to that of the object glass, the distance between the centre of the object glass and the plane of the stadia hairs being 1·11 to 1·12 times the common focal length. If the horizontal axis of the telescope intersects the optical axis midway between the objective and reticule, the design secures that distances derived as a constant times the staff intercept represent distances from the instrument axis within 1 part in 1,000 for sights of from 20 to 600 ft. The telescope is therefore for practical purposes anallatic Messrs A. and E W Taylor have produced an internally focussed telescope which is perfectly anallatic, but three lenses are introduced between the objective and diaphragm, so that a larger aperture is required to compensate for the loss of illumination

Instrumental Fittings to Simplify Reductions.—Whether the telescope of a tacheometer is, or is not, anallatic, a good deal of labour is involved in the determination of the horizontal and vertical distances. Certain instruments and attachments are available in which fittings are provided for the purpose of facilitating the reduction of observations. These include the Beaman Stadia Arc, the Stanley Compensating Diaphragm, and the Jeffcott Direct-Reading Tacheometer.

The Beaman Stadia Arc.—The reductions would be simplified if the only values of θ used were those for which either $\cos^2\theta$ or $\frac{1}{2}\sin 2\theta$ is a convenient figure. The former varies too slowly for the small angles usually required, but a list of values of θ for which $\frac{1}{2}\sin 2\theta = \cdot 01, \cdot 02$, etc., can be prepared as follows:

$\frac{1}{2}$ Sin 2θ	θ to nearest second			$\frac{1}{2}$ Sin 2θ	θ to nearest second		
	°	′	″		°	′	″
·01	0	34	23	·06	3	26	46
·02	1	8	46	·07	4	1	26
·03	1	43	12	·08	4	36	12
·04	2	17	39	·09	5	11	6
·05	2	52	11	·10	5	46	7
				etc.	etc.		

If these particular angles were used, the vertical component V, for a multiplying constant of 100 and no additive constant, would be s, $2s$, $3s$, etc.

The idea is applied in the Beaman Stadia Arc, which has been used to a considerable extent in tacheometers and plane table alidades, and has increased in popularity in recent years. It is fixed to the vertical circle, and consists of a scale engraved with the above angles on either side of zero up to about $26^\circ\ 33'\ 54''$, for which $\frac{1}{2}\sin 2\theta = \cdot 40$. The scale is figured in terms of $100 \times \frac{1}{2}\sin 2\theta$, and is read against a fixed index mark. To avoid possible confusion between elevations and depressions, the zero is commonly marked 50, so that 50 must be subtracted from every reading. No fitting is required to enable fractional parts of the scale to be read, since for every sight a graduation of the scale is to be brought opposite the index by means of the vertical circle tangent screw.

Let it be supposed that a staff is sighted with an instrument having the tacheometric constant 100 and the stadia arc zero 50, and that the tangent screw is adjusted to bring a graduation of the stadia arc, say 36, exactly opposite the index. Let the observed axial hair reading be 6·10, and the staff intercept 4·95.

The stadia arc reading $= 36 - 50 = -14$.

The vertical component $= -14 \times 4{\cdot}95 = -69{\cdot}3$ ft.,

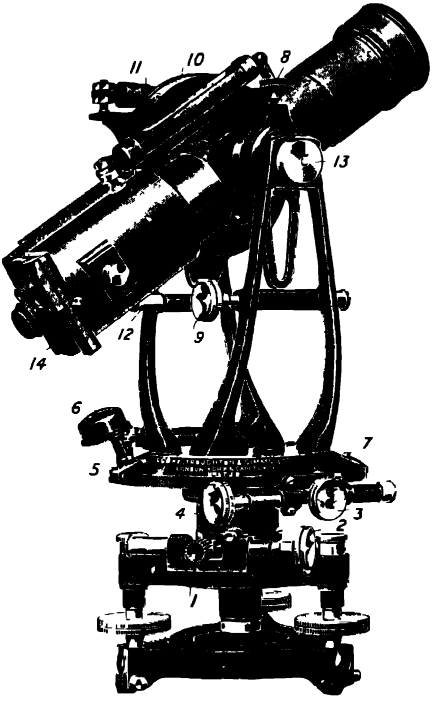
and the staff point is below the trunnion axis by $69{\cdot}3 + 6{\cdot}1 = 75{\cdot}4$ ft.

To facilitate the calculation of horizontal distances, the stadia arc also carries a scale of percentage reductions to be applied to the distance readings, $\frac{fs}{i}$. In the above example, the distance scale will read 2·0, so that

$$\text{the horizontal distance} = 495 - 2 \times 4{\cdot}95 = 485 \text{ ft.}$$

The Stanley Compensating Diaphragm.—This diaphragm is designed to enable the horizontal component of a sloping sight to be read directly. Platinum stadia pointers are mounted in such a way that the interval between them can be adjusted by turning a micrometer head carrying a scale of vertical angles. The observer, having read the inclination of the line of sight on the vertical circle, sets the micrometer head to that angle, and obtains the horizontal distance as 100 times the staff intercept. The diaphragm is made in two patterns, for vertical and normal staff holding respectively.

The Jeffcott Direct-Reading Tacheometer.—This instrument (Fig. 301) invented by Dr. H. H. Jeffcott* enables the horizontal



1 *Lower Clamp*
2 *Lower Tangent Screw*
3 *Upper Clamp*
4 *Upper Tangent Screw*
5 *Coarse Vernier*
6 *Achromatic Doublet*
7 *Fine Vernier*
8 *Telescope Clamp*
9 *Telescope Tangent Screw*
10 *Cam Housing*
11 *Cam Control Level*
12 *Slow Motion Screw for Do*
13 *Focussing Screw*
14 *Eyepiece Slide*

FIG. 301.—JEFFCOTT DIRECT-READING TACHEOMETER.

and vertical components to be read directly without the necessity for measuring the vertical angle. The diaphragm carries a fixed platinum iridium needle point and two movable points, for distance and height respectively. The latter are mounted at the ends of levers which are actuated by cams operated by the tilting of the telescope, so that the setting of the pointers is automatic. The proportions are such that 100, or in an alternative pattern 200, times the staff intercept between the fixed pointer and the movable right-hand, or distance, pointer is the required horizontal distance, and 10 times the intercept between the fixed pointer and the left-hand, or height, pointer gives the vertical component The range of operation of the cams is 30° elevation or depression

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* H. H. Jeffcott, "A Direct-Reading Tacheometer." *Trans. Inst. C.E. of Ireland*, Vol. XLI.

To take a reading, the axial needle should preferably be set at a whole foot graduation of the staff In the observation illustrated in Fig 302, the readings are 4·13, 5·00, and 6 49, so that the horizontal distance is 149 ft , and the vertical component is + 8·7 ft , the height pointer moving upwards from the fixed pointer for angles of elevation, and downwards for angles of depression When the vertical angle exceeds about 8°, the height pointer is not seen simultaneously with the others, and the eyepiece, which is mounted on a slide, is moved up or down until the height reading is obtained

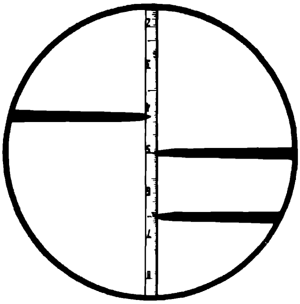

FIG 302

In addition to confining the reduction work to the computation of the reduced levels, the instrument economises time in reading and booking through the elimination of vertical circle readings The standard instrument has no vertical circle, but a circle can be fitted if desired. Useful accessory features include the placing of a horizontal circle vernier, reading to 1 min , at about 30° to the left of the line of sight, the opposite vernier reading to 20 sec A reading of the coarse vernier alone is sufficient in tacheometry, and, as it is observed through an achromatic doublet (page 64), bearings are read without the surveyor's having to move from the telescope

Particulars of the adjustment of the instrument are published by the makers, Messrs Cooke, Troughton and Simms, Ltd

The Stadia Rod.—An ordinary levelling staff may be used, but the graduation of some patterns is not sufficiently bold for tacheometry, as sights are frequently of much greater length than is usual in ordinary levelling Fig 303 illustrates graduations typical of the many patterns which have been proposed The rod has a length of from 12 to 15 ft , and is either telescopic, hinged, or in separate pieces fitting together as in the Scotch levelling staff Lightness is a desideratum, and rigidity may be secured by means of a stiffening piece screwed to the back or by side strips projecting a little beyond the graduated face.

For movable hair observations, diamond-shaped or circular vanes may be attached to the back of an ordinary staff . for moderate lengths of sights, patterns painted on the staff itself are sufficient. A centre vane or mark should always be provided at a known distance from the foot of the staff. It is useful to have two intercept

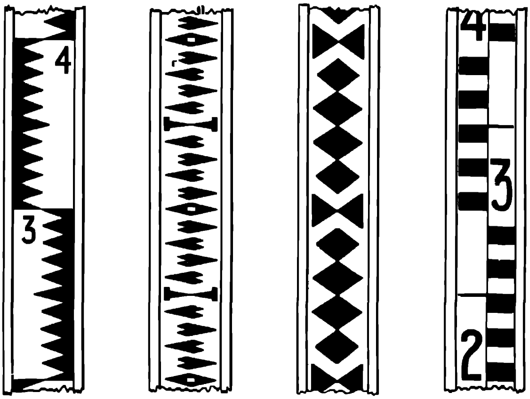

Fig 303 —Stadia Rod Graduations

distances of, say, 5 ft and 10 ft , distinguished by different patterns of targets, the smaller being used for short sights In this way the alteration of the hairs necessary to suit widely different lengths of sights is reduced

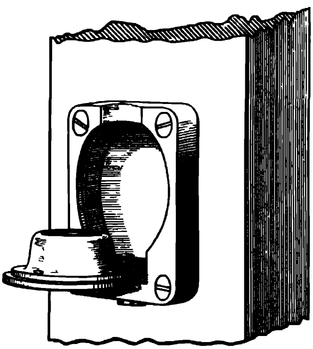

Fig. 304.—Staff Plumbing Level

If the staff is to be held vertically, it is desirable that some appliance should be provided to enable the holder to see if he is maintaining it in the vertical position One method is to attach a pendulum plumb-bob to the back, such that when the staff is vertical the point of the plummet lies in the centre of a projecting ring fixed to the staff A neater method is to fit a small circular spirit level, which may be hinged to fold up (Fig. 304) For normal holding, a staff director is usually attached to the rod A common form consists of a tube fitted with cross wires which provide a line of sight at right angles to the staff The latter is inclined until the telescope of the theodolite appears bisected by the wires. The tube may be fitted with lenses forming a small telescope to assist the staffman in setting the rod for long sights. If, however, in these circumstances he can sight the upper part of the instrument roughly, the error of inclination of the staff will be negligible, so that the use of a telescopic director is a needless refinement Strictly, the director should be attached to the rod at the reading of the centre hair, but it is sufficient to place it at a convenient height for the staffman

Instrumental Constants.—Taking first the case of the external focussing telescope without an anallatic lens, the expression, $\frac{fs}{i}+(f+c)$, and its modifications involve the two constants, f/i, and $(f+c)$, the values of which must be known for the instrument used The actual distance, i, between the stadia hairs is not required, since the ratio f/i can be obtained without direct measurement of i

The constant f/i is by far the more important of the two, since it is a multiplier It is usually arranged by the maker to be 100, but its actual value should be determined as carefully as possible The $(f+c)$ term is not strictly a constant, since the distance c will vary a little for different lengths of sight owing to movement of the objective in focussing This variation will, however, seldom exceed a small fraction of an inch, since very short sights are comparatively rare, and it is quite allowable to regard c as constant. The value of $(f+c)$ varies from less than 1 ft to nearly 2 ft, according to the size of the telescope

To determine the values of the constants, a base line of about the same length as the longest sight to be used in the survey is measured on fairly level ground with a steel tape Pegs are driven along it at intervals of, say, 200 ft The instrument is set at one terminal, and readings are taken with the rod held at the various distances The fact that there are two constants to be determined suggests the use of pairs of simultaneous equations Thus, if D_1, D_2, etc represent distances from the instrument, and s_1, s_2, etc, the corresponding staff intercepts, then

$$D_1=\frac{fs_1}{i}+(f+c),$$

$$D_2=\frac{fs_2}{i}+(f+c), \text{ etc}$$

If horizontal sights are impracticable, the corresponding formula for inclined sights will be applied The equations are solved in pairs for f/i and $(f+c)$, and the average values are adopted

A simpler, and rather better, method is to determine $(f+c)$ by direct measurement on the instrument. The length f is measured between the centre of the object glass and the plane of the cross hairs when the telescope is focussed on a remote object c is best measured with the telescope focussed for an average length of sight. The results of the observations of the several measured lengths are now employed for the determination of an average value of f/i. The reduction is somewhat simplified if the instrument is placed at a distance $(f+c)$ beyond the end of the base line

The accompanying table shows a typical set of readings with level sights. The constants are worked out for the half-intervals between the axial hair and the stadia hairs, as well as for the whole interval Although it can be shown that distances derived from half-interval observations are not strictly correct in the case of highly inclined sights, these subsidiary constants are used in the reduction of observations in which the whole stadia interval cannot be used, as (*a*) when an obstacle intervenes in the line of sight of one of the stadia hairs, (*b*) when a very long sight is taken so that

R T C 5 in Transit No 3 $(f+c)=1.4$ ft

Distance in feet 1.4 +	Readings			Constants		
	Lower	Centre	Upper	Whole Interval	Lower Interval	Upper Interval
200	3.30	4.32	5.32	99.01	196.08	200.00
400	6.67	8.71	10.70	99.26	196.08	201.01
600	3.46	6.53	9.51	99.17	195.44	201.34
800	4.79	8.86	12.85	99.26	196.56	200.50
1,000	2.99	8.10	13.08	99.11	195.69	200.80
			Average	99.2	196.0	200.7

both stadia hairs do not cut the staff If the stadia hairs are equidistant from the middle hair, these constants will each have a value of twice that for the whole interval, but, since the intervals may not be exactly equal, the constants should be evaluated If they differ sensibly, care must be exercised in observing with a half-interval to avoid confusing between the two constants in the direct and reversed positions of the telescope.

An important source of error in the determination of the value of f/i, and one which affects all stadia work, is that caused by the difference in the effect of atmospheric refraction on the two sight lines through the stadia hairs Near the surface of the ground the density of the strata of air may vary rapidly at different distances from the ground, the nature and extent of the variation depending upon the relative temperature of the atmosphere and the ground. The effect is most marked during the midday hours in summer The density then decreases downwards, and a ray of light situated at less than about 4 ft from the ground is convex towards the earth. Professor L S Smith, of the University of Wisconsin, has shown that the effect on the lower sight line is much greater than that on the upper, and that readings taken at the time of maximum change of refraction give too small staff intercepts, but that the effect is negligible when the lower ray is 3 to 4 ft. above the ground In consequence, the value of the constant, deduced from observations taken during the midday hours is likely to be too great for the reduction of sights taken in the morning or evening, when the variation in atmospheric density is very small; and similarly a constant measured in the morning or evening is too small for observations in the middle of the day.

To determine that value of the constant f/i by which observations taken at any time of day may be reduced with sufficient accuracy, several observations of the base lengths should be made at different hours of the day Furthermore, the conditions under which the constant is measured should be as nearly as possible those obtaining in the survey, particularly as regards weather and the nature of the ground over which the base line is laid out

The same general methods and precautions are applicable to the determination of the constants for a movable hair instrument. In this case, the multiplier constant f/p is usually arranged by the maker to have a value of 1,000 It is not unusual to find that the micrometer readings are not exactly zero when the stadia hairs are brought to the axial hair The amount of index error, e, to be applied as a correction to the sum of both micrometer readings may be deduced from the observation of the measured distances by expressing each in the form, $D = \frac{fs}{(n+e)p} + (f+c)$ The index error for each stadia hair may be similarly obtained from the same expressions for D, if s is the half intercept, and n and e are respectively the reading and index error of one of the micrometers.

For the evaluation of the single constant of an anallatic telescope, observations are taken over several measured distances with the same precautions as before In the case of the internal focussing telescope, the aim should be to obtain values of the multiplier and the additive term which will suit for the determination of other than short distances These are deduced by solving simultaneous equations corresponding to a series of sights ranging from, say, 200 to 800 ft If the values so derived are used as constants, the errors introduced at measured distances of 20 to 100 ft can then be determined, and, if appreciable, they may be eliminated by making the additive term a variable for short sights In the case of telescopes designed to be virtually anallatic (page 378), it is practically impossible to detect any departure from the anallatic condition, since, with a constant multiplier, the additive term may never exceed an inch.

Value of Constant.—Although the constant C in the general formula, $D = Cs + C_1$, is designed to be a round number, it is a difficult matter for the surveyor to space the hairs exactly at the required interval after a breakage Change of conditions further increases the likelihood that the value of the constant may be found other than was intended

Since the reductions are considerably simplified if C is exactly 100, instruments have been fitted with means for adjusting the stadia interval so that the hairs may be set to yield this value of the constant. Experience has shown, however, that there is greater possibility of error with adjustable hairs, by their accidental disturbance, and that better results are obtained with permanently fixed hairs. In the case of the anallatic telescope, the constant is adjusted by motion of the anallatic lens, and the adjusting screw must be adequately protected against accidental movement

When the constant of a non-adjustable instrument is found to differ from 100, and the difference is too great to permit the assumption of a value of 100, two methods are employed to lessen the arithmetical work of reduction: (1) the use of a table of reduction; (2) the use of a stadia rod specially graduated to suit the instrument.

(1) The first method is the more satisfactory The table required is one showing the values of Cs for various values of s, and can be prepared in a few minutes It is sufficient to tabulate Cs for values of s increasing by 0·1 ft. Intermediate values are obtained with sufficient accuracy by taking each unit in the second decimal place of intercept as equivalent to 1 ft of distance, since the constant will have a value in the neighbourhood of 100 Otherwise, a subsidiary table can be prepared for $s =$ 01, ·02, etc , ft The tabulated distances fall to be increased by C_1 unless an anallatic telescope is used. This term is commonly included in drawing up the table, but, unless for normal staff holding, the tabulation of Cs only is preferable on the whole.

(2) The second method consists in graduating the rod so that when the observed intercepts are multiplied by 100 the correct values of Cs are obtained This method has the serious objection that, since the rod is not graduated in feet, the determination of elevations involves additional calculation. It is also undesirable in that the same rod must always be used with the same instrument and that a change in the value of the stadia interval necessitates repainting of the rod

In the case of the movable hair instrument, it is equally desirable to have the value of the constant $C = f/p$ a round number. Since in this case s is a constant, the quantity Cs should be made a round number, and this is easily effected by adopting a suitable value for s Thus, if C is found to have the value, 1,019, the targets should be fixed on the staff at a distance of 9·813 ft. apart Then, for level sights, $D = \frac{10{,}000}{n} + C_1$, and the reductions may be performed with the aid of a table of reciprocals. There is not the same objection to making the rod suit the instrument in movable hair tacheometry as in the fixed hair system, since the middle target from which elevations are obtained is placed at a known distance from the foot of the staff.

Observing with the Tacheometer.—In sighting the stadia rod, the pointing may be made to any part of the graduation. Readings are estimated to ·01 ft. When the instrument constant has a value of 100, it is usual to dispense with the booking of the individual stadia hair readings and to read the staff in terms of Cs, or 100 times the actual intercept. The mental subtraction of the lower reading from the upper is facilitated by setting one of the hairs to a whole foot graduation. The reading of the axial hair must, however, be noted. The reduction is somewhat simplified if this hair is directed to the point on the rod corresponding to the height of the instrument axis above the ground. This will not be possible when the lower part of the rod is hidden, and should not be done if it brings the lower stadia reading near the ground. When the method is employed in conjunction with that of reading the intercept in terms of *Cs*, the latter is first observed with one of the stadia hairs at that whole foot graduation which brings the axial hair approximately to the instrument height. Thereafter, the axial hair is brought exactly to the instrument height graduation, and the vertical angle is noted A small error is introduced by the circumstance that this angle differs slightly from that at which the intercept was read, but its effect is negligible except when locating station points by highly inclined sights.

When intervening obstacles to sighting prevent the three hairs being brought simultaneously upon the staff, the intercept between the axial hair and one of the stadia hairs is observed, and the appropriate constant is used If the constants for the half-intervals are unknown, the telescope is then tilted to enable the intercept between the axial hair and the other stadia hair to be read, the mean of the two vertical angles being used in the reduction.

In observing with the movable hair instrument, the centre target is first bisected with the axial hair. The micrometer milled heads are then simultaneously turned to move the stadia hairs from the positions they happen to occupy until they bisect the other targets To read the distance of each hair from the centre, the number of complete pitches is counted on the comb scale by reckoning from bottom to bottom of the notches. The fractional parts are given on the drums, and the sum of the two complete readings is the quantity *n*.

Relative Merits of Vertical and Normal Rod Holding.—The vertical system proves the more convenient on the whole, and is generally preferred. Normal staffing is, however, sometimes adopted, and a comparison may be drawn between the two systems.

Ease of Reduction of Observations.—The distance and elevation formulæ are somewhat simpler for vertical than for normal holding, and published reduction tables and diagrams are based on the former. Practically, there is little difference in this respect since, except in the case of large values of θ, it is usually allowable in the normal system to assume $h \sin \theta = 0$ and $h \cos \theta = h$.

Facility of Holding.—Provided the rod is fitted with a spirit level, the vertical direction is more easily determined and maintained than the normal. Normal holding by the use of a staff director necessitates the rodman sighting the instrument, and this is at times impracticable in rough country, but the method has the advantage, particularly evident in high wind, that the rod can be swung as in ordinary levelling.

Effect of Careless Holding.—Errors of distance and elevation caused by deviation of the staff from its intended vertical or normal position are very much more serious with vertical than with normal holding. This would prove an important objection to vertical staffing, were it not that the errors are easily kept within allowable limits by the use of a spirit level or plummet.

Accuracy of Measurements by Stadia.—The principal sources of error in stadia observations fall into three classes: (1) Errors in reading the staff intercept; (2) Errors of instrument and value of the constant; (3) Errors due to natural causes.

(1) Since the actual intercept is multiplied by a constant of about 100, an uncertainty of ·01 ft. in the reading produces an uncertainty of 1 ft. of distance. In estimating the reading of each hair to the nearest ·01 ft., the greatest error which would occur under perfect conditions of sighting is ± ·005 ft., and the average error ± ·0025 ft. The average error in the estimated value of the intercept is therefore $\pm \sqrt{\cdot 0025^2 + \cdot 0025^2} = \pm \cdot 0035$ ft., corresponding to ± ·35 ft. of distance. Practically, such a good estimate of the intercept cannot be expected. Accuracy of reading depends greatly upon the length of sight, definition and magnifying power of the telescope, fineness of the hairs, elimination of parallax, clearness of the atmosphere, graduation of the rod, steadiness of holding and accuracy of the vertical or normal position of the rod, and the observer's personal factor. If the resulting uncertainty is expressed as a ratio of the distance, its value will be comparatively great for very short sights and also for distances beyond the range of clear sighting for the particular instrument used. The length of sight giving minimum value of the proportional error will generally lie between 400 ft. and 700 ft., depending upon the quality of the instrument.

In observations with the movable hair instrument on a staff with prominent targets, errors of sighting are reduced at long distances; but up to about 800 ft. the results are likely to be somewhat inferior to those obtained from fixed hair observations.

(2) The accuracy with which distances and elevations are measured is dependent upon the adjustment of the altitude level and the elimination or determination of index error, as well as upon the accuracy of reading of the vertical circle. The effect of such errors as are likely to occur in the vertical angle measurement is relatively unimportant in distance determinations. Errors of levelling, however, may have serious effects on the elevations, and attention must be paid not only to the adjustment of the altitude level but also to maintaining the bubble central during observations.

Distance and elevation are affected by using an incorrect value for the constant, and the resulting errors can be kept within reasonable limits only by determining the constant from time to time under the same conditions as obtain on the survey Appreciable error may be introduced by alteration in the value of the constant through change of conditions

(3) The chief natural sources of error are wind and the effect of the difference of atmospheric refraction on the two stadia lines of sight In high wind the rod cannot be held steady and plumb, and good reading is impossible To avoid the second error, the lower sight line should not be made to pass within 3 ft of the ground for any considerable portion of its length This precaution becomes specially important during the midday hours In consequence of the disturbed state of the lowest strata of air, it is not unusual for distances to be determined more accurately in moderately rough country than over very flat ground

Experience shows that the error in distance from a single observation should not, on an average, exceed about 1 in 500 for moderate lengths of sights, and that the error of elevation need not exceed about ·3 ft for moderate inclinations Since many of the errors to which the observations are subject are of a compensating nature, the closing error of a stadia traverse is proportional rather to the square root of the distance than to the distance The closing error in traversing by stadia is usually less than ·10$\sqrt{D}$ ft , where D is the length of traverse in feet The closing error of elevation in circuits between bench marks depends greatly upon the roughness of the country, but on average ground can be kept well within ·01$\sqrt{D}$ ft

THE TANGENTIAL SYSTEM

Distances and elevations may be deduced from staff readings taken by a theodolite without any additional fittings whatever. Fig 305 shows the simplest case, that in which the ground is sufficiently level that the staff may be read with a horizontal line of sight The reading at B is observed with the telescope levelled, and that at A by means of a sight inclined at θ to the horizontal Denoting the difference of the readings by s, then

$$H = s \cot \theta$$

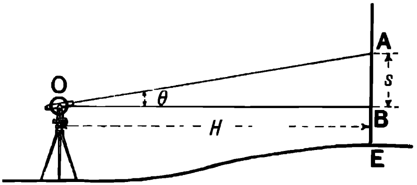

FIG 305.

The elevation of E is derived from the reading B as in ordinary levelling

When the ground will not permit of a horizontal sight, two vertical angles, θ and ϕ (Fig. 306), must be measured. For long sights the observations may be taken to prominent targets or marks on the staff at a known distance s apart. Otherwise a stadia rod is used, the two graduations observed being preferably an exact number of feet apart

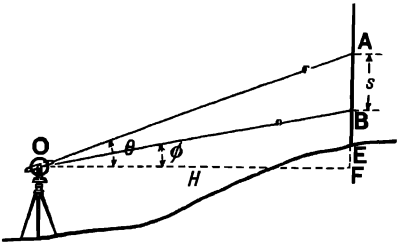

FIG 306.

If θ is observed to the upper, and ϕ to the lower point on the staff, the reduction formulæ are as follows:

$$\text{Since AF} = H \tan \theta, \text{ and BF} = H \tan \phi,$$

$$(\text{AF}-\text{BF}) = s = H\,(\tan \theta - \tan \phi),$$

$$\text{whence } H = \frac{s}{(\tan \theta - \tan \phi)}, \text{ for angles of elevation,}$$

$$\text{and } H = \frac{s}{(\tan \phi - \tan \theta)}, \text{ for angles of depression.}$$

Knowing H I, the elevation of the axis of the instrument above datum, the elevation of E is given by

$$\text{Reduced level of E} = \text{H I} + \text{FB} - \text{EB},$$

$$= \text{H.I} + H \tan \phi - \text{EB}, \text{ for angles of elevation,}$$

$$\text{and} = \text{H I} - H \tan \phi - \text{EB}, \text{ for angles of depression.}$$

The evaluation of $\dfrac{s}{(\tan \theta - \tan \phi)}$ is somewhat laborious as, since two angles are involved, a table of the values of $(\tan \theta - \tan \phi)$ would be cumbersome. The reduction is simplified by adopting a length of 10 ft for s and using a table of reciprocals. Various other suggestions have been made for lessening the labour of reduction, and a number of modifications of the theodolite have been devised so that part, at least, of the work may be performed mechanically.

Use of Particular Values of θ and ϕ.—If the inclined sights are taken to two targets on the staff, there is no choice as to the values of θ and ϕ, but if the staff is fully graduated, particular angles may be set on the vertical circle, and the variable s obtained from the staff readings Reduction is facilitated by using only those angles the natural tangents of which are ·01, ·02, ·03, etc. If two consecutive angles from a list of these are employed for an observation, $(\tan \theta - \tan \phi)$ is always ·01, so that

$$H = 100\,s,$$

and the quantity, $H \tan \phi$, required in the calculation of the levels, amounts to s, $2s$, $3s$, etc., according to the angle ϕ used. The method therefore enables reductions to be performed mentally.

By reference to a table of log tangents, a list of the required angles may be prepared as follows ·

Tangent.	Angle to nearest second.			Tangent.	Angle to nearest second.		
	°	′	″		°	′	″
·01	0	34	23	06	3	26	1
02	1	8	45	07	4	0	15
·03	1	43	6	·08	4	34	26
04	2	17	26	09	5	8	34
05	2	51	45	·10	5	42	38
				etc.		etc	

The difficulty of setting off these angles with sufficient accuracy by vernier explains why this system has not been applied in practice to any great extent The method is a feasible one with micrometer reading of the vertical circle It is, however, possible to have the vertical circle fitted with a special scale the graduations of which are placed at the above angles on either side of zero, while the figures represent 100 times their tangents.

The Omnimeter.—Eckhold's Omnimeter (Fig 307) was the earliest of the special instruments for tangential tacheometry, but it is now little used, and is mainly of historical interest.

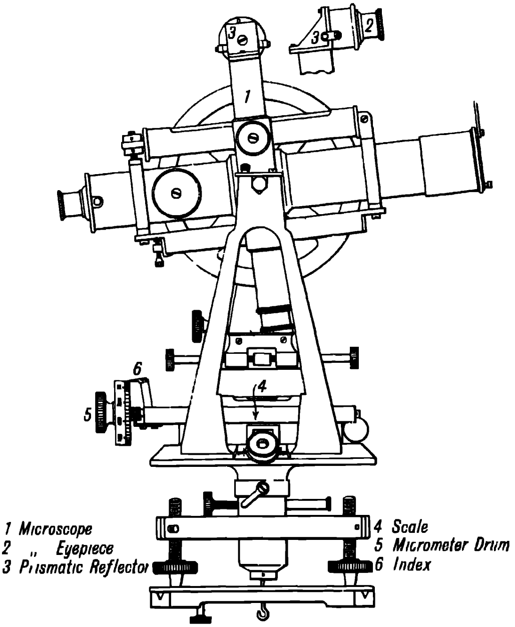

FIG. 307 —OMNIMETER.

It consists of a transit theodolite fitted with (1) a powerful compound microscope permanently fixed with its axis accurately at right angles to the optical axis of the telescope and passing through the horizontal axis, (2) a graduated scale placed upon the vernier plate, the reading of this scale being performed by means of the microscope As usually arranged, the scale is 4 in long, and is divided into 200 parts For the purpose of measuring fractional parts of these divisions, the scale can be given a small longitudinal motion by a micrometer screw of 1/50 in pitch, so that the graduation corresponding to the approximate reading may be brought into the line of sight of the microscope Fractional parts are given on the micrometer head to 1/500th of a scale division. The zero of the scale is in the middle of its length, and should be the point sighted by the microscope when the telescope is horizontal As before, an observation consists in making pointings to two graduations or targets on the staff, preferably 10 ft apart, but, in place of reading θ and ϕ, the corresponding readings of the scale are noted For convenience in reading, the microscope should be fitted with a prismatic reflector in the later forms of the omnimeter the eyepiece of the microscope is alongside that of the telescope

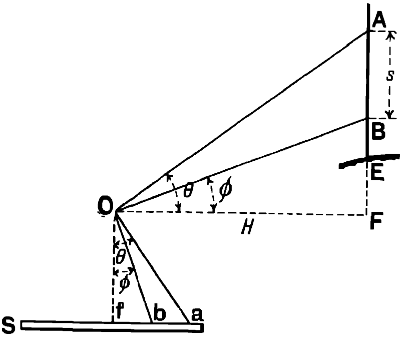

Fig 308

Reduction —In Fig 308, O represents the horizontal axis, and S the scale, of which the length and distance from O are greatly exaggerated Oa and Ob are the lines of sight of the microscope normal to OA and OB, those of the telescope The zero of the scale is at f vertically below O

Let n_1, n_2 = scale readings at b and a respectively,

d = length of one scale division,

c = constant of the instrument $= \frac{\text{Of}}{d}$

Since the triangles of OABF are similar to those of Oabf,

$$\frac{\text{AB}}{\text{OF}} = \frac{\text{ab}}{\text{Of}}, \text{ so that}$$

$$H = \frac{\text{Of} \times s}{d(n_2 - n_1)} = \frac{cs}{(n_2 - n_1)}$$

For the vertical BF, required in computing the elevation of E, we have

$$\frac{\text{BF}}{\text{OF}} = \frac{\text{bf}}{\text{Of}},$$

$$\text{so that BF} = \frac{n_1 dH}{\text{Of}} = \frac{n_1 H}{c} = \frac{s n_1}{(n_2 - n_1)}$$

The instrument is designed to have c a round number, but the value of the constant should be determined by repeated observations of measured distances.

The Bell-Elliot Tangent-Reading Tacheometer is an improved form of omnimeter in which the scale is graduated to read directly the natural tangents of the two vertical angles

The Gradienter.—If the tangent screw actuating the vertical circle of a theodolite is provided with a micrometer head for the measurement of parts of a revolution and a scale for counting the whole turns, the fitting is called a Gradienter It is principally used in setting out gradients, but is also employed in tacheometry. The pitch of the screw must be accurate, and is commonly such that a line of sight originally horizontal is moved through $\tan^{-1} \cdot 01$ by one revolution, so that the horizontal hair is shifted 1 ft on a vertical staff at a distance of 100 ft In some instruments this angular motion is produced by two revolutions of the screw, but in either case the smallest division on the drum corresponds to ·01 ft on a staff 100 ft distant

In the case where a horizontal sight can be taken, the staff is first read with the telescope level The screw is then given one turn (or two turns in the case of the finer pitch), and the staff is again read Then
$$H = 100\,s,$$
where s is the difference of the readings

For inclined sights, the staff may be held vertically or normal to one of the lines of sight The former is the usual method The formulæ derived below for that case may easily be modified for normal holding In making an observation with the staff station higher than the instrument (Fig 309), the telescope is first directed towards a low point B on the staff, and the vertical angle θ is read The line of sight is then moved through $\tan^{-1} \cdot 01$ by the gradienter screw, and the intercept AB = s is noted Alternatively, if A and B are fixed targets, the number of turns, n, necessary to move the line of sight from one target to the other, is read on the drum

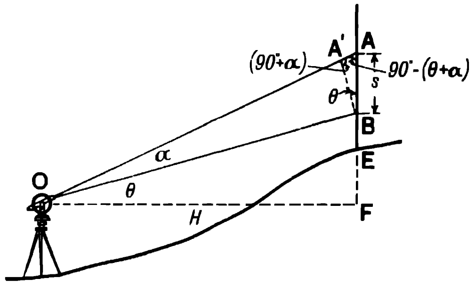

FIG. 309

To obtain the general formulæ, let the pitch of the screw be such that one revolution moves the line of sight from the horizontal through c ft on a vertical staff 100 ft. distant, and, in the case of Fig 309, let n turns of the gradienter move the line of sight from B to A through $\alpha = \tan^{-1}\dfrac{nc}{100}$.

Draw A′B perpendicular to OB

Then $OB = \dfrac{100A'B}{nc}$,

but, from triangle A′AB,

$$A'B = \frac{s\cos(\theta+\alpha)}{\cos\alpha} = s(\cos\theta - \sin\theta\ \tan\alpha) = s\left(\cos\theta - \frac{nc}{100}\sin\theta\right);$$

$$OB = s\left(\frac{100\cos\theta}{nc} - \sin\theta\right),$$

$$\text{and } H = OB\cos\theta = s\left(\frac{100\cos^2\theta}{nc} - \tfrac{1}{2}\sin 2\theta\right).$$

The vertical component, BF, of the sloping distance =

$$OB\sin\theta = s\left(\frac{100\cos\theta\sin\theta}{nc} - \sin^2\theta\right) = s\left(\frac{100\sin 2\theta}{2nc} - \sin^2\theta\right)$$

When θ is a depression, the above formulæ apply provided B is the higher of the two points on the staff If, however, θ is measured to the lower point, the observed angle must be decreased by $\tan^{-1}\dfrac{nc}{100}$ for insertion in the formulæ.

The use of a self-reading rather than a target staff facilitates reduction, for nc is then unity, and, since the second term can often be neglected, we have

$$H = 100\,s\cos^2\theta,$$

$$\text{and } BF = \frac{100\,s\sin 2\theta}{2}.$$

Fergusson's Percentage Unit System.—Mr J C. Fergusson, M.Inst.C E., has devised a novel system for the division of the circle with the object of reducing to a minimum all calculations involving trigonometrical functions An unequal division of the circle is adopted, angles being reckoned in terms of their natural tangents, expressed as percentages. The application of this system to tangential tacheometry facilitates the reduction of observations.

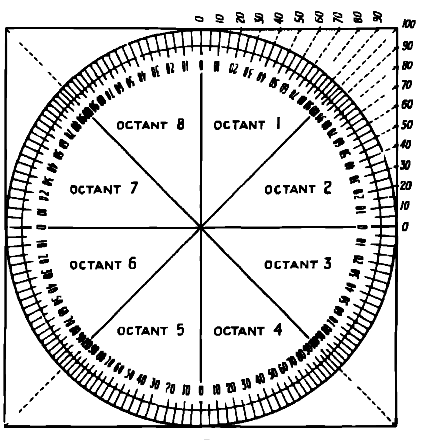

FIG. 310.

Fig 310 illustrates the method of division. A circle is inscribed in a square, and is divided into octants by the quadrant lines and by the diagonals of the square. Each of the eight tangents, of length equal to the radius of the circle, is divided into 100 equal parts. Lines from the centre to these points divide each octant into 100 unequal parts, which each subtend $\frac{1}{100}$th of the tangent and thus form telemetric units The points of division on the circle are numbered from 0 on the quadrantal lines to 100 at the diagonals. The magnitude of any angle between a quadrantal line and a line in the octant on either side is given on the circle in percentage units, and its value in these units measures the length of the perpendicular or tangent in terms of the radius

The percentage system is embodied in Fergusson's percentage theodolite, the circles of which are graduated both in degrees and percentage units. A vernier cannot be used for subdividing the unequal percentage divisions, and its place is taken by a spiral drum micrometer, by means of which readings can be taken to ·01 of a unit

In the application of the system to tangential tacheometry, the observations may be made either on a graduated rod or on one with fixed targets The former method is more convenient of reduction A reading is taken of a low point on the rod with the vertical circle set to a whole percentage unit The telescope is then elevated through an exact number, n, of units, usually 1 or 2, and the upper rod reading is noted Then, corresponding to the formula, $H = \dfrac{s}{(\tan\theta - \tan\phi)}$, we have

$$H = \frac{100s}{n}.$$

The elevation of the staff station may be determined as before by evaluation of $H \tan\phi$, which is readily obtained in this case. Alternatively, the line of sight is directed to the graduation representing the height of the instrument axis above the ground. If the vertical circle now reads $g\%$, this represents the gradient between the instrument station and the staff station, and their difference of elevation $= \dfrac{gH}{100}$.

The Szepessy Direct-Reading Tacheometer.—The objection to the tangential system, that two telescope pointings are required, has been overcome in a neat manner by Szepessy, of Budapest, who has patented a direct-reading instrument A scale of tangents projected upon a circle, as in Fig 310, is required This may be engraved on a glass plate of cylindrical form attached to the vertical circle cover, so that it does not rotate when the telescope is tilted. Light transmitted through the scale is received by the objective of a microscope attached to the telescope and having its optical axis parallel to that of the telescope and passing through the horizontal axis of rotation. The rays undergo reflection by prisms, and are brought to a focus in the plane of the telescope reticule, the eyepiece of the telescope serving also as the eyepiece of the microscope The image of the scale occupies nearly half of the field of view, and, when the telescope is directed towards a staff, the image of the latter is placed alongside that of the scale as in Fig 311

The scale of tangents is divided to ·005, and is figured at every ·01 in terms of 100 times the tangent, so that graduation 15, say, corresponds to $\tan^{-1} \cdot 15$ To take a reading, the vertical circle tangent screw should be adjusted to bring a scale graduation, preferably a numbered one, opposite the axial hair Since the scale graduations on either side represent angles whose tangents differ by ·01, the length of staff intercepted between them multiplied by 100 is the required horizontal distance The scale reading opposite the axial hair being 100 times the tangent of the inclination of the line of sight, the vertical component is given by the previous staff intercept times the scale reading at the axis In the case of Fig 311, the horizontal distance is 199 ft, and the vertical component is $1 \cdot 99 \times 15 = 29 \cdot 9$ ft The staff reading at the horizontal hair is noted as usual for application to the vertical component

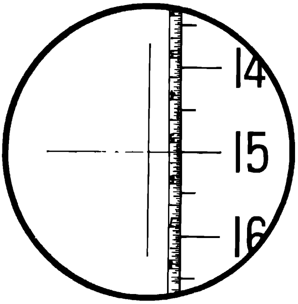

Fig 311

The instrument is made in England by Messrs E R Watts and Son, of London

The Subtense Bar Method.—In this method, distances are obtained by observation of the horizontal angle subtended by targets fixed on a horizontal bar at a known distance apart of from 2 ft to 20 ft The method is largely used in the Survey of India for measuring the lengths of traverse courses in rough country

The Indian subtense bar illustrated in Fig 312 is 11 ft long. It is mounted on a tripod, and is levelled by means of the small spirit level *2* and quick levelling head. The sight rule *1* affords a line of sight perpendicular to the bar, which is thereby set normal to the line of measurement When aligned and levelled, the bar is

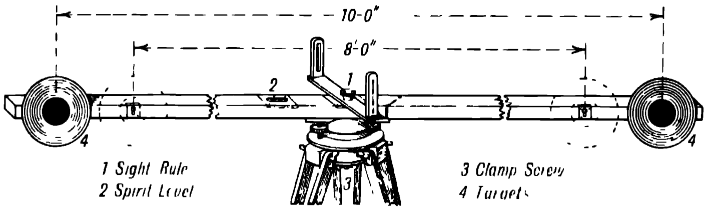

Fig 312—Indian Subtense Bar.

clamped by the screw *3*. The targets are of $8\frac{1}{2}$-in. diameter, and are painted red on one side and white on the other, in each case with a 3-in. black centre The fittings are such that the discs can be placed either 10 ft apart with the red faces showing or 8 ft. apart with the white faces showing, so that the colour seen indicates the interval being observed

Observations may be made by means of a theodolite fitted with a horizontal eyepiece micrometer The method then resembles stadia tacheometry with movable hairs, and the sloping distance is similarly obtained. An alternative system, due to Col Tanner, is used in the Indian Survey for traversing It consists in measuring the subtended angle on the horizontal circle of the theodolite, the method of repetition (Vol. II, Chap III) being used to obtain the necessary refinement of angle reading with a vernier instrument. The smallness of the angle makes it possible in most cases to make the repetition by alternately manipulating the lower and upper tangent screws without unclamping

In computing the distance in terms of the known subtense length s and the subtended angle a measured in this way, no reduction to the horizontal is required, since a is a horizontal angle, and we have $H = \frac{1}{2}\, s \cot \frac{a}{2}$. The approximate formulæ, $H = s \cot a$, and $H = \frac{s}{a'' \sin 1''} = \frac{206{,}265\, s}{a''}$, are more convenient and of ample accuracy The elevation of the subtense station may be derived by measuring the vertical angle to the bar In subtense traversing in the Indian Survey, however, it is found economical of time to measure the bearings and vertical angles between stations independently of the observations on the bar, which need not in consequence be plumbed exactly over the station mark

To apply the subtense bar system to contouring, it is sufficient to use a very simple form of bar. The alignment and levelling are simply estimated by the rodman, who holds the bar at a constant height above the ground.

Double Image Tacheometers.—There has been developed on the Continent of recent years a type of tacheometer, called the double image telemeter, of which there are several patterns A horizontal subtense staff is required, and this is supported by a vertical rod, which is also graduated. The optical arrangement of the instrument is such that two horizontal images of the staff are formed, one above the other, and these are seen to be displaced relatively to each other in the direction of their length by an amount proportional to the distance to the staff station The required displacement is effected by means of glass wedges. In a design by Wild the tacheometric angle is formed by deviating the rays from the staff through two similar wedges placed in front of the upper and lower halves, respectively, of the object glass In the Bosshardt-Zeiss* self-reducing instrument, one image of the subtense staff is formed directly by the upper half of the object glass, and the rays traversing the lower half to form the second image are first refracted through two wedges, which are rotated by the tilting of the telescope The tacheometric angle is varied in such a way that 100 times the displacement reading of the images gives the required horizontal distance directly. The reading of the fractional part of the displacement is obtained by means of a vernier on the staff and a rotative parallel glass plate placed in front of the upper half of the object glass.

Unlike the subtense bar method described above, the double image system is intended to be used principally for moderate lengths of sights. The results are of considerable precision, and the method is suitable for large scale surveying, but in work necessitating a large number of sights the use of a horizontal staff is inconvenient. The double image principle can be applied to vertical staff holding only if the telescope is horizontal It can therefore be so used with a level, and in such a case, by adjustment of the prisms, it is possible to read distances over moderately sloping ground.

Relative Merits of Tangential and Stadia Tacheometry.—In comparison with the stadia system, tangential tacheometry labours under the disadvantage that, except in the case of the Szepessy and double image instruments, two manipulations of the instrument and two sights are required for each complete observation. There is always a risk of disturbance of the instrument between the sights, as well as the possibility of a change in atmospheric refraction occurring in the interval Refraction error is least in the case of the horizontal bar method, which is well adapted for long sights, but in the vertical staff methods full advantage cannot be taken of the refinements of reading in the various instruments. Some of these instruments are also liable to instrumental imperfections, since they have not the simplicity of the stadia instrument with non-adjustable hairs. There is little difference between the two systems in the amount of reduction involved. In general, however, the tangential system must be regarded as inferior to stadia work, although comparative tests show that the difference in accuracy is not serious

FIELD WORK

General Arrangement of Field Work.—Tacheometric surveys are chiefly confined to detailed contouring, such as is required in location surveys for public works. The field work consists in locating representative points over the area.

If to the observations for the distance and elevation of a staff station there is added the measurement of its bearing from the instrument, the three co-ordinates thus determined serve to locate the point. By fixing in this manner as many points as are necessary, a survey may be made of the area within the limit of convenient observation from the instrument. Except in the case of very small surveys, it will be necessary to occupy several instrument stations so situated that all the required detail can be located from them. These form stations of a controlling framework, the precision of which governs that of the whole survey.

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* See R. Bosshardt, *Mesure Optique des Distances et Méthode des Coordonnées Polaires*, Geneva, 1930, for a full discussion of double image tacheometry.

When the tract to be surveyed is sufficiently narrow that the half-breadth is within sighting range of the instrument, the survey is controlled by a traverse approximately along the centre line of the strip In difficult ground, subsidiary traverses, closed or unclosed, may have to be run out at intervals to reach parts, such as wooded gorges, which cannot be surveyed from the main stations. Intermediate stations on long traverse courses have frequently to be occupied for the taking of topography. Control from a central traverse is that generally adopted in location surveys for lines of communication, the main traverse roughly following the proposed route.

When the survey is too broad to be based on a single traverse, the control may be furnished either by a trangulation or by a series of traverses. The former is more suitable when the survey embraces a considerable area A principal and a subsidiary system may be required, in which case the latter is of low precision and may be executed by plane table. For moderate areas, traversing is usually adopted. The arrangement may consist of a single main traverse from which numerous circuits are projected, or several approximately parallel traverses may be run out and tied together at intervals.

The measurement of traverse courses may be performed either by tape or tacheometrically, and the elevations of the instrument stations are obtained by ordinary or by tacheometrical levelling according to the degree of accuracy required. For large scale plans it is desirable to use tacheometer methods for locating detail only, but in small scale work they may be used exclusively provided due precautions are taken to limit the propagation of error. It is, however, always advisable to check the levelling at intervals by closures on to bench marks established by ordinary levelling.

Field Party.—A small survey on easy ground can be performed quite well by a surveyor and a staffman, but in rough country their progress is slow. Where long sights have to be taken, there is apt to be uncertainty in the location of the contours, as it is difficult for the surveyor to judge whether the staff is being held on representative salient points.

Large surveys are best executed by a party consisting of : (*a*) the chief of party ; (*b*) the instrument man ; (*c*) the recorder ; (*d*) two to four staffmen, (*e*) one or two labourers for clearing and porterage; (*f*) a draughtsman, if the survey is to be plotted in the field

The *Chief of Party* has to perform the following duties in addition to superintendence

(1) The selection of the instrument stations.

(2) The placing of the staffmen, the most suitable positions being ascertained by reconnoitring the area controlled by the instrument

(3) The preparation of a sketch showing the position of each staff station, the run of the contours relative to those, detailed measurements of buildings, etc A carefully made pictorial sketch is of great value in facilitating the plotting, particularly of rough country

The *Instrument Man* is responsible for the actual observations. The number of staffmen he is able to keep occupied depends upon his expertness,* but to a greater degree upon the time required by the staffmen to change their positions In close contouring over country with considerable detail, the observer may be fully occupied in sighting a single staff On the other hand, he may be able to keep six or eight staffmen employed if the required contour interval is large and the ground is precipitous

The *Recorder* acts as assistant to the instrument man He keeps the field book, entering the readings as they are called out His system of numbering or lettering of points observed must agree with that adopted in the chief's sketches When the instrument man is head of the party, the recorder must prepare the sketches

Staffmen.—When several staffmen are employed, the most experienced should be sent farthest from the instrument A definite code of signals, preferably by whistle, must be arranged, so that each may know when his rod is being sighted and when he is free to take up a new position.

Routine.—Taking the case of a location survey controlled by traversing, the first step is to obtain the elevation of the initial traverse station, A. This may be done either by independent spirit levelling or by tacheometric observations carried from a bench mark In the latter case, the procedure is analogous to that of ordinary levelling except that the alternate backsights and foresights are in general inclined to the horizontal, the elevations being derived from vertical angles and distances. This preliminary levelling is checked either by working back to the bench mark or by adopting the method, to be used in the traverse itself, of observing the difference of elevation between successive points by a backsight and foresight from each In this case the instrument is first set up either over the bench mark or near it.

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* Correspondence in *Engineering Record* for 1914 indicates that a thoroughly experienced observer, assisted by a recorder, can take over 1,000 sights in average country in a day of 8 hours.

When the instrument is set up at station A, the height of the axis above the ground is measured and noted. Orientation is effected in terms of the meridian adopted in the same manner as in ordinary traversing (page 164). The second traverse station, B, having meantime been selected, observations are made for its bearing, distance, and elevation. The instrument man then proceeds to locate such points round A as will enable the detail to be plotted and contours interpolated. The observations to those points are called side shots. In general, the accuracy required in their observation may be much lower than that necessary in the traverse, since errors cannot be propagated through them. It is usually sufficient to read the bearings to the nearest 5′, but, in contouring, the vertical angles should be read to the nearest 1′. The staffmen should be kept as far as possible in straight lines. Time is economised in sighting if they are placed roughly in radial lines from the instrument: mistakes in distance reading are, however, more easily detected if the lines are roughly perpendicular to the traverse courses. When the side shots at station A are completed, the bearing of B is observed anew both to check the previous reading and to test the orientation of the circle.

The instrument is now carried forward and set up at B, while the chief reconnoitres and selects the position of the next station, C. The first observation from B is upon A, not only for orientation, but also for distance and difference of elevation. The length of each traverse course and the elevation of each station are therefore determined twice. These observations should be reduced, and their consistency verified, as soon as possible. Any of the methods of page 166 is available for carrying forward the bearing, and the observations proceed at B as at A.

Notes.—(1) When the traverse distances and levels are obtained independently, it is still desirable to check them tacheometrically as above.

(2) Traverse courses are better kept sufficiently short that the distance can be accurately read. If long courses are introduced, the observations along them must be obtained by dividing the length into sections.

(3) Bearings of traverse courses should be obtained by sighting on poles marking the stations. When poles are not available, the edge of the staff is sighted for bearing.

(4) As in ordinary traversing, no opportunity should be neglected of taking cross bearings, and checks on elevations should also be secured where possible.

(5) The detail taken from an instrument station should not include points which can be better commanded from the succeeding station, but important points may be fixed by sights from both stations.

Note-keeping.—The tabular arrangement of the field book depends upon the system of tacheometry used and upon the character of the survey, but is largely a matter of individual preference. Sufficient columns should be provided for the entry of the results of the field observations and for use in the subsequent reductions. In the case of stadia book-keeping, the number of columns used ranges from 6 to 16.

The accompanying example will serve as a guide. In it the individual stadia hair readings are not entered, the distance reading representing 100 times the actual intercept. The reduced entries are for constants of 100 and 1·2, but details of the reductions are not shown, since the use of a table or diagram is supposed.

Inst Stn and Ht. of Axis	Staff Station.	Distance Reading	Axial Hair Reading	Bearing	Vertical Angle	Horizontal Distance	Vertical Component	Elevation	Remarks
A								261·3	Bearings from
4·2									Mag Mer at A
	B	532	4·2	93° 21′	+ 5° 16′	529	+ 48·7	310·0	
	1	316	4·2	326° 30′	− 3° 43′	316	− 20·5	240·8	Top of ridge
	2	205	8·5	320° 15′	− 6° 12′	204	− 22·1	234·9	Bed of stream
	3	218	4·2	357° 10′	− 0° 36′	219	− 2·3	259·0	Top of ridge

OFFICE WORK

Reduction of Observations.—Reduction formulæ for the various telemetric systems have been given, and, in describing the routine of performing reductions, it will suffice to consider the case of fixed stadia hair observations only

If the constant of the instrument is sensibly different from 100, the first step is to alter the distance reading proportionately before obtaining the horizontal and vertical components The labour of computing the latter quantities is reduced by the use of tables, diagrams, or mechanical aids. Some approximations are also justifiable If, as is usual, distances are required only to the nearest foot, the exact evaluation of $(f+c)\cos\theta$ is unnecessary, and it is commonly taken either as $(f+c)$ or simply as 1 ft In the case of nearly horizontal side shots, the reduction to the horizontal may frequently be omitted, the limits of vertical angle and distance up to which the correction may be ignored depending upon the character of the work It is usual to regard the distance reading as representing the horizontal distance for vertical angles up to 3° and distances not exceeding about 400 ft. If, in conjunction with this approximation, the $(f+c)$ term is ignored altogether, the two errors tend to compensate, and the limits may be extended Elevations may be taken only to the nearest foot, or may be required as close as 0·1 ft. In the former case, $(f+c)\sin\theta$ can be neglected for vertical angles up to about 15°, but in the latter only up to 2°. When the $(f+c)$ term is included, and the reductions are performed by calculation, the use of the approximate formula,

$$V=\left(\frac{fs}{i}+(f+c)\right)\tfrac{1}{2}\sin 2\theta,$$

saves time, and is allowable up to $\theta=15°$.

Reduction Tables.—Reduction by direct calculation is laborious, and the work is greatly facilitated by the use of a reduction table, of which various forms are published A common form gives for every minute of vertical angle the values of $\cos^2\theta$ and $\sin\theta\cos\theta$, or 100 times these values, the quantities being called "horizontal distance" and "difference of elevation" respectively. Such tables may include, for a few values of $(f+c)$, the values of $(f+c)\cos\theta$ and $(f+c)\sin\theta$ for every degree of vertical angle Another type of table gives the correction factor, $\sin^2\theta$, required to reduce the distance reading to the horizontal

Reduction Diagrams.—Observations may be reduced still more rapidly by the use of diagrams Various forms of these are published, but it is a simple matter to construct one on squared paper The accuracy of the reduction depends upon the scale of the diagram, which should be as large as possible

Fig 313 illustrates in outline the simplest form of diagram. It shows both horizontal corrections and vertical components, which are represented clear of each other to avoid confusion The range of 12° of vertical angle, as illustrated, embraces the great majority of observations, but it would be desirable to extend it to 20° or 25°. If this necessitates too small a scale for clearness, the diagram can be continued on a second sheet

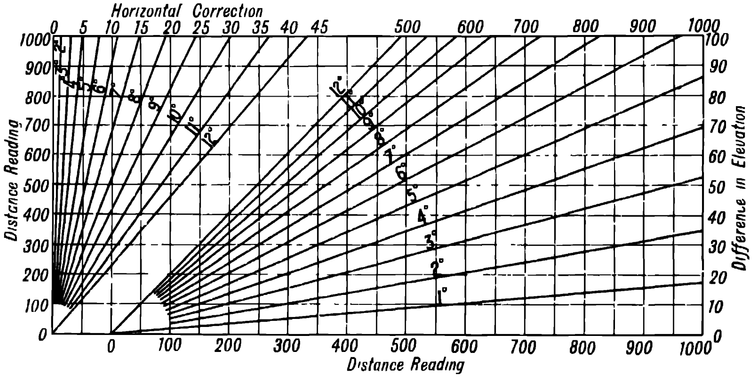

FIG 313.—STADIA REDUCTION DIAGRAM

To prepare the horizontal correction diagram, the scale of distance readings up to 1,000 ft. is set out along the left-hand vertical, and on the horizontal line at 1,000 the values of 1,000 $\sin^2\theta$ are marked off for vertical angles increasing by 10′, or by 5′ if the scale is sufficient. Straight lines joining the points so obtained to the origin give the corrections for other distance readings, since the correction varies as the distance. In constructing the diagram of vertical components, the range of the latter is limited to 100 ft. to keep the scale large Distance readings are set out along the bottom, and on the right-hand vertical are marked off the values of $\frac{1}{2}\sin 2\theta \times 1{,}000$ for every 5′ of vertical angle up to 5° 45′, the points being joined to the origin Thereafter, the positions of the radial lines are obtained by setting off on the top horizontal line the distance readings which give for each angle a vertical component of 100 ft

Fig. 314 illustrates the McCullough diagram, which gives horizontal corrections for all vertical angles On the left-hand and lower margins are scales of staff intercepts. The scales of vertical angles and of the required corrections extend round all four sides The correction corresponding to any vertical angle and staff intercept is obtained by first following along the horizontal or vertical line through the value of the angle until an intersection is reached with the vertical or horizontal line corresponding to the observed intercept. The figure given at either end of the diagonal through this intersection is read, and, according as the vertical angle appears on the left, upper, bottom, or right-hand margin, it is multiplied by ·01, 0·1, 1·0, or 10·0 to give the actual amount to be subtracted from the distance reading.

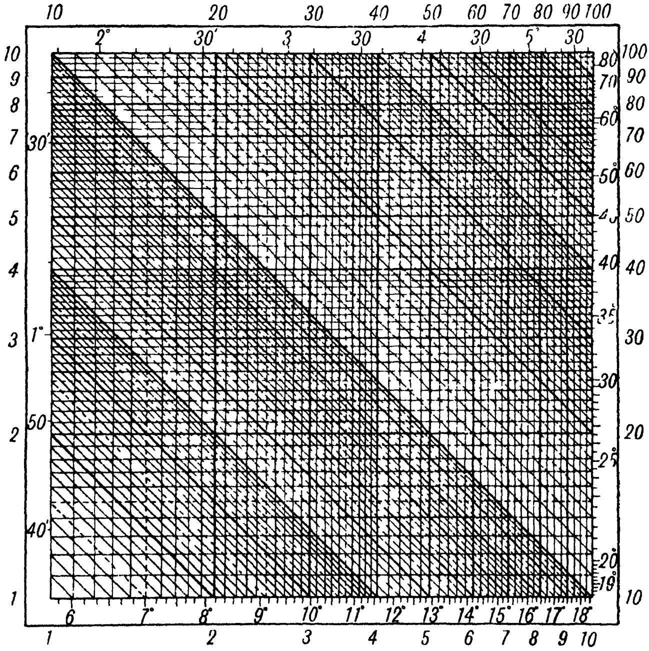

FIG 314 —STADIA REDUCTION DIAGRAM

Reduction by Mechanical Means.—Ordinary stadia observations may be reduced rapidly and with sufficient accuracy by using the stadia slide rule Various forms of these are in use Some are arranged to enable the horizontal and vertical distances to be read directly others give the horizontal correction

For the mechanical reduction of distances for plane table surveying with a tacheometric alidade, Dr Louis* has devised a method whereby the horizontal component may be plotted by the use of proportional dividers The scale on the dividers is one of vertical angles, and is such that when the centre mark is set to an observed vertical angle and the longer legs are opened out to scale the distance reading, the horizontal equivalent is given by the distance between the points of the shorter legs

Plotting.—The framework of triangulation or traverse should, if possible, be completely plotted and checked before any side shots are laid down. If, however, the framework is built up as the survey proceeds, it is preferable to have the plotting of detail keep pace with the field work so that in difficult country the run of the contours may be verified on the ground The main framework is best plotted by co-ordinates, but, when the measurements have been obtained by tacheometer, co-ordinate plotting is sometimes regarded as a refinement, and the protractor is used

In plotting the side shots, the bearings are first marked off by means of a circular protractor The centre is placed at the station point, and the circle is oriented with reference to the framework lines meeting at the station The draughtsman then proceeds to mark off points round the circumference corresponding to the bearings, which should be read out to him. Opposite each point is written the reference number of the side shot. The protractor is then removed, and the distances of the several points are scaled from the station in the directions of the appropriate marks The points so obtained are surrounded with a small circle, and against each is written its elevation Time is saved in plotting side shots by the use of one or other of the several forms of protractor fitted with a radial scale This can be set to any bearing, so that each point may be plotted in one operation. When the side shots are plotted, contours are interpolated to lie appropriately amongst them.

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EXAMPLES

1. A staff is held at distances of 100 ft and 300 ft from the axis of a theodolite fitted with fixed stadia hairs, and the staff intercepts by level sights are 0·99 and 3·00 ft respectively. Determine the constants of the instrument, and calculate the horizontal distance of the staff from the instrument when the intercept is 5·00 ft. if, in this case, the telescope is inclined at 10° to the horizontal, the staff being held vertically.

2 In the course of a tacheometrical survey the following observations were made for the normal cross-section of a stream, the instrument being set up on one bank with the telescope level:

Staff Point	Readings			Remarks.
1	4·62,	4·76,	4·90	edge of water
2	6·06,	6·27,	6·48	
3	7·25,	7·63,	8·01	
4	8·30,	8·82,	9·34	
5	8·36,	8·96,	9·57	
6	7·31,	8·05,	8·80	
7	5·67,	6·54,	7·41	
8	3·69,	4·75,	5·81	edge of water

The telescope was fitted with an anallatic lens, the value of the constant being 100. Compute the cross-sectional area of the flow at the time of the observations.

3. A tacheometer has a diaphragm with three crosshairs spaced at distances apart of 1/40 inch.

The focal length of the object glass is 9 inches and the distance from the object glass to the trunnion axis is 4½ inches. A staff is held vertically at a point the level of which is 80 feet A D. The telescope is inclined at 9 degrees to the horizontal, and the readings taken on the staff are 6·65 feet, 5·14 feet, and 3·63 feet.

Find the distance of the point from the telescope and the level at the telescope The height of the trunnion axis of the telescope is 4′ 6″. (Univ. of Lond, 1918.)

4. Find up to what vertical angle sloping distances may be taken as horizontal distances in stadia work, so that the error may not exceed 1 in 300. Assume that the instrument is fitted with an anallatic lens and that the staff is held vertically.

5. The cross and stadia hairs of a theodolite were replaced by a surveyor in the field after a breakage. The necessary instrumental adjustments having

been made, the following observations for the determination of the multiplying tacheometric constant were taken upon a vertical staff held at measured distances from the instrument :

Observation	Horizontal dist to staff in feet	Vertical angle	Stadia Hair Readings	
1	200	0	2·86	4·76
2	400	+ 1° 3′	4·00	7·83
3	400	+ 1° 12′	5·05	8·87
4	600	+ 1° 36′	4·00	9·76

Obtain the mean value of the constant given by these observations, that of the additive constant being known to be 1·2 ft (T.C D , 1930)

6 A theodolite fitted with stadia wires and having an additive constant of 15 inches is used for contouring At the first station the height of the instrument was 4 72 ft , and the following readings were taken :

Staff Station	Top Wire	Middle Wire	Bottom Wire.	Vertical Angle	Remarks
1	1·50	3·45	5·40	5° 14′	Theodolite set up over B M 223·50 ft above datum.
2	2·87	4·23	5·60	7° 23′	
3	3·95	5·96	8·00	8° 12′	

Calculate the heights of the staff at the stations above datum. Take the multiplying constant as 100. The staff is held vertically in each case. (Univ. of Lond., 1919.)

7. A tacheometer is set up at an intermediate point on a traverse course AB, and the following observations are made on a vertically held staff.

Staff Station	Intercept	Axial Hair	Bearing	Vertical Angle.
A	7·42	6·71	218° 37′	− 5° 30′
B	6·80	6·40	38° 37′	− 6° 20′

The instrument is fitted with an anallatic lens, and the constant is 100. Compute the length of AB and the reduced level of B, that of A being 226·8. (R.T.C., 1921.)

8. Observations taken by means of a theodolite fitted with stadia hairs gave staff intercepts of 1·95 and 4·89 with the telescope horizontal and the staff held vertically at horizontal distances of 200 feet and 500 feet respectively from the centre of the instrument. The instrument was then set over a station having a reduced level of 264 3, the height of the instrument axis above the station point being 4·0 feet. The readings of the three hairs on a vertical staff were 3·00, 6·21, and 9·42, the telescope being inclined at 8° below the horizontal. Calculate the reduced level of the staff station and its distance from the centre of the instrument.

9. The elevation of a point *X* is to be determined by observations from two adjacent stations of a tacheometrical survey. The staff was held vertically upon the point, and the instrument constant was 100. Compute the required elevation from the following data, taking the two observations as equally trustworthy :

Inst. Stn.	Ht. of Inst. above Stn	Elevn. of Stn.	Staff Point.	Vertical Angle.	Axial Hair Reading.	Stadia Intercept.
G	3·8	490·9	*X*	+ 2° 27′	7·56	8·12
H	4·1	582·1	*X*	− 4° 51′	6·93	6·87

(T.C.D., 1929.)

10 The elevation of a peg marking a survey station *A* is 334·6 ft. A tacheometer fitted with an anallatic lens, and having a constant of 100, is set over the station so that the line of sight is 4·1 ft above the peg An observation is taken on a vertically held staff with the line of sight directed downwards at 7° 45′ below the horizontal, and the observed readings of the three hairs are respectively 4·50, 7·37, and 10·24 The staff remains upon the same point while the instrument is transferred to station *B* A backsight taken on the staff with an angle of elevation of 9° 30′ gives readings of 3·00, 5·56, and 8·13 Calculate the elevation of station *B* if the height of the line of sight above the station peg is 3·6 ft.

11. Determine the gradient from a point A to a point B from the following observations made with a fixed hair tacheometer fitted with an anallatic lens, the constant of the instrument being 100 .

	Bearing	Readings of Stadia Hairs		Reading of Axial Hair	Vertical Angle.
To A,	345°	3·00	8·48	5·74	+ 15°
To B,	75°	2·50	12·20	7·35	+ 10°

(R.T.C., 1914)

12. The following notes refer to three of the observations in a tacheometric survey The elevation of the instrument station was 639·4, the trunnion axis of the telescope having been at 4·7 ft above the station.

Staff Station	Bearing	Vertical Angle.	Hair Readings		
1	97° 45′	− 6° 12′	3·19	4·70	6·21
2	103° 15′	0° 0′	7·00	8·92	10·84
3	105° 50′	+ 10° 42′	4·00	6·46	8·92

The instrument was fitted with an anallatic lens, the value of the constant being 100, and the staff was held *normal* to the line of sight. Find the horizontal distances to the staff points and their elevations.

13 Levels were carried from a bench mark to the first station *A* of a tacheometric survey by tacheometric observations. The instrument was fitted with an anallatic lens, and the value of the constant was 100. The following observations were made, the staff having been held vertically :

Inst. Stn.	Ht of Axis	Staff Point.	Vert. Angle.	Staff Readings.	Elevn.
1	4·4	O B. M.	− 2° 20′	4·00, 6·11, 8·22	206·3
		Change Pt.	+ 4° 36′	3·50, 5·07, 6·64	
A	3·8	do.	− 5° 12′	4·00, 5·90, 7·81	

Compute the elevation of station *A*. (T.C.D., 1929.)

14. A traverse survey is run along the foot of a piece of precipitous ground and the distance between two points A and B on the rough ground is to b obtained by observations from the traverse. A being visible from one of th traverse stations, X, and B from another station, Y, tacheometrical observa tions from these stations are taken upon a vertically held staff at A and B, a follows :

Traverse Station	Total Co-ordinates of Station		Staff Point	Bearing	Vertical Angle (Elevn)	Hair Readings
	Latitude.	Departure				
X	1,075·6 N.	2,843·2 E.	A	336° 42′	18° 23′	5·00, 7·12, 9·24
Y	839·3 N.	3,609·5 E.	B	12° 27′	15° 16′	6·00, 7·95, 9·91

The tacheometer is provided with an anallatic lens, and the instrumen constant is 100. Calculate the distance AB. (T.C.D.. 1927.)

15. Calculate the bearing and distance from A to D from the following notes of a traverse run by tacheometer The telescope was fitted with an internal focussing lens, the constant being 100, and the staff was held vertically.

Line	Bearing.	Vertical Angle	Staff Readings
AB	360° 0′	+ 5° 11′	3·00, 6·24, 9·48
BC	289° 54′	+ 2° 30′	3·13, 5·59, 8·05
CD	201° 12′	− 2° 43′	2·97, 6·49, 10·02

16. The following notes refer to a tacheometrical traverse run from a station A to a station E, the horizontal distance between which is known to be 2,465·7 ft. Taking this value as correct, determine the constant of the instrument, an anallatic lens being fitted in the telescope, and the staff having been held vertically.

Line.	Bearing.	Vertical Angle	Stadia Intercept.
AB	360° 0′	0	6·39
BA	180° 0′	0	6·39
BC	29° 36′	+ 2° 27′	5·12
CB	209° 36′	− 2° 25′	5·10
CD	350° 8′	− 1° 48′	7·01
DC	170° 8′	+ 1° 49′	7·02
DE	7° 54′	− 3° 12′	7·14
ED	187° 54′	+ 3° 12′	7·13

17 The stadia intercept read by means of a fixed hair instrument on a vertically held staff is 3·73 ft, the angle of elevation being 6° 27′. The instrument constants are 100 and 1·1. What would be the total number of turns registered on a movable hair instrument at the same station for a 5 ft intercept on a staff held on the same point, the vertical angle in this case being 6° 21′ and the constants 1000 and 1·4 ?

18. A staff held vertical is observed with an ordinary theodolite, and the angles of elevation from the horizontal of two well-defined lines on the face of the staff, which are 10 ft. apart, are observed. What is the horizontal distance of the staff from the axis of rotation of the telescope, when the difference of the tangents of the angles of elevation is 0·033 ? (Inst. C.E , 1905)

19. Two observations are taken upon a vertical staff by means of a theodolite, of which the reduced level of the trunnion axis is 154·3. In the case of the first the line of sight is directed to give a staff reading of 3·00, and the angle of elevation is 4° 58′. In the second observation the staff reading is 11·00, and the angle of elevation is 5° 44′. Compute the reduced level of the staff station and its horizontal distance from the instrument.

20. A surveyor, about to take a series of cross sections on rough ground by means of a theodolite, wishes to obtain his height of instrument by observing a staff held upon a bench mark which is at a lower level than the instrument. He takes two observations on the staff, the readings being 9·74 and 3·26, and the corresponding angles of depression 9° 20′ and 10° 45′. Calculate the elevation of the instrument if that of the bench mark is 193·5. (T.C D., 1925.)

21. An observation with a percentage theodolite gave staff readings of 3·12 and 7·86 ft. for angles of elevation of 4% and 5% respectively. On sighting the graduation corresponding to the height of the instrument axis above the ground, the vertical angle was 4·26%. Compute the horizontal distance and elevation of the staff station if the instrument station has an elevation of 213·1.

22. The horizontal angle subtended at a theodolite by a subtense bar with vanes 10 ft. apart is 18′. Compute the horizontal distance between the instrument and the bar.

Deduce the error of horizontal distance if the bar were 1° from being normal to the line joining the instrument and bar stations.

CHAPTER X

HYDROGRAPHICAL SURVEYING

HYDROGRAPHICAL surveying is that branch of surveying which deals with bodies of water. It embraces all surveys made for the determination of water areas, volumes, and levels, rate of flow, and the form and characteristics of underwater surfaces. The usual methods of applying the fundamental principles of surveying and levelling have to be adapted to the conditions, and some of the operations and apparatus are of a specialised character.

Scope.—Extensive hydrographical surveys, directed to the preparation of charts for the use of navigators, are undertaken by various countries Such surveys are conducted from specially equipped surveying vessels, and comprise the determination of depths available for shipping, the survey of currents, the location of shoals and other dangers, buoys, anchorages, and lights, as well as the mapping of conspicuous land features which will guide the navigator An examination of the operations forming the routine on board a surveying vessel is outside the scope of this volume,'but the civil engineer uses similar methods on a smaller scale in connection with the design and maintenance of certain classes of works. The relationship of hydrographical surveying to those branches of engineering which deal with harbours, docks, navigable waterways, coast protection, etc., is evident ; but the application of the subject to civil engineering is wider than might at first sight appear, since works connected with water supply, water power, irrigation, sewage disposal, flood control, land reclamation, viaducts, and river works generally, also involve the practice of hydrographical surveying.

That part of the subject which relates to the measurement of the discharge of rivers and streams, as required in water supply and similar projects, is dealt with in the latter part of the chapter. The more general branches of the subject may be classed as marine surveying, but they are just as applicable to inland waters as to the sea.

THE TIDES

The marine surveyor has to make such frequent reference to tidal movements that some knowledge of their general characteristics is indispensable. Detailed treatment of the subject is outwith the scope of this work, and the reader is referred to the bibliography on page 469, but a brief outline of the simpler phenomena is offered to explain the significance of some of the terms used in connection with the tides.

Tidal Theory.—Tides are periodical variations in the level of a water surface due to the attraction of celestial bodies The sun and moon are the principal tide-producing agents

No theory adequately explains all the phenomena of the tides. The most useful working hypothesis is the equilibrium theory, due to Newton, which forms the basis on which all subsequent work has been founded This is a static theory which interprets the fundamental causes and character of the more general phenomena, but does not afford quantitative results regarding the range nor time of the movements at a place.

Equilibrium Theory.—Newton's law of universal gravitation states that every body in the universe attracts every other body with a force acting in the straight line between the bodies and of magnitude proportional to the product of their masses and inversely proportional to the square of the distance between them In applying this law in the equilibrium theory, the chief assumptions made are (1) that the earth is completely enveloped by an ocean of uniform depth ; (2) that the inertia and viscosity of the water and its attraction for parts of itself are negligible, so that the ocean may be supposed capable of assuming instantaneously the figure of equilibrium required by the tide-producing forces

Tide-producing Force.—The moon is the principal tide generator, and the nature of tide-producing force is best followed by assuming that the earth and moon are the only two bodies in existence. These attract each other, the resultant force acting along EM (Fig 315), the straight line joining their centres Their separate positions are maintained by the earth and moon revolving monthly about G, their common centre of gravity Let this be the only motion taking place, the daily rotation of the earth on its axis being supposed annulled since it exercises no influence on tide-producing force.

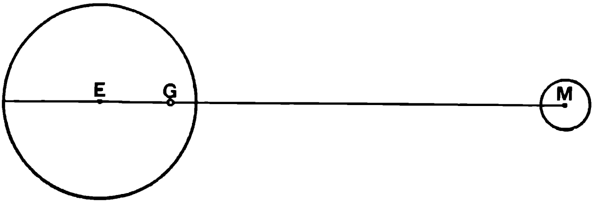


FIG. 315.

In consequence of the earth's revolution about G without rotation, all particles on the earth are subjected to centrifugal force of uniform intensity, and the directions of these forces are parallel to the line joining the centres of the earth and moon at any instant (Fig 316) While the total centrifugal force thus generated is balanced by the moon's attraction, the attractive forces are not uniformly distributed over the earth Since attraction is inversely proportional to the square of the distance, particles at A are subject to a greater force, and those at B to a smaller force, than the mean, the distribution being represented diagrammatically in Fig 317. On adding the two systems vectorially, the residual forces of Fig 318 are obtained These locally unbalanced forces are the tide-producing forces due to the moon

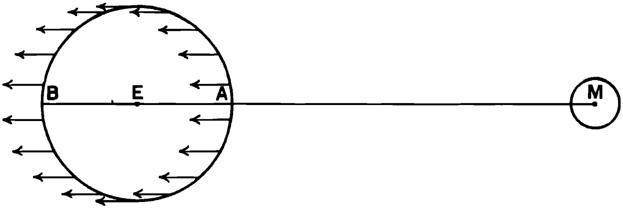


Fig 316

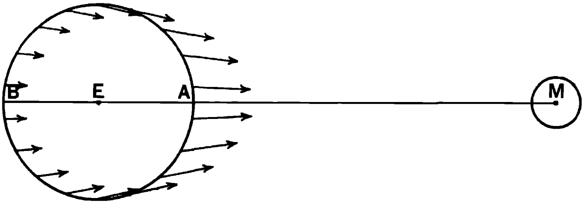


Fig 317

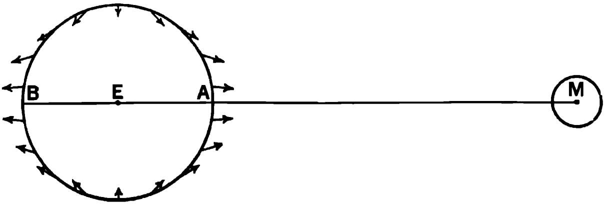


Fig. 318.

Equilibrium Figure.—In conformity with the assumption that the ocean is devoid of inertia and viscosity, it will assume under the action of these forces the equilibrium figure of a prolate ellipsoid of revolution with the major axis directed to the moon (Fig 319). Two lunar tides are thus simultaneously produced. That at A, under the moon, is termed the superior lunar tide, or tide of the moon's upper transit; that at B, the inferior or anti-lunar tide, or tide of the moon's lower transit. Low water simultaneously occurs at C and D.

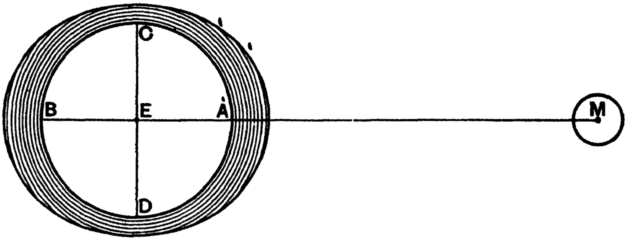


FIG 319.

Tidal Day.—To explain the recurrence of high and low water at a place, consideration must be given to the effect of the earth's rotation on its axis Let Fig 319 represent a section in the plane of the earth's equator, the moon being assumed in that plane As the earth rotates, a point such as A occupies the successive positions C, B, and D at intervals of 6 h , and returns to A in 24 h If the moon remained stationary, the major axis of the lunar tidal figure would maintain a constant position, and A would experience during a rotation a regular variation in the level of the sea, and would encounter high water at perfectly regular intervals of 12 h , with low water midway between

The effect of the moon's motion, assumed in the plane of the equator, may now be added In a lunation of 29·53 days the moon makes one revolution relative to the sun, *i e* from new moon to new moon This revolution is in the same direction as the diurnal rotation of the earth, from west to east, so that there occur 28·53 transits of the moon across a meridian during a period of 29·53 mean solar days The interval between successive transits, or the length of the lunar day, is therefore on an average 24 h 50 5 m The axis of the lunar tide figure maintains its direction towards the moon, and therefore passes through a point such as A every 12 h 25 m , which is the average interval between successive high waters

Solar Tides.—In the same manner, the sun, acting alone, generates a superior solar tide and an inferior or anti-solar tide. The sun, however, is a less powerful tide-producer than the moon. The tide-producing force exerted by a celestial body may be measured by the excess in attraction at A (Fig 319) over the mean, the deficit at B from the mean being of practically the same amount

Let E, M, and S = the masses of earth, moon, and sun respectively,
m and s = the mean distances from the centre of the earth to that of the moon and sun respectively,
r = the radius of the earth,
k = the constant of gravitation.

The tide-producing force of the moon on unit mass at A

$$= kM\left(\frac{1}{(m-r)^2} - \frac{1}{m^2}\right)$$

$$= kM\left(\frac{2r}{m^3}\right) \text{ to a first approximation.}$$

i e Differential attraction or tide-producing force is proportional to the mass of the attracting body, and is inversely proportional to the *cube* of the distance We therefore have

$$\frac{\text{Sun's tide-producing force}}{\text{Moon's tide-producing force}} = \frac{Sm^3}{Ms^3}.$$

Substituting $S = 331{,}000\ E$,

$$M = \frac{1}{81}E,$$

$$s = 92{,}800{,}000 \text{ miles},$$

$$m = 239{,}000 \text{ miles},$$

$$\text{Solar tide} = {\cdot}458 \text{ lunar tide.}$$

Since the moon and sun are acting simultaneously, the lunar and solar tides are superimposed, and the ocean assumes an equilibrium figure in obedience to the combined tide-producing forces.

Spring and Neap Tides.—At new moon the sun and moon have the same celestial longitude, and cross a meridian of the earth at the same instant The three bodies are in one plane, and the sun and moon are on the same side of the earth, the moon being said to be in conjunction (Fig 320) The crests of the lunar and solar tide waves coincide, and the high water level of the resulting tide is above the average, and its low water level below the average The tide is called *Spring Tide of New Moon*

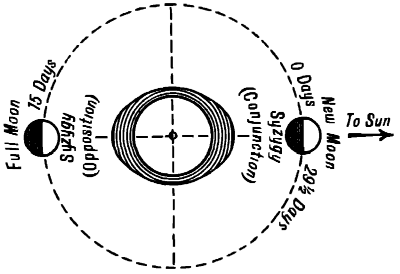


FIG 320 —SPRING TIDES

Thereafter, the moon falls behind the sun, and crosses the meridian about 50 minutes later each day In about 7½ days the moon is in quadrature, its elongation, or the difference between its longitude and that of the sun, being 90° (Fig. 321) The crest of the lunar tide then coincides with the trough of the solar tide High water level is below the average, and low water level above the average, the tide being called *Neap Tide of the First Quarter*

Full moon occurs at about 15 days from the start of the lunation. The moon's elongation is 180°, and the earth, moon, and sun are again in the same plane, but the moon is in opposition (Fig 320) The moon and sun again act together, and the variation in sea-level is great, the tide being *Spring Tide of Full Moon*

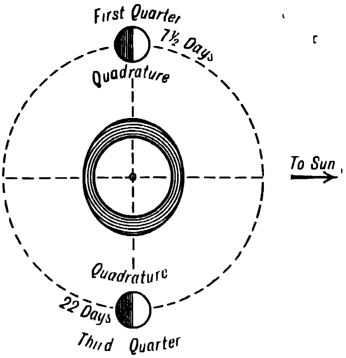


FIG 321 —NEAP TIDES

In about 22 days, the moon reaches quadrature with an elongation of 270°, and the conditions for small tides are repeated (Fig 321) The tide is *Neap Tide of the Third Quarter*

Finally, in about 29½ days from the previous new moon, the moon returns to the meridian of the sun, and the cycle is recommenced with the *Spring Tide of New Moon*

Priming and Lagging.—Besides giving rise to variation in the tidal range, varying relative positions of the sun and moon affect the regularity with which high water recurs at a place, so that the tidal day is not of constant length The tidal figure at any time consists of the superimposed lunar and solar tides, and, since the moon is the more powerful agent, the crest of the composite tide lies nearer the meridian of the moon than that of the sun It is therefore convenient to refer the time of high water to that of the moon's upper or lower transit When high water occurs at a place before the moon's transit, the interval between successive high waters is less than the average, and the tide is said to prime otherwise, it is said to lag

At new moon the crest of the composite tide is under the moon, and there is a normal tide without priming or lagging Between new moon and first quarter the axis of the resultant tidal figure lies as in Fig 322 The place A experiences high water before coming under the moon, *i.e* before the moon crosses the meridian of A, and the tide primes At quadrature the crest of the composite tide is again under the moon, and the tide is normal Between

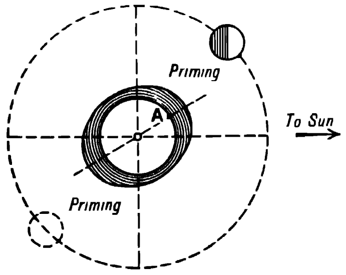


FIG 322

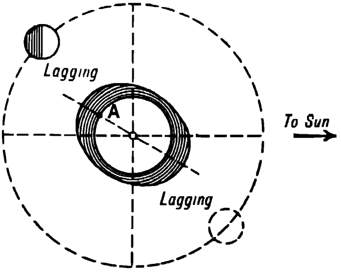


FIG 323

first quarter and full moon the conditions are as in Fig 323 A does not encounter high water until some time after it has passed under the moon, and the tide lags Normal tide prevails at full moon, and again at the third quarter. The tide primes in the third quadrant of the lunation, and lags during the fourth

The effects of varying relative positions of the sun and moon may be summarised as follows .

Moon in	Moon's Elongation	Moon's Phase	Moon's Approx Age Days	Tide	Range	Tide
Syzygy	0°	New	0	Spring	Great	Normal
	0°–90°				Decreasing	Priming
Quadrature	90°	1st Quarter	7½	Neap	Small	Normal
	90°–180°				Increasing	Lagging
Syzygy	180°	Full	15	Spring	Great	Normal
	180°–270°				Decreasing	Priming
Quadrature	270°	3rd Quarter	22	Neap	Small	Normal
	270°–360°				Increasing	Lagging
Syzygy	360°	New	29½	Spring	Great	Normal

Further Sources of Inequality.—Several other factors affect the range as well as the interval which elapses between successive high waters at a place The more important of these are the ellipticity of the orbits of the moon and the earth and the varying declination of the moon and sun

Effect of Ellipticity of Orbits of Moon and Earth.—The lunar orbit is not circular, as has been tacitly assumed, but is an ellipse with an eccentricity of ·055 The moon is said to be in perigee when nearest the earth and in apogee when most remote, the ratio between the least and greatest distances being $\frac{1-\cdot055}{1+\cdot055}$ Since tide-producing force is inversely proportional to the cube of the distance of the attracting body,

$$\frac{\text{range of perigean tide}}{\text{mean range}}=\left(\frac{1}{1-\cdot055}\right)^3=1\cdot17,$$

and

$$\frac{\text{range of apogean tide}}{\text{mean range}}=\left(\frac{1}{1+\cdot055}\right)^3=0\cdot84.$$

This disturbance has a period, between successive perigees, of the anomalistic month of 27·55 days, so that it does not bear a constant relation to the succession of spring and neap tides The tides, whether spring or neap, are increased at perigee and decreased at apogee by the superimposition of this " lunar elliptic " tide.

The distance between the sun and the earth likewise varies on account of the earth's orbit being an ellipse. The sun is in perigee on Dec. 31st and in apogee on July 1st, the period of the tidal disturbance being the anomalistic year of 365·26 days. The resulting inequality is small, since the eccentricity of the orbit is only ·0166, so that

$$\frac{\text{perigean range}}{\text{mean range}} = \left(\frac{1}{1-\cdot 0166}\right)^3 = 1\cdot 05,$$

and

$$\frac{\text{apogean range}}{\text{mean range}} = \left(\frac{1}{1+\cdot 0166}\right)^3 = 95$$

Effect of Declination.—The sun and moon have been assumed to remain in the plane of the earth's equator, but both bodies periodically vary their position on either side of the equator, or have declination. The sun's declination (Vol. II, Chap. I) attains a maximum value of 23° 27′ N. or S. at the solstices, and is zero at the equinoxes, the period of the motion being the tropical year of 365·24 days. The plane of the moon's orbit is inclined to that of the ecliptic at a mean angle of about 5° 8′, and the moon undergoes a periodic variation in declination on either side of the equator during the tropical month of 27·32 days, or time of the moon's revolution relatively to the First Point of *Aries* (Vol. II, Chap. I). The declination is N. for half of the month, and S. for the other half. The maximum N. or S. declination attained ranges from 23° 27′ + 5° 8′ = 28° 35′ to 23° 27′ − 5° 8′ = 18° 19′ in cycles of about 19 years.

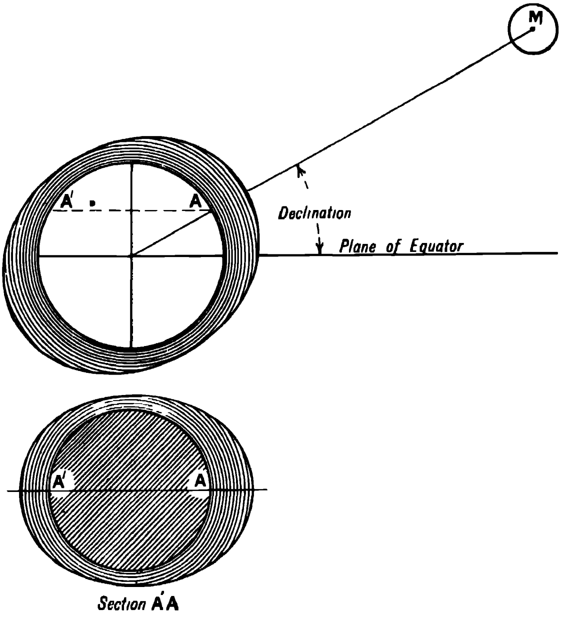


FIG. 324.

The effects produced by solar and lunar declination are similar, the latter being the more marked Fig. 324 illustrates a lunar declinational tide, the axis being towards the moon and therefore inclined to the equator At places on the same side of the equator as the attracting body the superior tide is greater than the inferior, the reverse being the case for places on the other side of the equator The conditions during a tidal day at the place A are exhibited on the section through A by a plane parallel to the equator. The inferior crest is encountered at A′, and the level of high water is then sensibly less than that of the previous high water. This phenomenon, termed *Diurnal Inequality*, vanishes when the attracting body is on the equator

Discrepancies between Theory and Observation.—In view of the assumptions involved in the equilibrium theory, close agreement between the results of theory and those of observation is not to be expected The ocean is assumed to be able to arrange itself in obedience to the tide-producing forces, but it is not explained how the adjustment is effected In reality, the time available for the formation of an equilibrium figure is insufficient on account of the inertia of the water, and, in consequence, the lunar and solar tides at a place do not occur at the instants of transit of the moon and sun The average length of the tidal day does, however, agree with that of the lunar day, and spring and neap tides occur at about the times of syzygy and quadrature respectively Inequalities due to declination and varying distance of the attracting bodies can also be traced

The theory does not accord with observation as to the amount of tidal movement On the assumption that the ocean completely envelops the earth, it can be shown that the greatest range of tide would be about 3 ft , but very much greater ranges than this actually occur The average range experienced at a place depends upon the form of the coast lines and the depth of the adjoining seas. On account of the irregularity of the obstructing land masses, the tide is heaped up in places, while the range is further increased by the attraction of the ocean for parts of itself and by the momentum of the water carrying it beyond the equilibrium position In consequence, the prediction of the tides at a place must be based largely on observation

Similar difficulties due to the influence of local conditions are encountered in the theories classed as dynamic or kinetic, which investigate the characteristics of the tidal oscillation These need not be detailed, and we may pass to a consideration of the main characteristics of actual tidal phenomena.

The Primary Tide Wave.—The Southern Ocean, extending southwards from about 40° S latitude, is the only great body of water which encircles the earth. It is free from large obstructions to the development of the equilibrium figure, except where it is restricted in width by the southern part of South America. In consequence, it is the only ocean in which the tides approximately obey the tide-producing forces of the sun and moon From the primary tide wave generated there, derivative waves are propagated into the Pacific, Atlantic, and Indian Oceans

Derivative Waves.—These waves proceed in a general north and south direction in the open oceans, and consequently are only indirectly traceable to the tide-producing forces of the sun and moon The direction of propagation is greatly influenced by the form of coast lines and the intervention of land masses The velocity of wave travel is great over areas of deep water, and may exceed 600 miles per hour, while the amplitude, or vertical range from crest to trough, is then not more than 2 to 3 ft The velocity is greatly reduced in shallow water and by contact with coasts. When the wave has to pass through shallow water, or is contracted in width, the water is heaped up on account of its momentum, and the range between high and low water levels is much increased The true wave motion may then assume the character of a wave of translation.

Age of the Tide.—On account of the direction of propagation of the tide wave, high or low water occurs at different times at various places on the same meridian If the assumptions of the simple equilibrium theory were fulfilled, the spring tide of new or full moon would occur at each place at the instant of the noon or midnight transit of the moon. Actually, the greatest spring tide arrives several tides after the transit at new or full moon The interval is called the age of the tide, and represents the time which elapses between the generation of the spring tide and its arrival at the place. It is obtained as the mean of several observations, and is usually reckoned to the nearest $\frac{1}{4}$ day Its value varies for different places up to a maximum of about 3 days

The progress of the tide wave and the varying age of the tide may be shown by means of a chart of co-tidal lines These are lines drawn through all places having high water at the same time Their form and distance apart exhibit the path of the wave and its varying speed

In the case of the British Isles, the age of the tide on arrival at Land's End and the west of Ireland is nearly $1\frac{1}{4}$ days. The wave divides, one branch running up the English Channel, and the other along Ireland and the west coast of Scotland, sweeping round the north of Scotland into the North Sea The former wave passes through the Strait of Dover with an age of $1\frac{1}{2}$ days. The latter attains that age off the north-east coast of Scotland in its passage down the North Sea, and finally, when the age is 2 days, it encounters off Harwich the succeeding tide from the English Channel.

Lunitidal Interval.—The interval of time which elapses between the moon's transit and the occurrence of the next high water at a place is called a lunitidal interval. If the times of several consecutive tides are observed, and are referred to the times of the moon's upper and lower transits, the value of the lunitidal interval will be found to vary because of the existence of priming and lagging The required local times of transit are derived from the times of transit at Greenwich given in the *Nautical Almanac* by applying 2m for every hour of longitude, adding if the place is west, and subtracting if east, of Greenwich

On plotting the lunitidal intervals for a fortnight on a base of times of the moon's transits, a curve similar to Fig. 325 is obtained. This curve of semi-mensual inequality of time has approximately the same form for each fortnight, and may be used for the purpose of rough predictions of the occurrence of high water at the place.

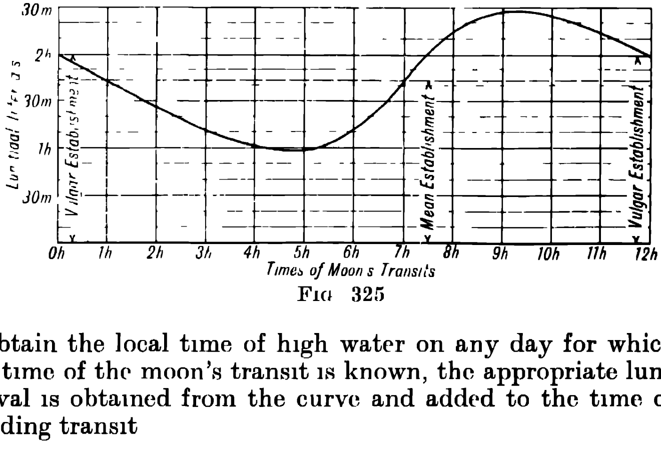


FIG 325

To obtain the local time of high water on any day for which the local time of the moon's transit is known, the appropriate lunitidal interval is obtained from the curve and added to the time of the preceding transit

Mean Establishment.—The mean establishment of a place is the average value of all its lunitidal intervals

When the mean establishment is known, a rough prediction of the time of high water may be made by deducing the lunitidal interval for that day from the mean establishment This involves the application to the establishment of a correction representing the amount of priming or lagging at the generation of the tide, the evaluation of which requires an approximate knowledge of the age of the tide at the place

The priming and lagging correction to mean establishment is as follows

Hour of Moon's Transit	0	1	2	3	4	5	6	7	8	9	10	11	12
Correction in Minutes	0	- 16	- 31	- 41	- 44	- 31	0	+ 31	+ 44	+ 41	+ 31	+ 16	0

To illustrate the manner of applying the correction, let it be required to find the time of afternoon high water at a place at which the mean establishment is 4 h 12 m and the age of the tide $1\frac{1}{2}$ days, if the moon transits at 5 h 46 m p.m on the day in question

Since the moon falls behind the sun by about 50 m in 24 h , at the birth of the tide, $1\frac{1}{2}$ days before, the moon crossed the meridian at which the wave was generated $1\frac{1}{2}\times 50$ m. = 1 h 15 m earlier, *i e* at 4 h 31 m

By interpolation from the table, the priming correction corresponding to 4 h 31 m is 37 m , so that

		h.	m.
Required lunitidal interval = 4 h. 12 m − 37 m	=	3	35
Time of moon's transit	=	5	46 p m
Time of high water	=	9	21 p m

The results given by this method of prediction are only approximate, since it takes account only of the moon's phase and makes no allowance for variations in declination or distance of the moon and sun At places with marked diurnal inequality the estimates may be seriously wrong, but otherwise phase prediction is sufficiently accurate for many purposes

Vulgar Establishment.—The value of the lunitidal interval on the day of full or change of the moon is called the vulgar establishment, or simply the establishment, of the place It is approximately the clock time at which high water occurs on those days It exceeds the mean establishment by an amount depending upon the age of the tide at the place, since the parent tide, generated a day or two previously, was in the second or fourth quadrant and therefore lagging The value of the establishment for nearly all ports and anchorages is published in the Admiralty Tide Tables under the name " High Water, Full and Change "

Mean establishment can be derived from vulgar establishment if the age of the tide is known, and the lunitidal interval for any day can then be obtained as before Thus, if the vulgar establishment of a place is 4 h 32 m , and the age of the tide there is $1\frac{1}{2}$ days, then at the generation of the tide the moon crossed the meridian at 12 h $-1\frac{1}{2}\times 50$ m $=10$ h 45 m The lagging corresponding to this hour of transit is, from the table, 20 m. The mean establishment is therefore 4 h. 32 m -20 m = 4 h 12 m

Height of Tides.—The rise of a tide is the vertical distance of the high water level above a fixed reference surface. The datum surface in general use is that of low water of ordinary spring tides for the locality

The range of a tide is the vertical distance from low water level to the succeeding high water level

Observation of the high and low water levels of successive tides at a place shows the gradual diminution in rise and range from spring to neap tides (Fig 326) Such a diagram could be used in conjunction with phase prediction of times to make rough predictions of heights.

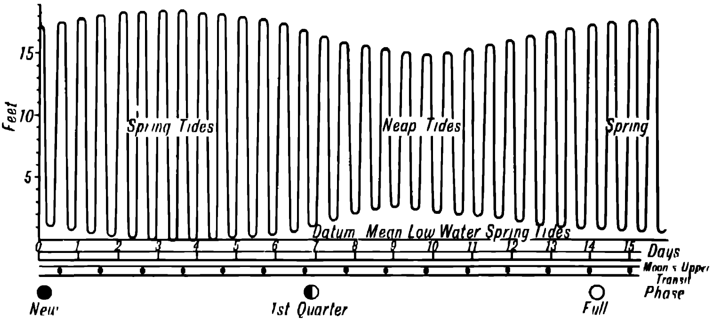


FIG 326

Rate of Variation of Level.—If the level of the water is observed every 15 or 30 m, it will be found that the rate of variation of height is small at high or low water and greatest at half-tide. On plotting the heights on a time base, the resulting tide curve is found on an average to approximate to the form of a harmonic curve.

For many purposes it is useful to be able to ascertain, in default of regular gauge readings, the approximate height of a tide of known rise or range at any time between high and low water

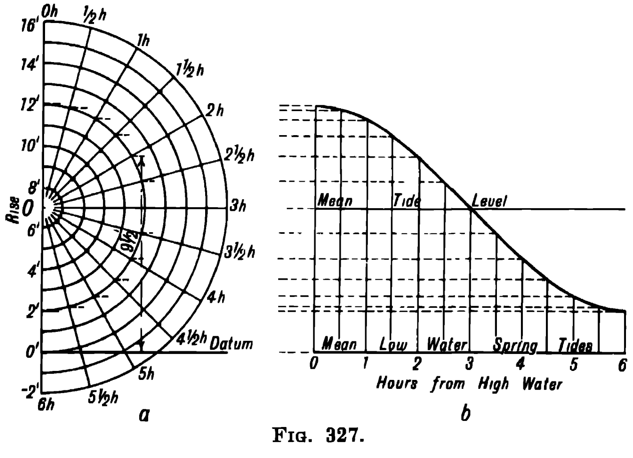


FIG. 327.

This is conveniently obtained from a diagram (Fig 327*a*) constructed as follows.

Draw a line to represent mean low water level of spring tides or other datum adopted. At the appropriate height above datum scale off a point O on mean tide level. With centre O, construct a series of semicircles of radii representing in whole feet the semi-ranges of all tides likely to occur at the place Divide the circumference of one of the semicircles into as many equal parts as there are half-hours between high and low water If the place experiences normal tides, the average interval between high and low water is 6 h 12 m , and for the present purpose the interval may be assumed to have a constant value of 6 h Draw radii through the points so obtained, and designate them as shown The elevation at any time of a tide of known range is approximately given by the ordinate from the datum to the point of intersection of the radius representing the time with the semicircle corresponding to the range.

In the example illustrated, the spring tide rise is taken as 14 ft To ascertain the approximate height at 2 h from high water of a tide having a rise of 12 ft , the height of the intersection of the radius marked 2 h with the semicircle through 12 ft is read, the result being $9\frac{1}{2}$ ft above datum The construction of the harmonic curve for this tide is shown at *b* (Fig 327)

The same results are obtained analytically from

$$H = h + \tfrac{1}{2} r \cos \theta,$$

where H = required height of tide above datum,
h = height of mean tide level above datum,
r = range of the tide,

$$\theta = \frac{\text{interval from high water}}{\text{interval between high and low water}} \times 180^\circ$$

Deviations from Normal.—At many places the above methods of estimating the times and heights of the tides are of little service on account of the influence of local conditions The amount of diurnal inequality is very variable over the earth It is small in European tides, but large in the Indian Ocean and parts of the Pacific and on the coasts of Australia and China At some places it is so pronounced that there is only one tide in 24 h for several consecutive days On the other hand, the tides may approach a place from two different directions When the two tides arrive at different times, the place experiences twice the normal number of tides, but if the difference in time is small, the effect is to make the tide practically stand at high water, it may be for several hours These phenomena occur at several places on either side of the English Channel

Tidal Streams.—On account of the inequalities in sea level caused by the tide wave, the action of gravity produces along the sloping surfaces an actual flow of water, called a tidal stream or current In the open ocean the height of the tide wave is so small that the stream is inappreciable, but in shallow seas the gradients are much increased, and the resulting currents become an important factor in navigation.

The speed and direction of a stream vary during the passage of the tide wave causing it The greatest velocity usually occurs at about half-tide, and there is a period of rest, or " slack water," at the turn of the tide This does not necessarily take place at the times of high and low water, since in general the flood stream continues to run after the sea level has started to fall, and the ebb stream for some time after low water The direction may remain fairly constant during flood and become reversed at ebb, but in many cases the direction undergoes a regular series of changes during flood and ebb The actual regime depends upon local circumstances, and may call for careful study by the engineer in connection with the design and construction of works (page 449)

Meteorological Effects.—*Wind*—Considerable variation may occur in both the height and time of tides through the action of wind in heaping up the water A strong wind blowing in the direction of the flood stream accelerates its progress, and increases the height of the tide The same wind retards the ebb On coasts swept by the wave high water occurs sooner, is higher, and keeps up longer than usual, while low water level is also above the average. The reverse effects occur when the wind blows against the flood stream. The effect of wind is least marked in the open ocean, and is most evident when the storm occurs in the region of a contracted area through which the tide pours.

Barometric Pressure —The atmosphere resting on the sea exerts upon it a varying pressure as indicated by differences in the height of the barometer at different places The sea level is depressed in regions of high pressure and is raised where the pressure is low, a difference of 1 in in the reading of the barometer corresponding to a difference of sea level of over 13 in Observation shows that the height of tides is affected thereby, but investigation is difficult since the effect is masked by the greater influence of the wind caused by a steep barometric gradient

Harmonic Analysis.—In order to predict successfully the heights and times of the tides at a place, allowance must be made for all the factors appreciably affecting the period and range of the wave If observations are continued at a place for at least a year, the effects of the principal perturbations are included, and contribute towards the resulting tide curves

These curves are capable of being analysed into a series of 20 to 25 simple harmonic curves, each of which represents a component oscillation induced by a particular characteristic in the action of the moon or sun or by periodic meteorological changes In making the analysis, the moon and sun are supposed replaced by several imaginary bodies having circular orbits in or parallel to the plane of the equator The constants of the individual curves are derived by a laborious process of averaging The data thus obtained can be used for constructing tide curves for the place for future years by combining the separate components. The position of the tide producers at the beginning of the year must be known, so that the various components may be correctly placed for combination. The labour of summation is eliminated by the use of Lord Kelvin's Tide Predicter, by means of which the tide curve is traced mechanically.

TIDE GAUGES

The vertical movements of tides are measured by means of a tide gauge When a prolonged record of the tides of a locality is required, the gauges at the observation station are of a permanent character. In the execution of a marine survey it is necessary to ascertain the variation of sea level during the operations, and a tide gauge must be established temporarily if there is no existing one in the vicinity.

Selection of Site.—The gauge may be required to record the tides at a particular point, in which case there may be little choice as to its position On the other hand, if it is desired to register the average conditions prevailing over a wide area, the site must be selected with a view to obtaining a representative record for the locality. Abnormal conditions prevail in shallow water, in narrow creeks and straits, and near the mouths of rivers Deep water and shelter from storms are the chief desiderata, so that frequently the most suitable situation is on the side of a bay with a wide entrance In the case of a survey covering a limited area, the choice is more restricted, and the gauge should be placed as near the work as possible, consistent with obtaining a trustworthy record

Forms of Tide Gauge.—Gauges may be classed as non-self-registering and self-registering. The former require the attendance of an observer to note the elevations of the water surface and the times of reading The latter are automatic, and produce a continuous record, or marigram.

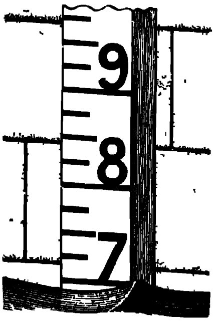


Fig 328. Staff Gauge.

Non-self-registering Gauges.—*The Staff Gauge* (Fig. 328) is simply a graduated board, 6 in to 9 in. broad, firmly fixed in a vertical position, and is the most commonly used form. Its length should be more than sufficient to embrace the highest and lowest tides known in the locality. The graduations and figures should be very bold, as it may be necessary to read them from a distance. The division is sometimes carried to tenths of a foot, but there is no advantage in closer graduation than quarter feet.

Piers, etc , form suitable supports for the staff, since the zero must always be under water If such a support is not available, the gauge must be erected below low water mark, and securely strutted or guyed. In such a case, if the tidal range necessitates a long staff, it may be preferable to arrange the gauge as a series of posts from above high water mark outwards, these being set by levelling so as virtually to constitute one staff. In setting any gauge, the zero may be fixed at a predetermined level, or, more usually, having erected the gauge, the elevation of its zero is observed by levelling and noted.

The *Float* or *Box Gauge* (Fig 329) is designed to overcome the objection that accurate reading of a staff gauge is at times difficult on account of the wash of the sea It is enclosed in a long wooden box about 12 in square, in the bottom of which are bored a few holes The surface of the water thus admitted is comparatively smooth, particularly when the holes are small and well below the surface The float carries a vertical rod, which may itself be graduated or which may carry a pointer over a fixed scale In the former case the graduations increase downwards, and the reading is obtained against a fixed index With the latter arrangement the rod may be made quite short, and the attached pointer is brought through a narrow continuous slit in the face of the box so that readings are obtained on a staff gauge attached to the outside of the box

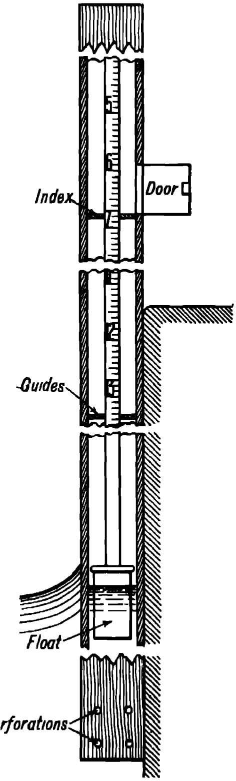


FIG. 329.
FLOAT GAUGE.

The *Weight Gauge* is suitable for situations where the above gauges would be liable to disturbance or would not be conveniently accessible for reading In the simplest form, a weight is attached to a graduated wire, chain, or tape, and observations are made by lowering the weight to touch the surface of the water and reading against a fixed mark Alternatively, the gauge may take the form shown in Fig 330, in which the chain carrying the weight passes over a pulley and along the surface of a graduated board, to which it is hooked when not in use. The chain is furnished with an index, and readings are obtained by unhooking the chain, lowering the weight to touch the water surface, and noting the graduation opposite the index The length of chain must be such that readings can always be obtained at the lowest state of the tide, but the graduated board need not be unduly long as a second and third index can be attached to the chain at intervals of, say, 10 ft In referring to these, the reading is increased by 10 or 20 ft, as the case may be

The reduced level of the water surface corresponding to zero reading must be determined This can be obtained by reading a levelling staff held with its foot opposite the bottom of the suspended weight and noting the corresponding gauge reading Otherwise it may be deduced from the level of the pulley, the length along the chain from the bottom of the weight to the index, and the horizontal distance between the vertical part of the chain and the zero graduation The value of the zero may change through stretching of the chain, which should be tested occasionally by steel tape, and adjusted if necessary

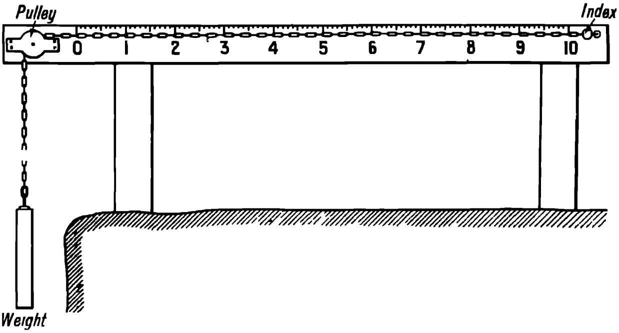


FIG 330 —WEIGHT GAUGE.

Self-registering Gauges.—Many types of automatic gauges have been constructed, but, while they vary in detail, the essential parts are similar in all forms A float, protected from the action of wind waves, has attached to it a wire or cord which is coiled round a wheel and is maintained at a constant tension by means of a counterweight or spring The vertical movement of the float, transferred through the float wheel, is reduced in scale by means of gearing, and is finally communicated to a pencil or pen which traces a curve on a moving sheet of paper In the commonest form the paper is mounted upon a drum which is rotated once in 24 hours by clockwork A week's record can be received on the sheet without confusion between the different curves In more elaborate gauges the recording mechanism is arranged to accommodate a band of paper sufficiently long to contain a month's record. The paper is paid out from one cylinder, passes over a second, against which the pencil or pen is pressed, and is wound upon a third drum.

In the housing of automatic gauges it is frequently necessary to lay piping from below low water level to a well, constructed under the building and in which the float operates. The effect of wave action outside is communicated but slightly to the water in the well because of friction in the pipe and the provision of a grid at its seaward end.

SHORE LINE SURVEYS

The survey of a shore line is usually performed by traversing along the shore, the edge of the water being located by taped offsets, tacheometer, or plane table, according to circumstances In the survey of narrow rivers both banks may be located by tacheometry from a traverse on one side In the case of wide rivers it will be necessary to traverse along both banks and connect the two systems at intervals. Otherwise a triangulation net is run along the river, as in Fig 331. In the most rapid shore line surveys the banks are sketched with reference to a compass traverse executed from a launch, linear distances being deduced from the speed of travelling

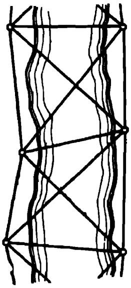


FIG. 331.

In the case of tidal waters it is necessary, for plotting on other than small scales, to locate both the high and low water lines These are taken for average spring tides The position of the former may be estimated roughly from deposits and marks, but to locate the lines carefully it is necessary to ascertain the elevations corresponding to high and low water of ordinary spring tides and trace the corresponding contours along the shore as in direct contouring Owing to the limited time available for the survey of the low water line, it is best located by interpolation from soundings

SOUNDING

The operation of sounding corresponds to that of levelling in land work, and has for its object the determination of the levels of the submarine surface This is accomplished by measuring from a boat or a launch the depth of water at various points, the measurements being termed soundings The water surface is thus used as a level surface of reference, which, however, in tidal waters undergoes continual change of elevation. But by observation of a series of tide gauge readings during the period of sounding this variation may be ascertained, and it is then possible to subject the observed values of the soundings to a process of reduction whereby the levels of the submarine surface may be referred to a fixed datum.

Scope.—Sounding is the most commonly required operation in hydrographical surveying, and is undertaken for engineering, navigational, and scientific purposes, these in many cases being closely interrelated For the design of such works as breakwaters, groynes, wharves, river viaducts, etc , a knowledge of the levels of the bottom over the site is obviously necessary. In the maintenance of harbours and docks, periodical soundings are taken to detect the progress of silting and so to ascertain where, and in what volume, material should be removed by dredging, and where it can safely be deposited. The measurement of river discharge and the examination of changes in the bed due to scour or silting also involve the running of lines of soundings between the banks

A very important application of sounding is in the preparation of charts exhibiting the depths available for navigation Harbour and river engineers are concerned with this aspect of the subject in the location and maintenance of navigable channels. In another direction sounding is applied in the investigation of scientific problems relating to the rate of accumulation of bars, growth of coral formations, etc

Soundings required for civil engineering purposes seldom exceed about 25 fathoms, and a description of the methods applicable to deep sea sounding (beyond the 100 fathom line) is outwith our scope. These observations may be accompanied by investigations of the nature of deep sea deposits and ocean temperatures, and are undertaken mainly for scientific purposes, but are of immediate practical importance in cable laying

Sounding Party.—Setting aside the preliminary work of establishing shore signals, the constitution of the party depends upon the manner in which the soundings are to be located

When the location is controlled by angular fixes taken on the boat, a full sounding boat complement comprises the surveyor, instrument man, recorder, leadsman, and oarsmen. The surveyor superintends the operations, sees that the boat is being kept on the proper course, and verifies the soundings as called by the leadsman The instrument man is responsible for the sextant or compass observations to shore marks. Usually the surveyor himself performs the duties of instrument man, but in locating by two simultaneous sextant observations on three shore objects, the chief and an instrument man share the work The recorder acts as timekeeper, and notes the soundings as called out by the leadsman as well as the results of angular and other measurements

When soundings are located by angular observations from shore, the instrument man remains there Two shore observers are required in certain methods. They measure bearings to the boat on receipt of a signal from it, and, unless the surveyor makes the signals, an extra man is required in the boat to act as signaller.

For cross rope sounding the instrument men and signaller are dispensed with, but a crew is required to man the reel boat.

In tidal waters gauge readings must be made at regular intervals during the sounding operations A trustworthy man is stationed at the gauge with instructions to note the reading at 10 or 15 minute intervals, his watch being set to correspond with that of the recorder. If a gauge is established close to the survey, it may be possible to have the readings observed through a field glass by a member of the boat party.

Equipment.—The essential equipment comprises a boat or launch, apparatus for taking the soundings, and instruments for their location The transit theodolite is used when the location is based upon angular measurements from shore, but observations from the boat are made by sextant or, less commonly, and with less accuracy, by prismatic compass.

Sounding Boat.—The chief qualities desired in a row-boat for sounding are roominess and stability. A flat-bottomed cobble is suitable for sheltered waters, but for sea work a boat of either the whale or ship's lifeboat type is more easily handled. Row boats for regular sounding may with advantage be provided with a well (Fig 332), through which the soundings are taken For harbour and river sounding two oarsmen are sufficient, but four are frequently necessary for sea work In localities where the currents are strong, much time is saved by the use of a motor or steam launch. The equipment of a sea boat should include a compass, grapnel, and spare oars.

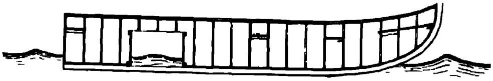


FIG 332.—SOUNDING BOAT

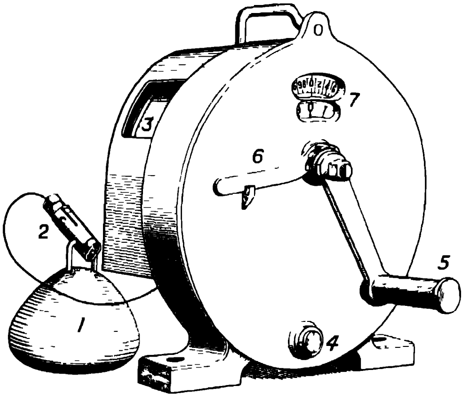
Sounding Rod.—The sounding rod is a pole of tough timber, usually circular in section, 2 to 3 in. in diameter, and 15 to 25 ft. long Long rods are awkward to manipulate when sounding in shallow water, and should be formed of two or three lengths screwed together so that the top may be removed when necessary. The rod is sometimes graduated in feet and tenths, but division to quarter feet only is generally preferable. To facilitate plunging it, the pole is weighted at the base, which is sometimes enlarged to prevent excessive sinkage into a soft bottom. If samples of the surface material of the bottom are desired, the base is made concave at the end, and the cavity is filled with tallow, to which the material adheres.

Lead Line.—The lead line consists of a graduated line or chain to which is attached the lead or sinker. A line of either hemp or cotton is commonly employed, but is liable to elongate with prolonged usage. To minimise this effect, the line must be thoroughly stretched when wet before graduation. A satisfactory method is to coil it tightly round a tree trunk or post, secure the ends, and wet it thoroughly. The shrinkage caused by the wetting induces tension, and, when dry, the line is stretched. The process should be repeated until the stretch becomes inappreciable, and the line is then soaked and graduated by attaching cloth or leather tags at foot intervals. Every fifth foot mark is distinguished from the others, and every tenth is made to exhibit its distance from the bottom of the lead. Such a line should have its length tested periodically

For regular sounding, a chain of brass, steel, or iron is preferred since, although subject to elongation through wear, it maintains its length much better than the line The links may be 1 in. long, and should be welded or brazed The attached tags are sometimes of brass, but may be of leather or cloth as in the line

For ordinary sounding, the weight of the lead sinker ranges from 5 to 20 lbs according to the strength of current and depth of sounding A weight of 8 to 10 lbs is sufficient for harbour work When samples of the submarine surface material are required, a lead is used with a recess in the bottom for the reception of tallow as in the sounding rod.

Sounding Machines.—The work of sounding is simplified by the use of one of the various forms of sounding machines suitable for engineering surveys. Fig 333 illustrates Weddell's Sounding Machine, which is manufactured by Messrs George Russell and Co , Ltd , of Motherwell The instrument is intended to be bolted over the well or to the gunwale of the sounding boat, and consists of a galvanised malleable cast iron casing carrying on a spindle a gun-metal barrel, 2 ft in circumference The lead, of about 14 lbs weight, is suspended by a flexible wire cord from the barrel, and can be lowered at any desired rate, the speed of the drum being controlled by means of a brake The indicating dials are driven by gearing from the spindle through a friction device, the outer dial recording the depth in feet, and the inner showing tenths of a foot. The lead is raised by means of the handle *5*, and it may be suspended at any height by means of a pawl and ratchet The standard machine is designed to record a maximum depth of 100 ft



1 *Lead Weight*
2 *Swivel and Hook*
3 *Barrel*
4 *Plunger for Zero Adjustment*
5 *Winding Handle*
6 *Pawl Arm*
7 *Recording Dials*

FIG 333 —WEDDELL'S SOUNDING MACHINE.

At the commencement of a series of soundings, the dials are set to read zero by means of the handle *5*. By depressing plunger *4*, the reading is held at zero, and the handle is again turned until the bottom of the lead is raised or lowered to touch the surface of the water. On releasing the plunger, the machine is ready for use, no correction to the readings being required to allow for the height of the machine above water level It is unnecessary to raise the weight to the surface after each sounding in sheltered water, but it should be wound up when moving from one section to another

The Sounding Sextant.—The form of sextant used in coast surveying is known as the sounding sextant (Fig 334), and differs somewhat from the astronomical sextant (Vol II, Chap II) It is of specially strong construction, and is fitted with a large index glass to minimise the difficulty of sighting from a small boat, due to its motion The horizon glass has no transparent part, and is entirely supported by a metal tray The object viewed directly is therefore sighted over the top of the horizon glass, and the reflected image is separated from the direct one by the small thickness of the frame The arc is boldly graduated, and is read by the vernier to single minutes The ring *5* carries a disc with pin-hole sight, but the instrument is also provided with a Galilean telescope of wide field. This is substituted for the disc when the sights are long, merely to afford resolution The telescope is non-adjustable, but the testing and adjustment of the perpendicularity of the mirrors to the plane of the arc and for index error are conducted as for the astronomical sextant, except that test sights may be taken on distant terrestrial objects instead of celestial bodies.

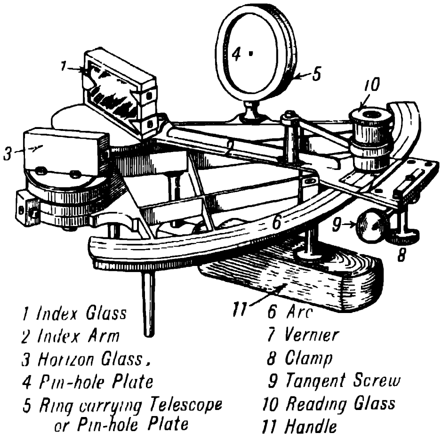


FIG. 334.—SOUNDING SEXTANT

Water Glass.—During sounding, an inspection of the submarine surface or of underwater construction is sometimes required, and may be made by means of the water glass or water telescope (Fig 335). The instrument consists of a watertight box of wood or sheet iron open at the top and having a sheet of plain glass fitted at the bottom. Two handles are provided for steadying the instrument. The observer, leaning over the side or stern of the boat and holding the instrument with both hands, lowers the bottom a few inches below the surface of the water, and looks in at the upper end. The clearness of the view presented depends upon the depth and transparency of the water and the brightness of the day.

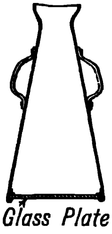


FIG. 335

Shore Signals.—Shore signals are required to mark ranges defining section lines and as points to which to observe in making angular measurements from a boat. For the former purpose, signals have in general to be specially set out and erected, but for angular observations existing landmarks may be available, especially in the vicinity of ports. It is, of course, necessary that objects, such as tall chimneys, spires, lighthouses, etc , used as signals, should be capable of identification on the map

Temporary signals consist of poles, piles of stones, or whitewashed marks on rocks. Unless the observations are made from short distances, ordinary ranging rods are too small, and longer poles of 2 to 4 in diameter, or battens, are required These are whitewashed or painted, and, if practicable, sufficient height should be given that they can be viewed against the sky They should be flagged, or have discs or spars nailed on at the top Different ranges may be distinguished from each other by using flags of various colours or by fixing timbers to form various geometrical figures, such as a triangle, cross, etc The reference number of each range is sometimes exhibited by nailing on spars to form Roman numerals when read laterally.

It may happen that one at least of a pair of range signals has to be placed in the water If the depth is such as to preclude the erection of an ordinary signal, a short flag-pole may be attached to a float formed of a block of light wood weighted at the bottom and anchored in position The action of current in displacing the buoy may, however, lead to considerable error in the direction of the range If there is no tide, this error may be avoided by guying the float from three anchor boulders, but in tidal waters guying is troublesome, as the guys must be effective at low water If there is a considerable tidal range, a long flag-pole must be used, so that a sufficient length may show above the surface at high water

Whitewashed marks are useful on rocky foreshores, and are suitable for observing to in making angular fixes from a boat For the accurate definition of ranges, it is generally inadvisable to use two such marks, unless they are at such elevations that they appear as one continuous line when viewed from a boat in the range.

Sounding by Rod.—In the absence of strong current, soundings are taken by rod up to depths of 20 to 25 ft , beyond which the lead line is more convenient. Currents may reduce the limiting depth for rod sounding to 15 ft.

Except when sounding through a well, the leadsman stands in the bow. He plunges the rod at a forward angle, depending on the speed of the boat, so that when it reaches the bottom it can be read in a vertical position. He must read the rod quickly owing to the onward motion of the boat, and the surveyor should satisfy himself by a glance that the sounding as called is correct Unless samples are being collected, the rod is then simply allowed to float up loosely in the hand, and is grasped at the right place for the next stroke

Sounding by Lead.—The leadsman in the bow of the boat heaves the lead forward such a distance that it will reach the bottom at the point where the sounding is required As the boat progresses, he takes in the slack, so that the line or chain is vertical when being read The soundings are, for engineering purposes, read to ¼ ft. by estimation As soon as the reading is called, the leadsman must coil in the line for the next cast in a manner which will prevent its becoming entangled.

Spacing of Soundings.—Since the desideratum that observations should be made at points of change of slope is impossible of fulfilment, soundings should be so spaced that no important irregularity in the submarine surface will go unrecorded With this object, it is best to sound along a series of straight lines at right angles to the shore line, so that the successive contours may be intersected normally or nearly so

The interval between lines of soundings and the spacing of the casts thereon depend primarily upon the object of the survey but also upon the nature of the bottom On a rocky foreshore the possibility of the existence of pinnacle rocks necessitates closer sounding than is required in the case of a soft bottom The spacing of soundings should not be made to depend altogether upon the intended scale of plotting, as the finished drawing may be made to exhibit only a selection of typical observations

For general engineering purposes, 100 ft is the most common interval between lines of soundings, and the casts may be made at 25 or 50 ft apart under favourable circumstances When close estimation of excavation is required, the lines may be as little as 20 ft apart, with soundings at 10 ft intervals

When indication is obtained of a sudden inequality in the bottom, additional lines should be interpolated to locate its limits These lines are sometimes projected radially from a buoy marking the suspected position of the irregularity, the operation being termed "starring" On the other hand, the interval between casts may generally be increased with safety as the water deepens.

Methods of Locating Soundings.—If the lines of soundings are defined by range signals to serve as steering marks, the position of the boat on the line can be determined by one measurement, linear or angular When the lines are not marked out, the boat is kept on an approximately straight course, and two observations are then necessary to locate its position from time to time Fix observations may be made . (*a*) entirely from the boat , (*b*) entirely from the shore ; (*c*) from both.

The methods of location are :

(1) By cross rope.
(2) By range and time-intervals.
(3) By range and angle from shore
(4) By range and angle from boat
(5) By intersecting ranges
(6) By two angles from shore
(7) By two angles from boat
(8) By one angle from shore and one from boat
(9) By tacheometry.

Cross Rope Sounding.—The most accurate method of locating soundings is that by cross rope It involves stretching across the line of sounding a rope marked off by equidistant tags, the soundings being taken opposite the tags The method is very suitable for sounding in harbours and across rivers if the sections do not exceed about 1,500 ft in length, but it can also be adapted to offshore work.

The Cross Rope.—For short lines a hemp rope will serve the purpose, but it cannot be heavily stressed, and for lengths over 500 ft. a steel strand wire rope of $\frac{1}{8}$ to $\frac{1}{4}$-in. diameter should be used. Phosphor-bronze rope is much more durable, but more expensive. The tags are of brass or leather, and are marked with their distances from one end They are spaced at 10, 20, or 25 ft intervals. The rope is wound upon a reel, which is mounted in a boat called the reel boat, usually similar to the sounding boat

Procedure.—The zero end of the cross rope is first made fast at one end of the section to a spike or other attachment previously located. The reel boat is then rowed across the line of sounding, unwinding the rope as it goes The course of the reel boat should be as straight as possible, and, if a range is not available in which the oarsmen can maintain themselves, their progress should be directed by the surveyor in the sounding boat waiting at the beginning of the section. If the reel boat is not rowed straight across the section, the uncoiled rope will lie in an irregular curve along the bottom, and will most probably foul when being tightened preparatory to sounding. On arrival of the reel boat at the other side, her anchor is taken ashore, and the men proceed to wind in the rope as tightly as possible. If anchoring is impracticable, the reel is taken ashore and spiked down.

On receiving a signal, the sounding boat now starts to cross the section. The oars are shipped, and the oarsmen propel the boat by hauling on the cross rope As each tag approaches the man in the bow, he warns the leadsman, who takes the sounding when the mark reaches him.

When a section is completed, the sounding boat is rowed to the starting-point of the next, while the reel boat proceeds back along the line, winding in the rope This proves more expeditious than dragging the rope along the bottom to the next section line.

Use of Floated Cross Rope —The rope, instead of being allowed to lie along the bottom as it is uncoiled, may be suspended from a series of floats These are usually pieces of light wood which are clipped on in the reel boat as the rope is first being paid out A float may be attached at each tag, and the 100 ft distances distinguished by specially painted floats When the rope is tightened, the points at which soundings are required are thus clearly visible. The sounding boat is propelled as before, but the rope is not taken on board.

The advantages of this method, as compared with the previous one, are ·

(*a*) The rope can be reeled in more tightly, so that the error due to vertical sag is reduced

(*b*) It is more rapid, since, in changing from one section to the next, the rope need not be reeled up, as there is no chance of fouling the bottom

It has, however, the disadvantages that

(*a*) It is quite unsuitable for situations where the operations are liable to sudden interruption by shipping

(*b*) The action of wind and current on the floats increases the lateral sag

Cross Rope Offshore Sounding —In calm weather or in sheltered situations, offshore sounding may be performed by cross rope with or without floats, provided the sections are comparatively short. The operations differ from those described above only in the manner of stretching the rope One end is fixed on shore, and the reel boat is anchored at sea with the other end, or, alternatively, the seaward end of the rope may be attached to a buoy or simply anchored

Location by Range and Time-intervals.—In this method the sounding boat is kept in range with two shore poles defining the section line, and is rowed at a uniform rate, the soundings being taken at regular time-intervals The method is not susceptible of great accuracy, but is useful in conjunction with other methods. It is seldom adopted alone except for comparatively short sections of known length, such as between the banks of a river or from a coast line to buoys of known position. If the total length of section is unknown, the precision of the method is greatly reduced on account of the influence of wind and current on the speed of the boat.

While proceeding along the section, the oarsmen must maintain a steady stroke so that equal time-intervals may correspond to equal distances The surveyor, watch in hand, warns the leadsman when to sound Alternatively, the soundings may be spaced by counting a constant number of oar strokes between each, or, in the case of a launch, a constant number of revolutions of the screw. When the total length of section is known, it is only necessary, for purposes of plotting, to divide it out into a number of equal parts. When the distance is unknown, the speed of the boat or launch must be very carefully rated beforehand by travelling over a measured length and noting the average number of oar strokes to a given distance or the actual speeds of the launch corresponding to various engine speeds.

Location by Range and Angle from Shore.—The boat is again steered in range with guide poles, and its position in the line is observed from a theodolite station on shore by measurement of its bearing or the angle between it and a located shore object The method is a useful one, capable of considerable precision, and proves simple in plotting It has the disadvantage that the surveyor in the boat has not the entire operations under his immediate control

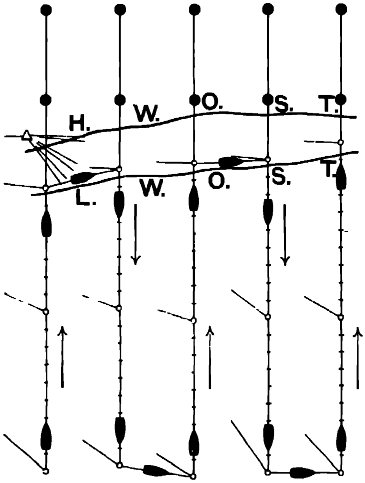


Fig 336.

Some surveyors fix each sounding by an angle, but this is not recommended as the rapidity with which the angles have to be read and booked may necessitate a notekeeper at the instrument, and is very likely to lead to confusion. It is sufficient to locate, say, every tenth sounding by angle and the intermediates by time-intervals Fig 336 shows angular fixes applied only to the end soundings and one midway in each section, and illustrates the course of the boat, seaward and shoreward on alternate sections

Suitable instrument stations should be set out, and their positions determined beforehand, due consideration being given to the quality of the intersections which will be made between the lines of sight and the range lines The nearer the intersection is to a right angle the better : a new instrument station should be occupied when the angle diminishes to about 30°.

The routine of the instrument man on setting up at a station consists in first orienting on the back station or on a reference point which can be reproduced on the plan. The line of sight is then directed towards the leadsman or the bow of the boat, which has been brought into position at the start of the section. The signaller holds up a flag for a few seconds to warn the instrument man to prepare, and on the fall of the flag the sounding and angle are observed simultaneously. The telescope is kept pointing towards the boat as it moves along the section, and the remaining angles are taken in the same way

The recorder in the boat distinguishes in his notes those soundings which are fixed by angle, but in transcribing the angle book notes into the sounding book at the close of the day's work, it may happen that there is difficulty in identifying particular angles. The possibility of confusion is eliminated if the time of each fix is noted both by the recorder and the instrument man, their watches having previously been compared. When frequent fixes are made on each section, the possibility of discrepancy between the records is increased, particularly through the omission of an angle observation. The time check may then involve reading to seconds, and unless the instrument man has a notekeeper, his work becomes very hurried An alternative and satisfactory method for the detection of mistakes is to make successive signals by flags of different colours. At each fix, the colour is noted in the angle book and also by the recorder

Notes —(1) If the lines of soundings have previously been drawn on the plan, suitable instrument stations may be selected, and note made of the sections to be dealt with from each so as to avoid acute intersections.

(2) There is no check on the accuracy of the fixes other than that afforded by the time-interval location As a precaution, the instrument man, before leaving a station, should check his orientation by again sighting the reference point

(3) It is unnecessary to read angles to parts of a minute In many cases it is sufficient to estimate readings to the nearest 5 min without consulting the vernier.

Location by Range and Angle from Boat.—In principle ,this method is similar to the last, but the angular fix is made between the range line and a shore object, and is observed from the boat by sextant (Fig 337) A compass bearing may take the place of the sextant angle, but proves less accurate

The boat is rowed outwards and inwards on alternate sections as before Fixes are taken at the ends of the sections and at as many intermediate soundings as are considered necessary, but the majority of the soundings are located by time-intervals between the fixes In observing with the sextant, the telescope is directed on the range signals, and the side object is brought into coincidence.

Compared with the previous system, this method has the advantages that

(1) The surveyor has better control of the work, since the party is not divided. Frequently the instrument man is dispensed with, and the angles are observed by the surveyor.

(2) The angles are booked by the recorder directly they are measured, so that there is less probability of mistakes in booking.

(3) At important fixes a second angle to another shore object may be observed as a check on the first when plotting (Fig. 337).

(4) On one section different shore objects can be used in the various fixes so as to maintain good intersections throughout.

There is little to choose between the two methods as regards accuracy and ease of plotting

Location by Intersecting Ranges.—Angular observations are avoided if the position of each sounding is defined by the intersection of two ranges The method is suitable for the location of a few isolated soundings, and proves highly accurate if the intersections are good

The system is sometimes employed when it is required to determine by periodical sounding the rate at which silting or scouring is occurring at a place. For such observations it is essential that the repeated soundings should be made at the same spots, and permanent range signals are established When applied to section sounding, as in Fig 338, precautions must be taken to obviate confusion between successive intersections, and a system of flagging the range poles is necessary.

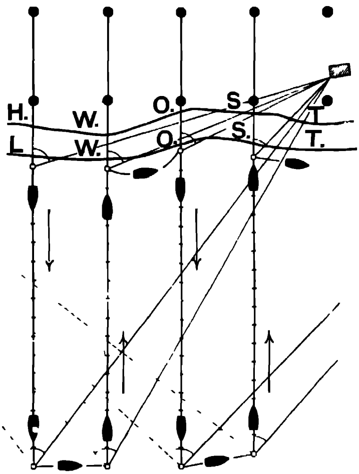


Fig 337

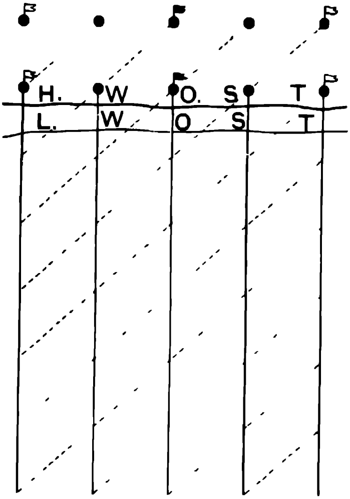


Fig 338

Location by Two Angles from Shore.—In this system the fix is made independent of a range by having simultaneous observations to the boat taken by theodolite from two shore stations, the soundings being located by the intersections of the sight lines. The method may be applied to the location of a few isolated soundings . if it is used on an extensive survey, the boat should be run on a series of approximate ranges

The routine followed by the instrument men is similar to that adopted in locating by a range and one shore angle They must observe simultaneously on the fall of the signal flag in the boat, and should note their watch times. New instrument stations must be occupied by one or both observers when the intersection angle falls below about 30°.

The principal merit of the system is the elimination of the preliminary work of setting out and erecting range signals It also avoids the difficulty, in localities where the currents are strong, of maintaining the boat exactly on a range. It is, however, a serious disadvantage that two instrument men are required on shore.

Location by Two Angles from Boat.—By observation of the two angles subtended at the boat by three suitable shore objects of known position (Fig. 339), the boat can be located by solution of the three-point problem (page 445) This method is largely used, particularly when periodical soundings are not required. It not only possesses the merit of concentrating the party, as in the range and one boat angle method, but, if a sufficient number of landmarks are exhibited on an existing map, no preliminary shore work is required

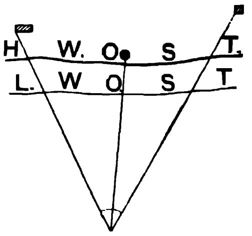


FIG. 339.

It is important that the angles should be observed as nearly simultaneously as possible, particularly for large scale plotting, and the surveyor and the instrument man should each have a sextant If the surveyor is observing alone, he should allow as little time as possible to elapse between his observations by using two sextants, which are successively set to the angles and read afterwards If the boat is stopped, a check can be secured by measuring the total angle subtended by the extreme objects Refined reading is not required, but considerable care is necessary in selecting suitable reference points on shore (page 447)

If the compass is used instead of the sextant, the position of the boat is defined by bearings to two shore objects, but with considerably less accuracy.

Location by One Angle from Shore and One from Boat.—This combination of the last two methods is not much used For convenience of plotting, one of the shore points sighted with the sextant should be the theodolite station.

Location by Tacheometry.—A tacheometric observation on a staff held in the boat, since it gives the bearing and distance from a shore station, affords a simple method of location, which, however, can be used only in smooth water Soundings have been located with considerable precision in river work by simultaneous observations from tacheometers on both banks. This fix affords a twofold check since each observation gives the position by angle and distance, while together they yield an intersection If the water is shallow over the area being sounded, the stadia rod may be dispensed with by reading the intercept on the sounding rod at the moment it is held on the bottom. Instrument stations should be selected near the edge of the water so that no correction for vertical angles may be necessary

Note-keeping.—*Recorder's Notes.*—These are entered in the sounding book, the form of which varies according to the system of location adopted. In any case columns are provided for measured depths, location observations, watch times, and remarks Columns are reserved for the subsequent entry of tide gauge readings from the gauge reader's notes, reduction corrections, and the reduced soundings.

The location measurements, if made from the boat, are simply entered as distances in the case of cross rope sounding (Fig. 340). With sextant observations, the shore stations must be sufficiently described, and the entries may be made in one column or separated as in Fig 348

The times of start and finish of each section line must be noted, as well as those at which intervening fixes are observed. In cross rope sounding, one or more intermediate times are entered opposite the corresponding distances In cases of interruption or delay, the times of stoppage and resumption must be noted, and fixes made at these times Any ranges which intersect the line of sounding are remarked in their proper places, as this forms a useful check in plotting

When the location is controlled from shore, the remarks column will show which soundings have been located, and, if successive signals are distinguished, the signals should be described

Instrument Man's Notes—The record kept by an instrument man observing on shore includes the angles or bearings taken to the boat, their watch times, and a note of the colour of the distinguishing signal Before leaving the ground, he should compare his entries with those in the sounding book for the discovery of possible discrepancies.

Gauge Reader's Notes.—These simply consist of a list of the gauge readings with the corresponding watch times It is usually sufficient to read to the nearest ¼ ft. at intervals of 10 or 15 minutes, but in special cases readings may be taken to 0·1 ft., and observed every 5 minutes.

Reduction of Soundings.—The object of reducing soundings is to convert the observed depths to the values they would have if measured from a water surface of unvarying and known elevation forming the datum of reduction. The reduced soundings are the reduced levels of the submarine surface in terms of the adopted datum.

The datum most commonly used is the mean level of low water of spring tides, written L.W.O.S.T. (low water, ordinary spring tides), or M.L W.S (mean low water, springs). The local value is adopted, as the datum has not a constant elevation over a wide area, but in practice this is no objection, and this datum is the most

. . . . . *Surveyor.*

Section No. 4. . . . . *Recorder.*

Date. . . . . .

Time	Gauge	Distance	Sounding	Correction	Reduced Sounding	Remarks
A M 10-3		25	5½	- 8¼	- 2¾	
		50	9		¾	
		75	13½		5¼	
		100	15¾		7½	
		25	18		9¾	
		50	20¼		12	
		75	21¾		13½	
10-5	14¼	200	22		13¾	
		25	22½		14¼	
		50	23¼		15	
		75	25½		17¼	
		300	26¼		18	
		25	27		18¾	
		50	27¾		19½	
		75	27¼		19	
		400	28½		20¼	
		25	29		20¾	Lt Ho. and C's Chy. Stk in range.
		50	27¾		19½	
		75	26½	- 8½	18	
		500	26		17½	
		25	26¼		17¾	
		50	24½		16	
		75	22		13½	
		600	20¾		12¼	
		25	17½		9	
		50	14¼		5¾	
		75	14		5½	
		700	11¾		3¼	
10-10	14½	25	10¼		1¾	

FIG. 340.

useful one for navigational charts It is commonly worked to in engineering surveys, but not exclusively, and H.W.O.S T., mean sea level, or any arbitrary, but defined, datum, such as dock sill level, may be used instead.

Reduction is performed by computing and applying corrections to the measured depths. In tideless waters a constant correction, equal to the difference of level between the actual water surface and the datum, could be applied to all the soundings of a series, but in tidal waters the amount of correction changes with the varying water level as read on the tide gauge.

Having interpolated the appropriate gauge readings, and entered them in the sounding book, the corrections are deduced as the differences between the gauge readings and the value of the datum as it would be indicated on the gauge These are entered as positive or negative quantities according as the latter is greater or smaller than the former. If, as is usual for engineering purposes, the reduced soundings are to be expressed to $\frac{1}{4}$ ft, corrections are applied for every $\frac{1}{4}$ ft. variation of tide level, and the observations are corrected in groups corresponding to this variation

Fig. 340 shows the reduction of a line of soundings located by cross rope. The datum of reduction is L W O S T., which corresponds to 6 ft. on the gauge used. The gauge readings were 14 and $14\frac{1}{2}$ at 10-0 and 10-10 a.m respectively, and the height $14\frac{1}{4}$ is interpolated for 10-5 The corrections are obtained by subtracting 6 ft. from the gauge readings, the $8\frac{1}{4}$ correction being applicable from the start of the section to 450 ft, which has been reached about midway between 10-5 and 10-10 The negative value of the first reduced sounding shows that point to be above the datum.

PLOTTING SOUNDINGS

Reduced soundings may be plotted in section form in the ordinary manner, provided the soundings have been taken in lines. This method of representing the variation in level of the bottom is required in river work, and is sometimes used in marine surveying for the design of engineering works For navigational and general engineering purposes the reduced soundings are shown in plan. They are exhibited as spot depths, the values of the reduced soundings being written at the points representing their positions The interpolation of contours increases the value of the chart to the navigator, and is a necessity for engineering purposes, as the plan then exhibits the topography of the bottom, and is available for the calculation of volumes of dredging.

Locating Soundings in Plan.—Having laid down the shore survey and from it the positions of landmarks and range signals, the section lines are drawn (in the case where the soundings have been taken in lines), and the positions of the soundings are spaced out.

In plotting cross rope soundings to a large scale, vertical and lateral sag of the rope may occasion difficulty in spacing the soundings when the total length of section is known In such a case, the distance should be divided up into a number of equal parts corresponding with the number of soundings. If the recorder has noted the rope distance at which the line of section is cut by the range of two landmarks represented on the plan, the spacing out can be performed in two parts with increased accuracy

The plotting of angular fixes and intermediate soundings does not require explanation, except in the case of location by two sextant angles from the boat This is an application of the three-point problem, and differs from its use in plane table surveying only in that the measured values of the two subtended angles are known.

The Three-point Problem.—The problem may be stated Given three points A, B, and C, representing the shore signals, and the values *a* and *b* of the angles APB and BPC, subtended by them at the boat P Required to plot P The problem may be solved mechanically, graphically, and analytically

Mechanical Solutions.—(1) *By Station Pointer*—The station pointer or three-arm protractor (Fig 341) consists of a graduated circle with one fixed and two movable arms, the fiducial edges of which pivot about the centre of graduation. The edge of the fixed arm passes through the zero division, and the movable arms can be set so that their fiducial edges subtend the observed angles with that of the fixed arm. For large scale plotting the arms can be extended by clamping on lengthening pieces.

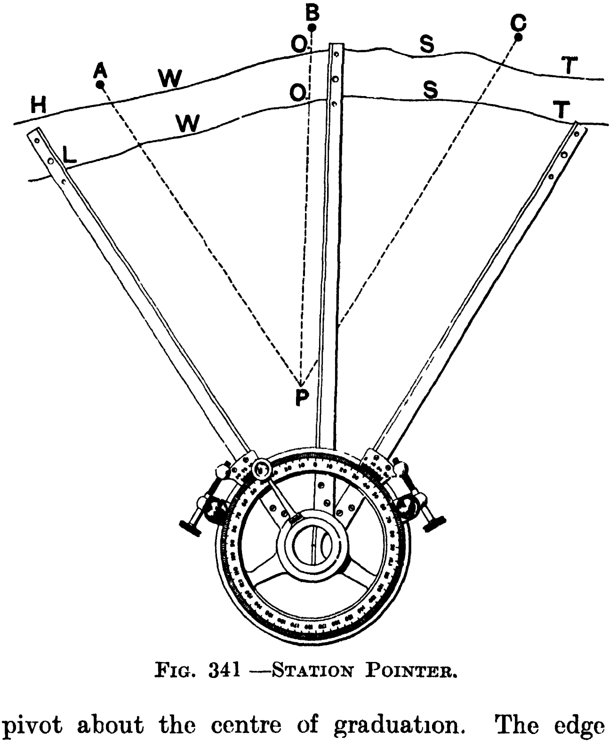


FIG. 341 —STATION POINTER.

To use the station pointer, set off the angles a and b, and clamp the movable arms. Move the instrument over the plan until the bevelled edges of the three arms simultaneously touch the points A, B, and C. The centre of the protractor then marks the position of P, which is recorded by a prick mark

(2) *By Tracing Paper Protractor.*—On a piece of tracing paper protract a and b between three radiating lines from any point Apply the tracing paper to the plan, and move it about until A, B, and C simultaneously lie under the lines ; then prick through the point P at the apex of the angles

Graphical Solutions.—(1) Join AB and BC (Fig 342), and at A and B set off AO and BO respectively, making angles of $(90^\circ - a)$ with AB on the side next P. From B and C similarly set off BO′ and CO′ at $(90^\circ - b)$ with BC With centre O, describe a circle through A and B, and, with centre O′, describe a circle through B and C The intersection P of the two circles is the point sought, for angle APB at the circumference $= \frac{1}{2}$AOB at the centre on the same chord But AOB $= 2a$, so that APB $= a$ similarly, BPC $= b$.

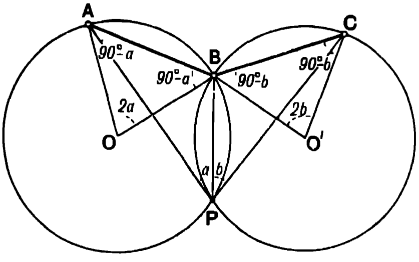


FIG. 342.

Note —Observed angles exceeding 90° have their excess over 90° set off on the side of AB or BC remote from P.

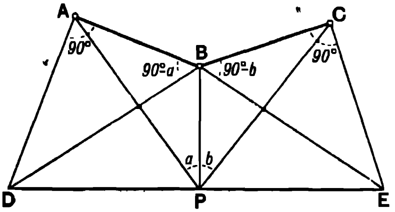


FIG 343.

(2) Join AB and BC (Fig. 343), and from B set off BD and BE making angles of $(90^\circ - a)$ and $(90^\circ - b)$ with BA and BC respectively and on the side next P. From A erect a perpendicular to AB to cut BD at D, and from C a perpendicular to CB to cut BE at E Join DE. From B drop a perpendicular BP on DE This will intersect DE at the required point P, for the quadrilaterals BADP and BCEP can be circumscribed by the circles used in the previous construction, so that APB = ADB $= a$, and BPC = BEC $= b$.

Note —Observed angles exceeding 90° have their excess over 90° set off on the side of AB or BC remote from P.

(3) Join AC (Fig. 344), and at A set off AD, making CAD $= b$ on the side remote from P From C similarly set off CD, making ACD $= a$. Let these lines intersect at D. Join DB. The circumscribing circle through A, D, and C cuts DB, produced if necessary,

at P, for APD = ACD = a, since they are circumferential angles in the same segment; similarly, DPC = b

In place of drawing the circumscribing circle, it is more convenient to set off a and b on either side of any point on DB and to draw through A and C parallels to these lines to intersect at P.

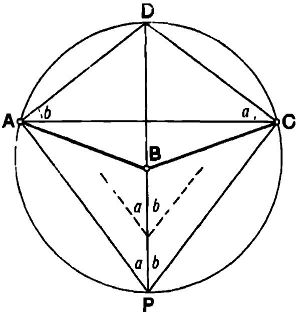


FIG 344

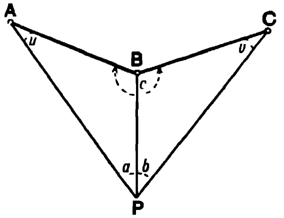


FIG. 345

Analytical Solution.—Given the lengths AB and BC and angles a, b, and c, the values u and v of angles BAP and BCP and the distances AP, BP, and CP may be computed trigonometrically (Fig 345)

$$\text{By sine ratio, BP} = \frac{\text{AB} \sin u}{\sin a} = \frac{\text{BC} \sin v}{\sin b},$$

but $u + v = (360° - (a + b + c))$ = say, s, so that, by substitution,

$$\frac{\text{AB} \sin u}{\sin a} = \frac{\text{BC} \sin (s - u)}{\sin b} = \frac{\text{BC} (\sin s \cos u - \cos s \sin u)}{\sin b},$$

whence

$$\cot u = \frac{\text{AB} \sin b}{\text{BC} \sin a \sin s} + \cot s$$

Angle v is now obtainable as $(s - u)$, and AP, BP, and CP by sine ratio

The Indeterminate Case.—If A, B, C, and P can be circumscribed by a circle, any point on the arc on which P lies subtends with AB and BC the observed angles a and b, and the position of P is indeterminate In this case it will be found that a station pointer or tracing paper protractor can be moved about on the plan without swinging the legs off A, B, and C In geometrical plotting the two circles of Fig. 342 merge into one, D and E coincide in Fig. 343, as do D and B in Fig. 344. Since $s = 180°$, the analytical method yields the indeterminate result that $\cot u = (\infty - \infty)$.

Care must therefore be exercised to select for observation such shore stations that the circumference of the circle through them will not pass through the boat.

Strength of Fix.—If P is near the circumference of the circumscribing circle ABC, the fix, although theoretically determinate, is unsatisfactory, since errors of observation and plotting have then considerable effect in displacing P Bad fixing is evidenced when in using the station pointer a small movement of the circle can be made without appreciably affecting the contact of the arms with A, B, and C (so that the experienced draughtsman finds the plotting of such a fix troublesome, while the beginner finds it simple). Bad fixes are indicated in the geometrical constructions by oblique intersection of the circles of Fig 342, by D and E falling close together in Fig 343, and by D falling near B in Fig 344 If plotting is carried on at sea, bad fixes can be discovered at once and eliminated by check observations Otherwise, the surveyor must exercise care in the selection of shore objects from which to fix, and the following precepts will serve as a guide

Good fixes are in general obtained

(1) When the three objects lie on a straight line or on a curve convex to the observer, the middle object being nearest, provided the angles are not both less than 30°.

(2) When the three objects lie on a curve concave to the observer, provided (*a*) the observer is within the triangle formed by them, (*b*) they are practically equidistant from the observer, and the observed angles are not less than 60°, (*c*) the observer is well outside the circumscribing circle

(3) When one of the outside objects is much more distant than the others, and the angle between the near objects is much greater than that between the middle and remote objects

Finishing the Sounding Plan.—The values of the reduced soundings are written up neatly in black on the points representing their respective positions In engineering plans they are given to the nearest $\frac{1}{4}$ ft, and in navigational charts to the nearest foot or $\frac{1}{4}$ fathom If the scale does not permit of showing all the soundings clearly, a selection is made, keeping in view the importance of the shallower soundings, and the sounding book is kept for reference

Contour lines are interpolated amongst the figures in the usual manner Since the submarine surface is in general characterised by flatter slopes than obtain on land, a smaller contour interval is necessary to exhibit its form The Admiralty charts display contours at 1, 2, 3, 4, 5, 6, 10, 20, 50, and 100 fathoms, but for engineering purposes contours are usually shown at 2 ft or 3 ft intervals, while on large scale plans an interval of 1 ft is sometimes adopted.

The contours are first sketched in pencil, and their positions relative to the soundings are verified by re-examination A careful inspection should be made for indications of suspicious areas, and additional soundings may be required to remove uncertainty regarding the direction of contours where sudden inequalities occur between sections Finally, the contours are inked in with Prussian blue An effective method of colouring the water area consists in applying successive washes of pale Prussian blue to the areas contained within each contour, so that the tint is gradually deepened towards the deeper zones

THE SURVEY OF TIDAL CURRENTS

Observations of the direction and velocity of tidal currents have to be undertaken by the engineer in connection with certain harbour and coast protection projects They are also required to aid in the selection of suitable points at which to discharge crude sewage into tidal waters, to ensure its being carried seawards Such observations must be made at all states of the tide, and a complete set should therefore extend over a series of spring and neap tides

Methods.—The most satisfactory method of ascertaining the direction and velocity of the tidal stream over an area is by immersing floats, which drift with the current, and are located from time to time If, however, it is required to determine the characteristics of a current at a particular point, the measurements may be made by current meter (page 457) A form of meter suitable for marine work is fitted with a compass which can be locked before the meter is brought up, and which registers the direction of the current. Description of meter observations is reserved for page 459

Floats.—The requirements of a float for tidal current observations are that (1) it should be carried along by the subsurface current, and therefore should present as little surface as possible to the action of wind and waves, (2) it should be easily identified from a distance

Fig 346 shows a wooden rod float, 3 or 4 ft in length, and weighted at the bottom with sheet lead The double float of Fig. 347 consists of a surface float from which is suspended a perforated cylinder The float and cylinder are of sheet iron, and the chain or wire connecting them is made adjustable in length so that the direction and velocity of a current at various depths may be indicated. A pail is a useful substitute for the cylinder shown. It may be loaded with stones until the float is nearly submerged. Flags attached

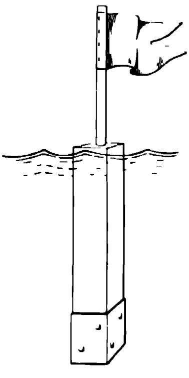


FIG 346
ROD FLOAT

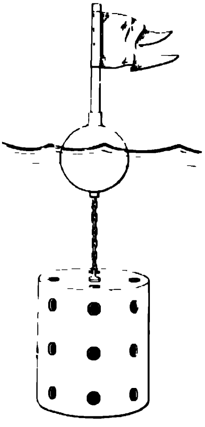


FIG 347.
DOUBLE FLOAT.

TIDAL CURRENT OBSERVATIONS AT

Date

Surveyor.

Recorder

Flood *Tide*

Float	When Immersed	Tide Gauge	Station	Angle	Station	Angle	Station	Time of Observation	Time between Observations	Distance Travelled	Speed of Current	Remarks
	a.m.							a m	h m	Feet	Knots	
No. 3 Red 6 ft long	8 45		B's Ch Stk	60° 56′	Lt ho	61° 47′	Cairn	8 45				Sea smooth
				Check	do	79° 52′	Farm ho					Gentle breeze S E
			do	23° 14′	do	73° 22′	Cairn	9 25	0 40	1,460	0 36	
			Cairn	75° 2′	Farm ho	36° 7′	Pole	10 42	1 17	3,400	0 44	
			Farm ho	47° 8′	Pole	34° 0′	Hut	11 45	1 3	2 430	0 38	
			do	45° 30′	do	37° 53′	do	p m 12 10	0 25	550	0 22	
				Check	Farm ho	83° 27′	do					
			do	42° 0′	Pole	36° 2′	do	12 29	0 19	240	0 13	
			do	39° 56′	do	32° 3′	do	1 0	0 31	550	0 17	
				Check	Cairn	82° 2′	Pole					Float withdrawn

Fig 348

to floats should not be larger than is strictly necessary : different colours should be used as a means of distinguishing the various floats of a series.

Surveying Course of Float.—The position of a float at various points of its course can be measured · (1) by two angles from a boat alongside the float ; (2) by an angle from the boat and one from the shore ; (3) by two angles from the shore The first method is the most satisfactory, and is that usually adopted The others are not recommended, as it is difficult to secure simultaneous observations unless the sea is very smooth

If the object of the survey is to ascertain the characteristics of the current at a particular point, successive floats are placed in the water on the upstream side of the point at intervals of about half an hour If the survey is to extend over a considerable area, the floats must be placed at different distances from the shore In either case, the boat or launch takes up a position alongside each in turn, and sextant angles to three shore objects are observed at each position, the time of each observation being noted When the floats are moving steadily, it is sufficient to observe their positions at about 1 hour intervals, but at the time of change of current they should be located every 15 minutes or so to define clearly the period of flood or ebb tide

Notekeeping.—The notes will include not only the results of the sextant observations, but also a record of the wind, weather, and tidal conditions during the survey, as these are necessary for the interpretation of the results Fig 348 shows a form of field book provided with columns for the entering up of the computed velocities. A separate page is reserved for each float being followed The appropriate tide gauge readings are subsequently entered from the gauge reader's notes.

Plotting and Reducing.—The sextant fixes are plotted mechanically or geometrically. From the scaled distances between the points so obtained and the watch times of the fixes, the speed of the current from point to point is computed, the results usually

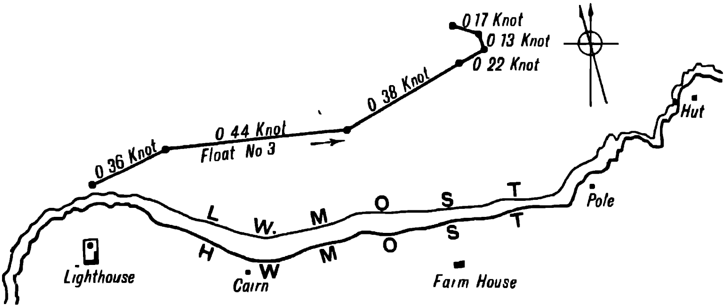


FIG 349.

being expressed in knots (of 6,080 ft per hour) The speeds are recorded in the current book, and are also written up in plan as shown in Fig 349 It is useful to have the tide curves for the period of the observations plotted for reference

STREAM MEASUREMENT

The measurement of stream discharge is an operation which falls to be performed by the engineer in connection with the design of water supply, irrigation, and power schemes For certain special purposes a single measurement may be all that is required, but, since stream flow fluctuates from day to day, costly mistakes may occur by basing designs on insufficient observations A complete investigation should extend over a considerable period, and should include measurements of the maximum and minimum flow and the duration of each at a time.

Methods.—The methods of measuring the discharge of a stream at a place may be classed as follows

(1) By measuring the area of cross section of the stream at the place and determining the mean velocity of flow

(2) By introducing a weir or dam across the stream and observing the head

(3) By chemical means

The results are most commonly expressed in cubic feet per second or gallons per minute or per day.

Gauges.—For the comparison of measurements made at different times, the result of an observation should be accompanied by a statement of the stage of the stream in terms of the elevation of its surface at the place This is obtained from the reading of a gauge erected at the discharge station and connected by careful levelling to a permanent bench mark

In its simplest form the gauge may consist of a stake firmly driven into the bed and carrying a nail of known level from which measurements are made to the water surface Otherwise, the gauge takes one of the forms described on page 426, but with the graduation sufficiently close to enable readings to be estimated to 0 01 ft if required The float gauge is particularly useful Oscillations of the water are most effectively damped out by admitting the water to the float chamber through a cock, and refined readings may then be made against a vernier fixed on top of the chamber The self-registering form is sometimes used to obtain a continuous record of stage. The most precise reading is possible by the use of the hook gauge (Fig 350), which is employed in refined

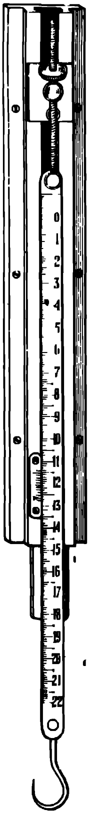


Fig. 350. Hook Gauge.

observations for the measurement of small heads over weirs The sharp-pointed hook is carried by a scale which can be moved relatively to a fixed index and vernier by means of a screw To take a reading, the hook is lowered into the water, and is then slowly raised by the screw until the point just touches the surface The reading can be estimated to the nearest 0 001 ft , if the water surface at the hook is thoroughly protected from wave action

AREA-VELOCITY METHOD OF DISCHARGE MEASUREMENT

To avoid abnormal results, the site at which the measurement is to be made —called the discharge station—must be selected so that the stream lines are as regular as possible The stream should be nearly straight for some distance on either side of the station, and the channel should be free from obstructions, and have its bed fairly uniform in shape and character

The cross section is measured by ordinary levelling or by sounding. Soundings are located by cross rope or by means of a steel tape stretched across the stream, and are taken at intervals of from 2 ft. upwards If the width is great, location by the methods used in offshore sounding (page 435) will be required It is advisable to check the results by making the measurements twice

The average velocity of the current is obtained

(1) By means of floats
(2) By current meter
(3) By calculation from the measured inclination of the surface

Distribution of Velocity.—The velocity of the stream lines varies throughout the cross section in a manner depending upon the shape of the section, the roughness of the bed, and the depth of water A typical distribution of velocity over the cross section is illustrated

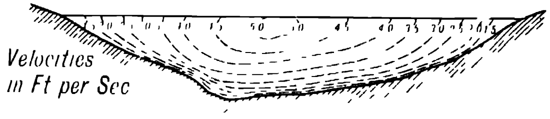


Fig 351

in Fig 351 by means of curves of equal velocity The minimum velocity occurs at the bed, due to the retardation produced by friction the maximum velocity occurs a little below the surface and away from the banks

Fig 352 shows the typical manner in which velocity is distributed along a vertical Observation of numerous streams shows that the curve approximates to a parabola the axis of which is horizontal and coincides with the stream line of maximum velocity. This line is usually situated between the surface and 0·3 of the depth at the vertical, and approaches the surface as the depth decreases.

In the case of moderately smooth channels, the mean velocity along the vertical has a value of from 0·7 to 0·95 of that of the surface, the coefficient increasing with increase of velocity and depth The ratio is less than 0·7 for channels having very rough beds or much obstructed by weeds The stream line having a velocity equal to the mean velocity in the vertical occurs at about 0·6 of the depth The mean velocity is also closely given by the mean of the velocities at 0 2 and 0·8 of the depth These figures are applicable to normal cases only The distribution of velocity is changed if the stream is covered with ice or if the water is being drawn through a sluice

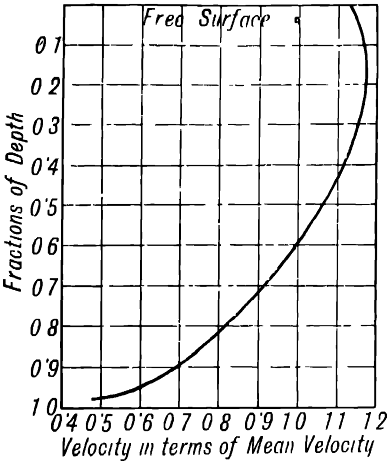


FIG 352

The distribution of surface velocity or of mean velocity in the verticals from one side of the stream to the other depends upon the shape of the cross section If the bed were very regular, the variation would approach that of a parabolic law with the maximum velocity in mid-stream Practically, the distribution of velocity across a stream cannot be satisfactorily predicted In order to measure the mean velocity throughout the cross section by floats or meter, it is therefore necessary to divide the area into a number of sections bounded by verticals and to measure the mean velocity in each

Floats.—Floats may be classified as (*a*) surface floats, (*b*) double or sub-surface floats, (*c*) rod floats.

Surface Floats —Surface floats are most commonly employed, but they should not be used in wind They are particularly suitable for rough determinations and for gauging streams in high flood, when the use of other methods is difficult. Where the course of the float will be easily seen, it is sufficient to use a flat piece of wood or a corked bottle weighted so that the surface exposed to the wind is not too great. In circumstances where such objects are difficult to observe, a larger sealed vessel carrying a flag should be used.

Double Floats —The form of double float of Fig 347, as employed in sea work, is suitable for use in deep rivers, but the chain should be replaced by a thin wire to present a minimum of area to the action of stream lines of different velocity from those at the depth of the submerged float Even so, when the sub-surface float is adjusted to the required depth, such as that of mean velocity, the indications are likely to be affected by the speed of the surface float and by the exposed area of the wire when the depth is great. Such floats are unsuitable for use on small streams.

Rod Floats.—These consist of wooden rods or hollow tubes weighted at the bottom so that they float vertically, with only a short length exposed above the surface of the water. The total length is such that the clearance between the lower end and the bed of the stream is as small as can be secured without danger of fouling the bottom. A number of rods of different lengths is required to suit the various depths across the stream. Rod floats are intended to indicate directly the mean velocity in the vertical, but are likely to give too large results because, on account of the necessary clearance at the bottom, they are not acted upon by the stream lines of least velocity. This error is greatest, and rod floats are unsuitable, when the bed is irregular, since the clearance is then excessive over part of the run Francis investigated the effect of clearance in the case of a rectangular channel, and deduced the formula,

$$v_m = v_r\left(1{\cdot}012 - {\cdot}116\sqrt{\frac{c}{d}}\right),$$

where v_m = the mean velocity in the vertical of the rod,
v_r = the velocity of the rod,
c = the clearance,
d = the depth of the stream

Rods are likely to give better results than other types of float. They are very satisfactory if the bed is smooth, as in artificial channels.

Measurement by Floats. Field Work.—To measure velocity by means of a float, observation is made of the time it takes to travel a measured distance down the stream. A base line is set out on one bank as nearly as possible parallel to the axis of the channel, and of length from 50 to 300 ft At each end a line is set out across the stream at right angles to the base, and is marked by ranging poles or, in narrow streams, by a rope. The cross section of the stream is measured at each of these ranges, and, if the base length exceeds 100 ft, intermediate cross sections are taken at equal intervals of 50 or 100 ft The area to be used in computing the discharge is the average area over the length of the run

To determine the mean velocity, a number of float observations is required at different points across the width of the stream. When the ends of the run are marked by ropes, distances from one bank are conveniently marked by means of equidistant tags affixed to the ropes When rod floats are used, it will be necessary to select lengths to suit the depth of water in which each run will be made. In the case of double floats, the wire must be adjusted so that the centre of the immersed float is situated at 0·6 of the depth. The float is placed in the water at some distance above the upper range, and, when it crosses that range, the time is taken, and note is made of the position of the float relatively to the tags The same observations are made when it reaches the lower range The time of the run is sometimes taken by stop watch The operations are repeated with the float at various positions from one bank to the other

If the stream is too wide to be spanned by a rope, the passage of the float across a range is timed by sighting along the marking poles For the greatest accuracy a theodolite is used The distance of the float from the bank as it crosses each range is determined by angular observation A theodolite or other angular instrument is used at the mid-point of the base line or at any other point whose position is obtained from the base line and from which good intersections are available on the ranges As the float approaches the upper range, the line of sight is kept upon it, and, when it crosses the range, an assistant stationed there makes a signal, at which the bearing of the float is read Both observers should take the watch time The assistant then proceeds to the lower end of the base, and the float is again located and timed at its passage across the second range Alternatively, a theodolite is set up at each end of the base The observer at the upper one signals the instant of the passage of the float across that range, and the other locates it as before At the lower range the duties of the observers are reversed In the case of very wide rivers, the location of floats is best performed by sextant observations from a boat as in tide work

Measurement by Floats. Calculation of Discharge.—The mean cross section throughout the run is first determined from the results of the section measurements at the upper and lower ranges and at the intermediate positions, if any Since the width of the stream should be nearly constant over the base length, the same number of observations usually defines the form of the bed at each cross section Each cross-sectional area is therefore divisible into the same number of trapezia, the parallel sides of which are the observed depths, and their width the constant distance between the soundings Triangles may take the place of trapezia at the sides, and the width of these may vary from one section to another The dimensions of the partial areas for an average cross section are readily obtained by averaging the dimensions of the corresponding areas of the individual sections

The next step is to obtain the mean velocity throughout each of those partial areas The velocity of a float is the distance between the ranges divided by the time of run, notwithstanding that the path of the float may not be parallel to the base line. In the case of a surface float, the mean velocity in the vertical throughout its path is obtained by multiplying the velocity of the float by a coefficient of from 0 7 to 0·95 according to the conditions (page 453), but surface floats should not be used unless the value of this factor can be carefully determined, preferably by means of a current meter.

The velocity thus derived represents the mean velocity in the average cross section along a vertical whose position in the width of the stream is the corresponding mean position of the float throughout its run This is obtained by averaging the observed positions of the float at which it passed the two ranges

On a base representing the width of the stream the average positions of the different runs are set off, and from them ordinates are erected to represent the mean velocities given by the floats The resulting curve exhibits the variation of mean velocity across the stream, and is utilised to interpolate the value of the mean velocity for each partial area These are represented by the ordinates corresponding to the positions of the centroids of the areas, which in the case of trapezia may usually be taken with sufficient accuracy at the middle of their width The product of each partial area by its mean velocity gives the discharge through each section, the sum of which is the discharge of the stream

The Current Meter.—As a means of measuring the velocity of flowing water, the current meter (Fig 353) proves more convenient

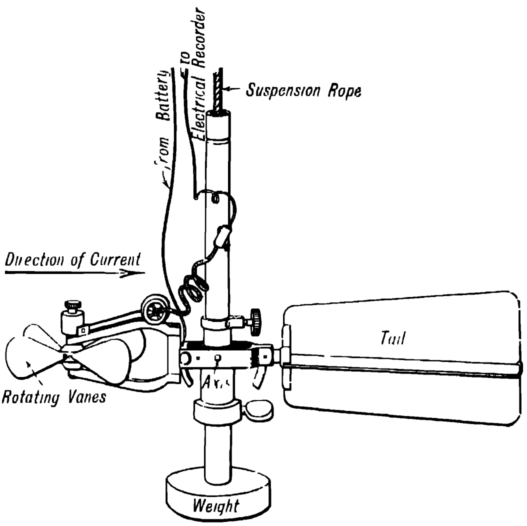


FIG 353 —CURRENT METER

than floats, and with careful usage gives better results Various forms differ considerably in detail, but the instrument consists essentially of a spindle mounted on a fork and carrying a wheel with cup-shaped or helical vanes, which is rotated by the action of the flowing water The spindle is vertical in some forms and horizontal in others, and is constructed to rotate with a minimum of friction The number of revolutions made by the wheel during the time the meter is in operation is recorded by gearing or other means. The instrument is suspended by a rope, wire, or jointed tube, and the weight of the mechanism is balanced by a two- or four-bladed tail, which keeps the instrument facing the current. A lead weight may be fixed at the bottom to assist in maintaining the meter in position. The weight is sometimes made torpedo shaped, and may be provided with a rear blade Provision is made for the attachment of additional weights when the instrument is used in swift currents

Several methods are employed for counting the revolutions of the wheel In a simple form of meter the number of revolutions is shown on a dial on the instrument itself When the meter is immersed at the required depth, the recording mechanism is thrown into gear by the operator pulling a cord. After a noted interval it is released by a similar pull, and the meter is then drawn to the surface and read. It is usually more convenient to be able to note the revolutions without having to pull up the meter at every observation, and this is effected in various ways In the acoustic type the gearing is so arranged that a tap is made at every fifth or tenth revolution, and the sound is communicated to the observer through a tube In the electrical type the revolutions are indicated by a sounder consisting either of a telephone receiver or a buzzer. For prolonged observations and high velocities electrical registration is most convenient, the revolutions being recorded upon a dial or dials above the surface.

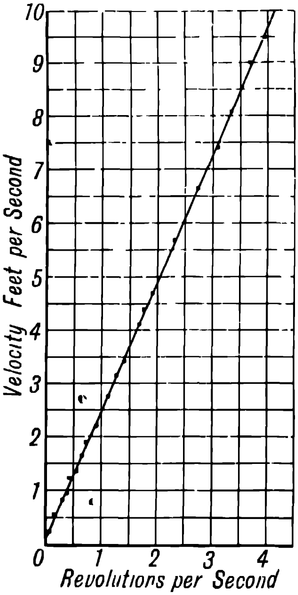


FIG. 354 —METER RATING CURVE.

Rating Current Meters.—Since the results given by meter represent the number of revolutions in a given time, it is necessary to know the relationship between the velocity of the current and the number of revolutions per second made by the meter. This ratio is obtained by rating the meter.

Rating is performed by running the meter at a uniform speed through still water over a measured distance and noting the number of revolutions and the time taken. Runs are made at speeds varying from the least which will cause the wheel to rotate to the greatest likely to be encountered in gauging. At specially equipped laboratories or rating stations the meter is suspended from a trolley, which is run at a sensibly uniform speed. In ordinary practice, rating is usually performed by hanging the meter from the prow of a dinghy, which is towed repeatedly along the course at different speeds

On plotting the mean velocities of the runs against the revolutions per second, a curve similar to that of Fig 354 is obtained. For all meters the curve is practically a straight line, and is taken as such. The slope depends upon the type of meter The ordinate corresponding to zero revolutions represents the velocity of current required to overcome initial friction and start the wheel From the rating curve the velocity corresponding to any observed number of revolutions per second can readily be obtained

In place of using the curve directly in the reduction of observations, it is more convenient to prepare a rating table giving velocities corresponding to revolutions per second This is compiled from the equation,

$$V = c_1 R + c_2,$$

where V = the velocity of the current in feet per sec ,
R = the number of revolutions per sec ,
c_1, c_2 = constants, the former representing the slope of the curve, and the latter the ordinate for zero revolutions.

The constants are obtained from the curve, or their most probable values may be computed directly from the rating observations.

Use of Current Meter.—In measuring discharge from velocities taken by current meter, one cross section only is required It is divided into partial areas as before, and the meter is used at different points across the width to give the mean velocity over each area. The velocity determinations are made either by using the meter in the verticals of the centroids of the partial areas or by taking observations at any points across the width, fixing their positions by angles or on a cross rope, and interpolating the values for the partial areas

If the water is sufficiently shallow, observations can be secured by wading, the meter being fixed to a graduated rod If there is a bridge conveniently near, and situated on a straight reach of the stream suitable for a discharge station, the meter should be suspended from successive points along it Otherwise, the observations are made from a boat, or, if repeated measurements are required, a cableway may be thrown across the stream, and observations taken from a suspended car or platform. When a boat is used, the meter is attached to a rod or tube which is held vertically and clear of the prow When the depth is considerable, the meter is suspended by a wire or rope, preferably from an outrigger provided with a pulley. The boat is headed upstream, and attention must be paid to keeping in the range at each observation. Unless the width is too great, the boat is brought into position and held against the current by ropes from each bank.

In taking an observation with the meter, the sounding is first taken, and the meter is then lowered to the required depth If the meter is one in which the recording mechanism is thrown into and out of gear by a cord, the interval between the two pulls is measured by means of an ordinary or stop watch and entered against the revolutions shown by the dial With acoustic or electric meters, the count may be taken over two periods of one or two minutes each

The object of the meter observations is to ascertain the mean velocity in the verticals at various distances across the width of the stream Different systems of measurement are available, according as one or several observations are made in each vertical or the integration method is adopted

Single Observations —Single observations are taken either at the surface or at the depth of mean velocity Surface measurements are not usually made in normal cases, but the method is useful in swift currents and in times of flood, when it is difficult to maintain the meter in a desired position much below the surface In taking the observation, the meter is held six inches or more below the surface in order that it may be wholly submerged and below the wave line The mean velocity in the vertical is deduced by multiplying the observed result by a coefficient having an average value of 0·9

Mean velocity is obtained directly by immersing the meter to a depth of 0·6 of the depth of the vertical This method is commonly used in ordinary circumstances, and, although the ratio, 0·6, is only an average value, the accuracy of the results is sufficient for most practical purposes

Multiple Observations —The best results are obtained by making measurements at a sufficient number of points in the vertical to enable the velocity curve to be plotted similarly to Fig 352 The area contained between the complete curve and the depth axis divided by the total depth equals the mean velocity This method is too laborious for ordinary measurements, but is that required in the evaluation of coefficients for the reduction of single point observations

Good results are obtained by two observations in each vertical These should be made at 0·2 and 0·8 of the depth of the vertical, the mean of the results giving a close approximation to the mean velocity

Integration Method.—In this method the meter is moved slowly and at a uniform rate along the vertical from the surface and back, the number of revolutions and time being observed The meter is thus exposed to all the velocities in the vertical, and these are integrated and averaged mechanically in the result Certain types of meter are not quite suited to the method since the vertical motion, unless very slow, causes the wheel to revolve. The correction to be applied to the indications of such meters may, however, be ascertained by moving the meter vertically at the same uniform speed through still water.

The integration method is sometimes extended to give the mean velocity over an entire cross section The meter is moved up and down at an angle of about 45° to the horizontal from one bank to the other and back (Fig 355) This system is suitable for the rapid gauging of streams of moderate depth

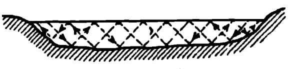


Fig 355

Measurement by Current Meter. Calculation of Discharge.—Except when the method of Fig 355 is employed, the total discharge is obtained as the sum of the discharges through the partial areas into which the soundings divide the cross section The mean velocities over those areas, if not directly observed, are derived by interpolation in the same manner as described for float measurements (page 456) In the case where the integration method is applied to give the mean velocity for the entire cross section, the discharge is simply the product of the observed velocity by the cross-sectional area

Thrupp's Ripple Method.—If a high degree of accuracy is not required in discharge measurements, rapid determinations may be made from surface velocities observed by the method described by Mr E C Thrupp* It is based upon the circumstance that, if a small obstruction is placed in the surface of a stream, ripples are formed if the velocity exceeds about 9 in per sec, and, as the velocity increases, the angle between the diverging lines of ripples becomes more acute To afford a simple means of measuring the rate of divergence, Mr Thrupp used two 3-in wire nails—about $\frac{1}{8}$-in in diameter—at a fixed distance d apart (Fig 356), and found that the velocity could be derived from the distance l from the base line so formed to the point of intersection of the last ripples He obtained for the velocity in ft. per sec.

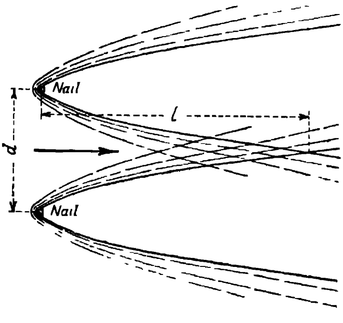


Fig 356

$$V = 0{\cdot}40 + 0{\cdot}206\,l, \text{ for } d = 6 \text{ in},$$
$$V = 0{\cdot}40 + 0{\cdot}28\,l, \quad \text{for } d = 4 \text{ in},$$

where l is measured in inches.

The method would appear to be quite as accurate as that of surface floats and much more convenient.

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* *Min. Proc. Inst. C.E*, Vol. CLXVII, p. 217.

Velocity by Formula.—In this method of estimating discharge—sometimes known as the slope method—measurement is made of the average cross-sectional area of the stream on a straight reach as well as of the longitudinal inclination of the water surface. To obtain the mean velocity of the stream, the formula in general use is that deduced by Chezy, *viz* ,

$$V = c\sqrt{ri},$$

where $V =$ the velocity in ft per sec ,
$r =$ the hydraulic mean depth,
$i =$ the inclination of the water surface,
$c =$ a variable coefficient

The hydraulic mean depth, or hydraulic radius, is the cross-sectional area of the stream divided by the wetted perimeter or length of bed under water The inclination is the ratio of the fall in a measured distance to that distance The value of the coefficient c depends principally upon the roughness of the bed, but also upon the inclination and hydraulic mean depth In practice, the value of c is commonly derived from tables or diagrams based upon the formula of Kutter and Ganguillet, *viz* ,

$$c = \frac{41{\cdot}65 + \dfrac{1{\cdot}811}{n} + \dfrac{0\ 00281}{i}}{1 + \left(41{\cdot}65 + \dfrac{0\ 00281}{i}\right)\dfrac{n}{\sqrt{r}}},$$

in which the coefficient n depends upon the character of the bed. The value of n varies from 0 020 for irrigation canals with well-trimmed beds in perfect condition to over 0·035 for canals in very bad order with much weed and stones For rivers very uniform in alignment, slope, and cross section, and with a smooth sandy or gravel bed, n is taken as 0·025 As the irregularities of the bed, etc. increase, n increases to 0·035, and may reach 0 050 if there is an excessive amount of weed or in the case of torrents bringing down much detritus.

In estimating discharge by formula, a straight reach of river should be selected with as nearly uniform a cross section as possible The fall and the distance between the points at which it is measured should be sufficiently great that the inclination can be determined without serious error. Cross sections are taken at intervals along this distance, and an average section is deduced. The slope of the water surface is measured by simultaneous readings of gauges placed one at each end of the reach and similarly situated with respect to the current. Gauge readings are taken to 0·01 ft., and the zeros of the gauges are connected by careful levelling

The results obtained by the slope method are inferior in precision to those in which the velocity is actually observed, principally on account of uncertainty in assigning a suitable coefficient in the formula and in measuring the slope. The method, however, is useful in making an isolated rough estimate of flood discharge from the flood marks left on the banks.

WEIR METHOD OF DISCHARGE MEASUREMENT

A weir is an artificial barrier built across a stream, which flows over it in a cascade of definite form, from the dimensions of which the discharge is computed. The weir method is specially applicable to the gauging of small streams when accurate results are required. The cost of construction usually prohibits its use on large streams, but in such cases existing dams are sometimes utilised in a similar manner

Varieties of Weirs.—The various forms of weirs differ in certain particulars affecting the rate of discharge over them.

Crest—Weirs constructed specially for the measurement of stream flow are of the sharp-crested type. In these, the crest, or edge over which the flow takes place, is virtually a line In the case of dams or broad-crested weirs, the water passes over a surface, and a different condition of flow obtains.

Shape of Opening—The opening or notch through which the water flows may be of rectangular, triangular, trapezoidal, or stepped form (Fig. 357) The rectangular weir is that most commonly used in stream gauging, the length of the notch being at least three times the head of water over it Other forms provide for an increase in the breadth of the flow as the discharge increases The triangular or V-shaped notch has the merit of giving a constant shape to the issuing stream at all heads It is well adapted for the accurate measurement of small discharges, but is not so suitable as the rectangular form for use in very shallow streams The trapezoidal form with side slopes of 1 in 4 was proposed by Cippoletti with a view to balancing by the increase of breadth upwards the loss due to increased contraction with increased head. The stepped form is designed to yield good measurements in dry weather and in flood The centre notch is sufficiently small that the minimum flow can be measured with greater accuracy than is possible with a very small head over a long crest.

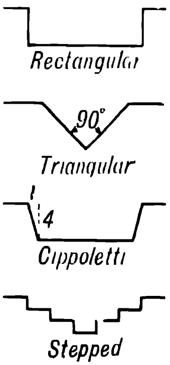

FIG. 357. FORMS OF WEIRS

End Contractions.—The form of the stream issuing from a rectangular notch depends greatly upon the position and size of the opening relatively to the cross section of the channel on the upstream side. In Fig. 358, which represents the most common arrangement, the breadth of opening or length of crest is less than that of the stream. The stream lines at the sides are sharply curved, with the result that the stream is contracted in width just after passing over the notch Such a weir is said to have end contractions When, as in Fig 359, the breadth of the weir is the same as that of the approaching stream, the weir is without end contractions Intermediate conditions arise when insufficient room between the sides of the notch and those of the stream prevent the proper development of the curved stream lines, and the end contractions are then incomplete, a condition to be avoided in gauging

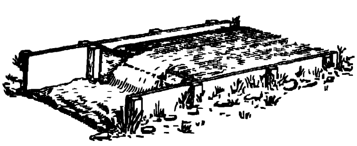

FIG 358
WEIR WITH END CONTRACTIONS

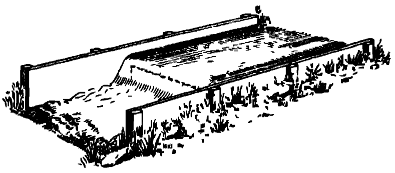

FIG 359
WEIR WITHOUT END CONTRACTIONS

Fig 360 illustrates diagrammatically the normal form of the issuing stream or nappe in longitudinal section The stream lines near the crest are curved upwards causing a vertical contraction, which is completely developed when the distance d is greater than $2H$ When the height of the crest is so small relatively to the head of water over it that the tail water is higher than the crest, the weir is known as a drowned or submerged weir

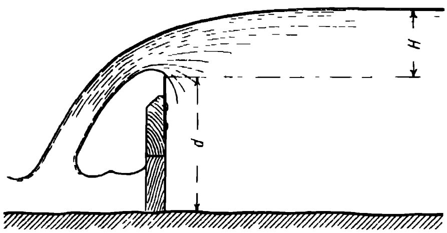

FIG 360

Construction of Weirs.—The site for a weir should be selected so that on the upstream side the channel is straight and free from great irregularities in order that the water may approach the weir with as uniform velocity as possible The weir will be constructed of stout uprights and planking or tongued and grooved boarding, and the wall must be vertical and normal to the flow Sheeting may have to be used to prevent leakage below or round the sides The timbers forming the crest and sides of the notch are chamfered on the downstream face, the edge not exceeding $\frac{1}{8}$ in in breadth Otherwise, an $\frac{1}{8}$ in thick metal strip is fixed, as in Fig 360 The crest, in other than V notches, should be set accurately horizontal If the weir is one with end contractions, these should be complete, so that the formulæ based on that condition may be strictly applicable. This is ensured by making the distance between the sides of the upstream channel and the sides of the opening not less than twice the head The use of the standard formulæ also assumes that the nappe is free from contact with the weir below the crest, to secure which there must be free admission of air under the nappe.

Measurement of Head.—The dimensions of the weir being known, it is only necessary to measure the head of water over it to enable the rate of discharge to be computed By the head is meant the difference of level, *H* (Fig 360), between the crest, or the bottom of the V in the triangular form, and the surface of the sensibly still water several feet above the weir and beyond the influence of the depression of surface caused by it This upstream distance should always exceed 3*H*

The measurement must be made with great care, more especially when the head is small. It is generally made by means of a stake driven into the bed at the still water and with the top at the level of the crest The head is then measured with a thin-edged scale or steel rule The accuracy of the observation is improved if the head is measured not in the flowing stream but in a small gauge pit in communication with the stream The connecting pipe, of 1 or 2 in diameter, must have its end at the stream placed at right angles to the direction of flow and at an adequate distance from the weir For the best results the hook gauge is used, the instrument being accurately set so that at zero reading the hook is at the same level as the crest Various forms of float gauge are also used, and recording mechanism may be added so that the varying level of the float can be plotted automatically.

Discharge Formulæ.—There are several formulæ available for computing the rate of discharge over weirs, and the reader is referred to text-books on Hydraulics for a discussion of the subject. The simplest and most commonly used formulæ are those of Francis

Rectangular Weirs —For a sharp-crested rectangular weir without end contractions, and neglecting the effect of the velocity of the approaching water, Francis obtained

$$Q - 3{\cdot}33\,LH^{3/2},$$

where Q = the discharge in cubic feet per sec
L = the length of the crest in feet,
H = the observed head in feet.

The effect of the velocity of approach of the stream is equivalent to an increase of head. This velocity, v, need not be measured directly, but is obtained approximately by dividing the approximate discharge obtained as above by the cross-sectional area of the approach channel where the head is observed. The velocity head, $h = \frac{v^2}{2g}$, was allowed for by Francis by putting

$$Q = 3{\cdot}33L[(H+h)^{3/2} - h^{3/2}].$$

The effect of velocity of approach is more commonly dealt with by increasing the observed head by ah, where a is an experimental coefficient having a value between 1·2 and 1·68, and which may be taken as 1·5, so that

$$Q = 3{\cdot}33L(H+1{\cdot}5h)^{3/2}, \text{ approximately.}$$

From the new value of Q a closer value for v can be obtained, and a second approximation may be made for Q, but this is seldom necessary since the velocity effect is small in most cases.

For a weir with end contractions, experiment shows that each complete end contraction shortens the effective length of the weir by about 0·1 H, so that for two end contractions,

$$Q = 3{\cdot}33(L-0{\cdot}2H)H^{3/2},$$

or, allowing for velocity of approach,

$$Q = 3{\cdot}33(L-0{\cdot}2H_1)H_1{}^{3/2},$$

where $H_1 = [(H+h)^{3/2}-h^{3/2}]^{2/3}$ or $= H+1{\cdot}5h$, according to the method of treating velocity head The velocity correction is usually insignificant in the case of weirs with end contractions, since the approach channel is of much greater sectional area than the nappe

Triangular Weirs.—In the case of a sharp-edged notch, with apex angle θ, the discharge formula has the form,

$$Q = c \tan \tfrac{1}{2}\theta\ H^p,$$

the velocity head being negligibly small The values of p given by different investigators range from 2·47 to 2·5, and c varies from 2·48 to 2·56 For ordinary measurements it is sufficient to adopt

$$Q = 2{\cdot}5 \tan \tfrac{1}{2}\theta\ H^{2.5},$$

which, in the common case of a right-angled notch, gives the easily remembered expression,

$$Q = 2{\cdot}5\,H^{2.5}.$$

Correction for velocity of approach is seldom necessary, but can be made as before.

Trapezoidal Weirs.—The discharge from a trapezoidal weir is the discharge over a rectangular weir of the same length of crest, and having end contractions, plus that over the triangular notch which would be formed by the sloping sides. If, therefore, $\frac{1}{2}\theta$ is the inclination of the sides to the vertical,

$$Q = 3{\cdot}33(L-0{\cdot}2H)H^{3/2}+2{\cdot}5\tan\tfrac{1}{2}\theta\ H^{3/2}.$$

By selecting $\frac{1}{2}\theta$ to make $2{\cdot}5\tan\frac{1}{2}\theta = {\cdot}666$, we have

$$Q = 3{\cdot}33\,LH^{3/2}.$$

Cippoletti suggested $\tan\frac{1}{2}\theta = \frac{1}{4}$, and adopted

$$Q = 3{\cdot}367\,LH^{3/2}.$$

Dams.—When an existing dam or broad-crested weir is to be utilised for the measurement of stream flow, the effective length and levels of the crest are measured. The dimensions of the cross section must be ascertained since the coefficient to be used in computing the discharge is derived from published results of experiments on dams of similar form. As with sharp-crested weirs, the head is measured to the surface of the still water behind the dam If, at the time of gauging, part of the flow is discharged through or round the dam, the quantity so discharged must be separately measured

The accuracy of the results is much inferior to that attained with sharp-crested weirs It depends greatly upon the selection of a suitable value for the discharge coefficient and also upon the condition of the dam, particularly as regards uniformity of crest and apron

CHEMICAL METHOD OF DISCHARGE MEASUREMENT

This method consists in introducing into the stream, at a uniform and accurately known rate, a fairly concentrated solution of some chemical, and comparing the analyses of the water before and after its introduction The percentage of the chemical found in the water (or the increased percentage, if the chemical is already present in the untreated water) bears the same ratio to its percentage in the solution as the volume of solution introduced per second bears to the discharge per second of the stream

Or, let P = percentage of chemical in the solution used,
p = percentage of chemical in the water samples,
Q = flow of stream in cub ft. per sec ,
q – flow of solution in cub ft per sec.

Every second there is added $\frac{Pq}{100}$ cub ft of chemical.

$$\text{Its dilution in the water samples} = \frac{Pq}{100Q} = \frac{p}{100},$$

$$\text{or } Q = \frac{Pq}{p}$$

The success of the method is dependent upon several factors The chemical should be one for which a very sensitive reagent is available, and a sufficient quantity must be used that the dilution may not be greater than will permit of the analyses of the water samples being made with the required accuracy. Thorough mixing of the solution with the stream is essential On other than very narrow streams it should be introduced at several points across the width, the flow of solution being continuous during the test. Samples of the untreated water are taken at the station where the solution is introduced, and, at some distance downstream, samples of the much diluted solution are collected from all parts of the cross section after a sufficient interval has elapsed for the flow to arrive there. The distance between the stations must be great enough to ensure complete mixing. Conditions favourable to mixing are irregularity of alignment and bed, and the distance between the stations may then be shorter than in the case of a stream with very uniform flow. The method is particularly suitable for turbulent streams with rocky beds.

CONTINUOUS MEASUREMENTS OF STREAM DISCHARGE

Investigations of stream discharge made in connection with the design of water schemes should be continued over several years to ensure a reasonably accurate estimate of the extremes of flow. The necessary observations are made either by means of weirs or by the area-velocity method

By Weir.—Continuous records are most easily and accurately secured by the weir method, and simply require the employment of an intelligent and reliable person to read the head daily A single daily observation is sufficient during periods when the head is changing slowly, but, when it is subject to rapid fluctuations, several readings should be made at intervals throughout the day * Continuity in the record of head is secured by the use of a self-registering gauge

A continuous measurement of discharge may be obtained mechanically by means of a weir recorder. In this instrument a float rotates a cylindrical drum on which is cut a spiral groove, the curve of which is deduced from the weir formula. A lever engages with the groove and actuates a pen, which traces out a continuous graph of rate of discharge The apparatus may also be fitted with an integrating mechanism whereby the total flow up to any time may be exhibited

By Area-Velocity Method.—In the application of this method, the discharge is measured on several occasions by current meter or floats at times of low, average, and flood stages With gauge heights as ordinates, and discharges as abscissæ, the observed discharges are plotted, and through the points so obtained a curve, known as the discharge or station rating curve, is drawn This curve, or a table prepared from it, then serves to give the discharges corresponding to gauge readings taken daily by an observer or recorded by an automatic gauge The method is dependent upon permanence of the stream bed and stability of the gauge. In streams with shifting beds the discharge curve may not remain applicable throughout a long investigation, but may require modification from time to time in the light of additional gaugings.

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* For an example of the comparative results given by one and four daily readings see "Investigations Relating to the Yield of a Catchment Area in Cape Colony," by E. C. Bartlett, M.Inst C.E., *Min. Proc. Inst. C.E.*, Vol. CLXXXVIII.

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ANSWERS TO EXAMPLES

CHAPTER III (page 199)

1 BC, 16° 53′, CD, 339° 10′
2 1,632 ft, 244° 1′
3 2 6 ft
4 Total co-ordinates *B*, + 706 6, + 212 9, *C*, + 1,206 4, – 39 7, *D*, + 2,099 1, – 61 8 ft
5 858 9 ft S, 177 0 ft W.
6 1 7 ft
7 57° 17′, 95° 30′, 673 7 ft
8 3,756 9 ft
9 636 4 ft
10 1,204 ft, 359 10′
11 1,662 1 ft
12 1° 59′
13 86° 41′ 18″
14 927 4 ft
15 (*a*) 1,507, 128, (*b*) 1,295 7, 258° 33′ 36″, (*c*) 40° 9′ 36″
16 254 8 ft, 249° 12′
17 538 5 ft, 44° 26′
18 374 1 ft, 180°
19 13° 34′ 40″
20 Latitude of EF should be 383 8, 1 5 ft
21 Line DE should be 100 ft longer
22 101° 44′
23 79 3 ft towards G, 1,070 7 ft
24 279° 57′ 49″, 1,521 5 ft.
25 223 ft. and 216 ft from *A* and *J* respectively.
26. 347 9 ft
27 1,565 ft, 150° 23′.
28 8 5 ft
29 B, 23,786 ft N, 39,845 ft W, C, 22,756 ft. N, 40,862 ft. W.
30 3 4 ft.
31 407 81 chn, 2 316 chn
32 4 7 ft.
33. 841 ft. N, 13,911 ft. E
34 529 4 ft
35. 6,957 ft.
36 114° 16′.
37 2,907 ft
38 *AB*, 85° 24′, *BC*, 161° 37′, *CD*, 262° 44′; *DE*, 348° 28′; EA, 6° 29′.
39. Corrected min and sec of lines in given order: 22′ 40″, 36′ 00″, 14′ 10″, 49′ 40″, 43′ 00″, 51′ 00″, 10′ 00″, 6′ 30″, 56′ 20″, 35′ 30″, 17′ 00″, 6′ 30″, 47′ 40″, 26′ 10″, 5′ 40″, 57′ 00″, 43′ 00″, 36′ 00″, 12′ 20″, 22′ 40″.
40 Corrected latitudes; 620·7, 512 2, 1,028 4, 633·4, 821·9 corrected departures, 183 4, 925 1, 729 5, 1,568 3, 118 9
41 Corrected latitudes; 1,425 9, 503 0, 617 5, 921·6, 589 7, 199 9 corrected departures, 0, 712·1, 638·1, 219·1, 427 4, 703·7.
42 Corrected latitudes; 341·3, 41 3, 235 4, 268 5, 244·9, 338·1, 77 5, 382 0, 520·2: corrected departures; 42 7, 114 8, 338 1, 97 6, 290·7, 187 3, 361 5, 220 6, 220 9

CHAPTER IV (page 234)

1 Reduced levels 75·40, 68 75, 71 41, 76·54, 75 79, 80 01, 84 50, 81 94.
2 06 ft
3 05 ft.
4 A, 116 75; B, 115 77.
5 05 ft.
6 13 82 ft
7 01 ft, 03 ft.
8 6′ 2½″, 5′ 1⅜″, 5′ 0⅛″, 6′ 7″, 5′ 11½″, 7′ 5½″, 5′ 8⅝″.
9 Pegs too low by 06, 02, ·08, 03, 01, 0, ·04, 0, 0, ·02 ft.
10 7 23 ft, 2,115·08 chn.
11. 6 16 ft. at 500 ft. in excavation; 5 87 ft. at 1000 ft in embankment.
12. Instrument is not in adjustment; 100·00.

CHAPTER VII (page 312)

1 7 ac 1 rd 6 pl
2 175 cub ft per sec
3 17 7, 9964
4. 9 ac. 2 rd 2½ pl
5. 488 sq ft.
6 38 8 ft . 28 7 ft , 593 sq ft
7. 2 ac 2 rd 5 pl
8 12 ac 3 rd 38 pl
9 BX = 195 3 ft , CY = 166 4 ft
10 DG = 113 5 ft
11 2,294 cub yd
12 2,281 cub yd.
13 313 sq ft , 1,170 cub yd
14 12,701 cub yd
15 141 cub yd
16 1 in 4 47 , 70 4 cub yd
17 330 cub yd , 267 cub yd
18 − 31 6 cub yd
19. 4,069 cub yd
20. (*a*) 784 million gal , (*b*) 787 million gal , (*c*) 486 9

CHAPTER VIII (page 364)

1 38° 49′.
2 318 + 91 2, 340 + 82 2
3 1,053 ft
5 (*a*) 6 85, (*b*) 1 14, (*c*) 13 21, (*d*) 313 + 45, 326 + 66, 320 + 05
6 Total length 15,012 2 ft , chainages, 4,696 3, 5,046 1, 11,008 5, 11,368 5
7 133° 6′ 30″, 132° 9′ 0″, 131° 12′ 0″, 130° 14′ 30″, 130° 0′ 0″
8 11 59 chn , 32′ 40″, 1° 15′ 40″, 1° 58′ 40″
9 382 14 chn , 394 26 chn. ; 13 7 links
10 680 47, 691 08, 710 56 chn
11 36′ 10″, 1° 12′ 10″, 1° 48′ 20″, 2° 24′ 20″
12 2½°, 490 4 ft , 969 ft
13 4,082 ft
14. 66 2 ft , 120 7 ft , 1,863 ft
15 3·47 chn , 5 18 chn.
16 21 33 chn., 252 + 22
17 19 64 chn , 11 34 chn , 159 43 chn
18. 48 2 chn
19. (*a*) 8·16 chn , (*b*) 529 45 chn , (*c*) 12·53 chn
20 380 4 ft , 219 6 and 380 4 ft from B and C respectively.
21 40 82 chn , 23 6 links
22 891 7 ft., 15 1 ft from *A* away from *C*, 27 1 ft from *B* towards *C*
23 145 6 ft towards A
24 13,340 5 ft , 126 5 ft
25 2,000 ft , 110 1 ft towards C
26 2 links towards C, 1 26 chn. away from E
27 0° 16′, 1 97 chn
28 9 02 chn
29 (*a*) 5° 48′, (*b*) 352 34, (*c*) 3 48 chn
30 19 1 ft
31 2° 7′ 40″, 6 99 chn
32 16 3 links towards centres
33 57 26 chn , 100 11 chn
34 11 51 chn , 16 85 chn
35 17 63 chn , 14 57 chn
36 480 ft
37 6 88 chn , 2 70 chn , 2 32 chn
39 82 of theoretical cant
40 4 in , 145 ft , 5 3 in
41 207 ft for radial acceleration of 1 ft per sec ^{3}, 6½ in
42 236 42, 235 78, 235 46, 235 45, 235 77, 236 40, 237 35

CHAPTER IX (page 407)

1 99 5 and 1 5, 484 ft
2 454 sq ft
3 531 ft , level of axis 169 3 or 1 0 ft
4 3° 18′
5 104 3
6 260 3, 258 9, 279 6 ft
7 1,407 ft , 223 3
8 171 7, 643 4 ft
9 521 6
10 173 1
11 1 in 39 3.
12 1, 299 7 ft , 606 8 , 2, 384 0 ft , 635 2 . 3, 484 6 ft , 729 1.
13 286·1.
14 1,022 ft
15 282° 9′. 732 6 ft
16 99·1
17 13 421
18 303 ft
19. 202 8, 593 ft
20 245 0.
21 474 ft , 233 3
22 1,910 ft., 0 3 ft

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